SOLUTIONS OF EXERCISES

IN

MESSRS. HALL & STEVENS' SCHOOL GEOMETRY.

PARTS I. & II.

BY

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Definition of signs occurred in the exercises

1. **The sign** $+$, which is read "plus," signifies that the quantity which comes next after it, is to be *added* to that which goes before.

2 **The sign** $-$, which is read "minus," signifies that the quantity which comes next after it, is *to be subtracted* from that which goes before

3 **The sign** $\times$, which is read "into," is placed between two quantities to denote that the first quantity is to be multiplied by the second number.

4 **The sign** $\triangle$ is read "triangle"

5. **The sign** $\angle$ is read "angle"

6 **Surd** —A root which cannot be obtained exactly is called a surd. The symbol $\sqrt{}$ is the corruption of the letter *r* the first letter of the word *root* or *radix*.

7. **The sign** $=$, which is read "is equal to," is placed between 2 expressions to denote that they are equal to one another.

8 **The signs** $>$ and $<$ are used to denote respectively *greater than* and *less than*.

9 **The sign** $\therefore$ denotes *therefore*. The sign $\because$ denotes *because* or *hence*.

10. **The sign** $\parallel$ denotes "parallel to."

PREFACE.

According to the new and revised University rules, students preparing for the Matriculation Examination are required to take up Hall and Steven's School Geometry as a portion of their Mathematical Course, which covers not only Theorical solutions of all the Propositions and exercises, but at the same time treats of and requires practice in the Practical and Graphical methods of solutions also Consequently students feel great inconvenience in preparing their daily lessons in Geometry, for they have not been so long accustomed to do such work

The students of the several classes repeatedly asked the author to prepare more elaborate and suggestive solutions with figures to help them in their daily work, for solutions printed up to date by different persons are only hints for teachers and without any figure hence, these solutions together with figures are prepared for the students in order to explain them the method of drawing figures and solving exercises at home.

The answers to the Practical exercises are derived from actual measurement and calculation and are therefore nearly correct As it is impossible either to measure or to draw a figure accurately with the help of an ordinary set of instruments, the results and answers obtained are therefore approximate

As the proof sheets were not sent by the press while the book was printing for corrections, great many mistakes and omissions have crept in, but to remedy this defect a seperate list of errata is annexed herewith In the second edition all attempts will be made to remove all these defects and to add more convincing and clear proof for some of the exercises not treated more elaborately in the present one.

The author's thanks are due to Pandit Shyamlal and Sons of Agra who undertook the duty of publishing these solutions merely for the help of the students preparing for the several public Examinations who are unable to buy most costly publications

THE AUTHOR.

ERRATA.

Page	Line.	For.	Read
3	14	$=145$	$=145^\circ$
11	13	ABC ACB and	ABC and ACB
11	19	CA and AC	CA and AO
12	22	$AC = B = 7$ cm.	$AC = b = 6$ cm
13	24	AD	CD
14	14	draw	drawn
18	3	angle AB	less than AB
21	33	or supplimentary	or supplimentary to the $\angle$ DEF.
22	16	$ACO = DBO$	$ACO = \angle BDO$.
22	23	angle ACB	angle ACB. $\therefore$ the $\angle$ ADE $=$ the $\angle$ AED
23	5	and A meets them the alternate $\angle$s	and AC meets them then the alternate $\angle$s.
23	17	Now from XY	Now join XY,
23	18	XO and BY	XB and BY
25	5	The angle ACD	The angle ACB.
25	18	DE	FE
25	19	DE	FE
25	19	DY	FY
25	30	of sides $D=$	of sides, and $D=$
25	34	$80+360=8\times180D=$	$8D+360^\circ=8\times180^\circ$, $\therefore D=$
26	2	$=10 \times 180\ D=$	$=10\times180^\circ\ \therefore\ D=$
26	8	$\angle\ C=3\ C$	$\angle\ C=3x$
26	9	$.\ 6x=180^\circ$	$\therefore 6x=180^\circ$.
26	9	$\therefore x=30^\circ$	$\therefore x=30^\circ$.
26	9	$\therefore$ angle $A=30^\circ$ angle B	$\therefore$ angle $A=30^\circ$, and angle B
26	12	$180^\circ \therefore x=36^\circ \therefore \angle A=36^\circ$	$180^\circ \therefore x=36^\circ. \therefore \angle A=36^\circ$,
26	13	$\therefore x=20\ \angle\ A=20^\circ$,	$\therefore x=20.\ \angle\ A=20^\circ$,
26	14	each of the	and each of the
26	22	$x-y=60^\circ$	$x-y=60^\circ$.
26	22	add $\dfrac{x-y=60}{2x=222}$	add $x\times y=162^\circ$ $\dfrac{x-y=\ 60^\circ}{2x=222^\circ}$
27	18	$\therefore n\ \angle s+4$ rt $\angle s=2n$ rt $\angle s$	$\therefore n\ \angle s+4$ rt $\angle s=2n$ rt. $\angle s$
28	6	As all the angles	12 As all the angles

Page	Line	For	Read
			(Here Exer 4 begins)
29	11	AB is ∥ to CD &c	4 AB is ∥ to CD &c
29	16	4 The int ∟s	The int ∟s
29	29	DB and DC	OB and OC
29	33	DBC, DCB and BDC	OBC, OCB and BOC
29	35	BDC	BOC
31	17	BC opposite to the = sides	BC opposite to the = angles
31	24	BPO = BQO,	BPO = ∟BQO,
		PBO = QBO,	PBO = ∟QBO,
32	3	AB = AC -	AB = AC
32	34	FOP	FOQ
32	36	FOP	FOQ
33	6	DEP	OEP
33	23	12 There are	11. There are
34	10	AB and AC or A and C	AB and AC or ∟ *a* and *c*
34	11	AC or B	AC or *b*
34	15	the A and C	the ∟s A and C
35	9	to the equal angles	to the equal angles are equal
36	4	∟ CFD	∟ CFP
36	5	∟ CFD	∟ CFP
36	7	28	50
36	31	The distance	14 The distance
		Light-house ∟	Light-house L
37	6	two ∟s ABC	two △s ABC
37	15	BAD = ∟ CDA	BAD = ∟BCD
38	11	DC and BC,	DC and BC of the other,
40	1	AB coinciding	AD coinciding
40	10	△ DCD	△ BCD
40	13	6 Join DB	7 Join DB
40	32	OP ∥ OQ	OP = OQ
41	17	∥ EF and BF	∥ DF and BF
41	29	AD = BC	AD = BC
		BE = BC	BE = BC
42	1	∟s = 180	∟s = 180°
43	12	QBQ	QBQ′
43	14	QBO′	Q′BO,
43	14	Q′BO	∟ Q′BO,
43	15	Q′BO,	∟ Q′BO,
		∟ QOB	∟ Q′OB,
43	23	CAD	∟ CAD
43	33	The yatch, &c	15 The yatch, &c
43	35	A and B	and at B

Page	Line	For.	Read
44	9	4 rt ∟s	—4 rt ∟s
44	17	9000 − 360°	900° − 360°
44	21	= 188	− 180°
44	30	(i) AP moves	18 (i) AP moves.
45	27	ZY = YV	YV = ZY
45	36	∴ ZV	ZY
46	16	st line joins	st line which joins.
47	28	AXQ	AX′Q′
47	32	OX = ½ BQ	OX′ = ½ BQ′
47	32	½ (AP × QQ′)	½ (AP + QQ′)
47	38	OD 8 cm.	0·8 cm.
48	2	B, P, &c.	O, P, &c.
48	14	AD and DC	AD and BC
48	18	(AD + BC) AD	(AD + BC). AD
50	5	in the CD	in CB
50	31	DS′	DS
51 } 52 }		(all the lines are not in	scale consult the figure part)
52	17	is 37½ miles,	is 37½ miles, and is
52	31	the CBP	the ∟s CBP
54	26	AD = BR	AO = BR
55	26	This case	(iv.) This case
56	7	A the point	At the point
56	25	At the point P	At the point A
56	30	the ∟ L (const)	the ∟ (90° − M.) (const.)
57	1	∟ CA′B′	∟ C′AB′
57	10	△ at the point	△ At the point
57	17	180° − L	180° − L,
57	24	∴ AC + CD	AC = CD
58	14	Draw a st line	18 Draw a st. line
58	22	BD = $c - b$	19· BD = $c - b$
58	23	BC = A = 7 cm.	BC = a = 7 cm
58	31	C = 7 + 1 = 8 cm.	∴ c = 7 + 1 = 8 cm.
59	5	ACD	ACB
59	14	AB is the	2. AB is the
59	21	AB = 3″	3. AB = 3″
59	22	CD = OD	CO = OD
60	12	angle BO	angle BOA
60	20	5 cm	4 4 cm.
60	22	Place the	5 Place the
61	3	2 (52 + 3)	2 (5 2 + 3)
61	11	Only the four	6. Only the four

Page	Line	For.	Read
64	2	(i) Let AB and CD	6 (i) Let AB and CD
64	9	EFGH	E, F, G, H
65	19	Bisect AB	11 Bisect AB
65	28	(i) P is the	12 (i) P is the
65	36	∴ PM + N	∴ PM + PN
67	17	also a distance	also at a distance
67	34	and DE	or DE
68	14	(i) Take points	19 (i) Take points
68	22	at S at S	at S At S
68	26	Let S and S′	20 Let S and S′
68	26	PS = S′P′	PS = S′P′
68	27	SP + S′P′	S′P + S′P′
69	18	SS′	S, S′
69	25	called. Hyperbolos	called Hyperbolas
69	31	at A	At A
70	17	OB, OR	OB, and OR
70	18	and OR =	and OP =
70	19	OR	PR
70	31	O as centre	From O as centre
70	34	terminate	terminating
71	1	if OP	when OP
71	6	BF	DF
71	22	DE, again OD	DE Again OD
71	28	the △	the sides of the △
72	17	BD	BO
73	3	one yard	$\frac{1}{3}$ yard
73	9	one inch	one sq inch
74	4	$a \times b$ = area	But $a \times b$ = area
74	18	a rectangle AB	a rectangle AB
79	27	= 4 5 cm.	= 3 2 cm
79	28	3 4	3 2
79	28	14 28	13 14
79	29	AC = b	AC or b
80	10	= C =	= c =
80	16	6′30″	6 3″
80	20	$\frac{\text{area}}{\text{base}}$ or base = $\frac{\text{area}}{\text{altitude}}$	$\frac{2\text{ area}}{\text{base}}$ or base = $\frac{2\text{ area}}{\text{altitude}}$
80	21	$\frac{60\text{ sq in.}}{20''} = 4''$	$\frac{80\text{ sq in} \times 2}{20''} = 8''$
80	22	$\frac{10\ 4\text{ sq cm}}{1\ 6\text{ cm}} = 6{\cdot}5$ cm.	$\frac{10\ 4\text{ sq cm} \times 2}{1\ 6\text{ cm}} = 13$ cm
80	26	3 36″ = sq in	3 36″ sq in

Page.	Line	For	Read.
81	13	and that	and EF
82	6	ABX on the	ABX are on the
82	7	CBX on the	CBX are on the
82	10	AX or AC	AC or AC
82	14	BE	DC
82	15	DC	BE
82	23	exer 11	exer 11 on page 65
83	20	0 68	0 62
83	21	68′	·62″
83	21	1 258″	1 147″
83	22	$\frac{1}{2} \times 370 \times 68 = 12580$	$\frac{1}{2} \times 370 \times 62 = 11470$
85	32	KLXOM	$KL \times OM$
86	14	or BC produced	(omit these words)
86	31	$4{\cdot}26775\ p=$	$42\ 6775\ \therefore p=$
86	last	$169 - 29 - 140$	$169 - 29 = 140$
89	18	5 25″ sq in	5 25″ sq in.
89	26	$\frac{1}{2} \times$ diagonal^2	$\frac{1}{2}$ the product of diagonals.
90	9	25 5	15·5
91	6	SQPR	$SQ \times PR$
91	10	angles	angle
93	12	3 6 sq in	3 6 sq in
93	17	AF diagonals	AF, diagonals
93	22	intersection the	instersection of the
94	32	C=3 4′	c=3 4″
96	35	Bare	B are
97	1	BD DC	BD, DC
97	31-32	(remove these lines)	
98	2	and PQ^2	for PQ^2
98	28	D	O
98	29	OQ	OG^2
98	last	B	D
99	15	perpendiculars	perpendicular
99	29	$50^2 196^2$	50^2 or 196
99	33	$101^2 400$	101^2 or 400
100	20	Problem	Problem 16
100	21	$1 + 1 - 2$	$1 + 1 = 2$
101	21	$BD = \sqrt{c^2 - p^2}$	(iii) $BD = \sqrt{c^2 - p^2}$
103	4	$\frac{DC + BD = 41}{2DC = 41} - \frac{559}{41} = \frac{422}{41}$	$DC + BD = 41$ $2\ DC = 41 - \frac{559}{41} = \frac{1122}{41}$
104	20	65	127
105	22	D	O
106	21	9 2	9 1
106	22	$92 = 28\ 06$	$9{\cdot}1 = 27{\cdot}76$

Page	Line	For	Read.
107	24	meeting ED	and produce it to meet ED
108	11-12	O is the point of intersection of PQ and ZC	*Read this in the beginning of the next line just after 10 in line 13*
109	3	PB	DB
109	11	Bisect	Divide
109	20	4 units	6 units
109	21	6 units	4 units
110	15	DP = 17	OP = 17
110	27	7 (*iv*)	(*iv*)
110	31	(*i*) PP′	8 (*i*) PP′
111	16	O	P′
111	17	XO	the ∥ to OX through P′
111	17	O	P′
111	21-22	O	P′
111	25	8 (*ii*)	(*ii*)
111	36	PP′ + PP″	PP′ and PP″
112	12	D	O
113	3	(5, 12)	are (5, 12)
113	5	B	D
113	6	at 2	at E
114	18	AP	DP
114	18	DF	DP
114	19	DP = 7 − 2 = 5	DP = 3 + 2 = 5
114	31	30 units of area	21 units of area
114	33	DC − 11 − 3 = 8.	DC = 11 − 3 = 8
114	34	∴ BE = 5	BF = 5
115	5	5 + 3 = 8	5 × 3 = 15
115	7	= 8 + 12.5 = 20.5	= 15 + 12.5 = 27.5
116	2 to 4	Join BD &c &c	Omit these lines, and proceed as given in Ex 23
116	7	(20, − 5)	(11, 1)
116	8	= 20 − 7 = 13	= 2 + 11 = 13
116	11	$\sqrt{5^2}+12^2$	$\sqrt{5^2+12^2}$
116	32	∥s	line ∥ to X′OX,
118	17	BP′	BP
118	17	AC	AE
118	22	4 rt $\angle$s	2 rt $\angle$s
118	30	AD	AP
118	32	inter $\angle$s	inter opposite $\angle$s
119	3	∴ the $\angle$AEC	and the $\angle$ AEC
119	5	= 2, the $\angle$BCE	= 2 $\angle$ BCE
121	24	△ABC	△ ADC

SOLUTIONS OF EXERCISES

IN

HALL AND STEVENS' GEOMETRY.

PART I.

PAGE 13.

Theor. 1, 2

1 OP a st line revolves round the point O in another st line AB In the beginning OP has its position as OB, and by revolution makes $\angle$s of different magnitude.

2 Construction is the same as given above.

SOLUTIONS OF EXERCISES

IN

HALL AND STEVENS' SCHOOL GEOMETRY.

PART I.

PAGE 13.

(THEOR. 1 AND 2.)

No. of Exercise

1

Prop. No 1.

rt ∟ = 90°, $\frac{1}{2}$ of rt. ∟ = 45
Sup. ∟ of 45° = 135°

Prop. No 2.

$\frac{4}{3}$ of rt ∟ = $\frac{4}{3}$ of 90°
= 120
Sup. ∟ of 120 = 60°

Prop. No 3.

Sup ∟ of 46
= 180 − 46 = 134°

Prop No 4.

Sup ∟ of 149° = 31°

Prop No 5

Sup ∟ of 83 = 97°

Prop No 6

Sup ∟ of 101° − 15′
= 78° − 45′.

2.

Prop. No 7.

$\frac{2}{5}$ of rt ∟ = $\frac{2}{5}$ × 90° = 36°
Comp. ∟ of 36° = 90 − 36 = 54°

Prop. No 8.

Comp. ∟ of 27° = 63°

Prop No. 9.

Comp ∟ of 38° − 16′ = 51° − 44′.

Prop. No. 10

Comp. ∟ of 41° − 29′ − 30″
= 48° − 30′ − 30″

No. of Exercise.

Prop No. 11.

3. St. lines AB and CD intersect each other at O, the ∟COB is a rt. ∟.

As the ∟s AOC, and COB = 2 rt. ∟s [Theo. I]

But ∟ COB is a rt ∟ [Hyp.]

∴ ∟AOC is also a rt. ∟.

Since CD is a st line ∵ ∟s COA and AOD, and also the ∟s COB and BOD are 2 rt ∟s. [Theo. I.]

But each of ∟s AOC and COB is a rt ∟

∴ The supplement of the ∟COA, i e, the ∟AOD = rt ∟.

Similarly the supplement of the angle COB, i e, the ∟BOD = a rt ∟

Each of ∟s AOC, COB, BOD and AOD is a rt ∟

Prop No 12.

4. The ∟ABC is = ∟ACB BC is produced both ways to D and E. The ∟ ABE shall be = to ∟ ACD

Since the ∟ ABE is supplement to the ∟ ABC, and the ∟ ACD is the supplement to the ∟ ACB and the ∟ ABC = ∟ACB. [Hyp]

∴ The ∟ABE = ∟ACD for they are supplementary to equal angles

5. The sides AB and BC in the figure given above are produced to F and G

The ∟CBF is supplementary to ∟ABC and the ∟BCG is supplimentary to the ∟ACB, and ∟ABC = ∟ACB.

∴ the ∟CBF = ∟BCG, for they are supplementary to equal ∟s

Prop No. 13.

6. Since the ∟s. AOB, BOC are = to 2 rt ∟s

∵ OX bisects the ∟AOB. AOX = BOX.

Similarly BOY = COY

∴ ∟XOY = ∟s COY + ∟AOX

But the ∟s COY, YOB, BOX, XOA are equal to twice the angles YOB + BOX = 2 rt. angles. ∟YOX (i e, ∟YOB + BOX) = 1 rt. ∟.

7. It has been proved in the above figure that the ∟YOX = one rt. ∟.

∴ The remaining ∟s COY and AOX are together equal to one rt. ∟, and consequently the ∟s. YOC and AOX are complimentary.

8. Since st. line OX makes with st. line CA two alternate $\angle$s COX and AOX which are equal to two rt $\angle$s [Theo I]

$\therefore$ $\angle$s COX and AOX are supplementary to each other. But the $\angle$AOX = $\angle$BOX [Cons —]

$\therefore$ $\angle$COX is supplimentary to $\angle$BOX.

In the same manner it can be proved that $\angle$s AOY and BOY are also supplementary.

9 In the figure to exercise 6 the angle AOB = 35°. The $\angle$AOB is bisected by OX $\therefore$ $\angle$AOX = BOX = $17\frac{1}{2}$°.

But the $\angle$s AOX and COY have been proved in the previous exercise to be complimentary to each other.

$\therefore$ $\angle$COY = rt. $\angle$ − $\angle$AOX = 90° − $17\frac{1}{2}$° = 72° − 30'.

or $\angle$s COB + $\angle$BOA = 2 rt $\angle$s.

and COB = 180° − $\angle$BOA = 180 − 35 = 145.

$\therefore$ COY = $\frac{1}{2}$ of COB = 72° − 30'

PART I.

PAGE 15

(Theor 3 or Euclid I 15.)

Prop No 14.

No of Exercise.

1. The minute hand OA of a clock completes one revolution round the dial in 60 minutes, and at the same time st line OA revolves round O and thus by completing one revolution it turns through four rt. $\angle$s = 360° $\therefore$ in one minute the minute hand turns $\frac{360}{60}$ = 6 degrees.

$\therefore$ (*i*) in 5 minutes it turns 5 × 6 = 30 degrees

(*ii*) in 21 minutes it moves 6 × 21 = 126 degrees

(*iii*) in 43 5 minutes moves 43·5 × 6 = 261 degrees

(*iv*) in 14 minutes 10 sec moves $14\frac{1}{6}$ × 6 = 85 degrees

(*v*) The minute hand will take $\frac{66}{6}$° = 11 minutes to cover 66 degrees

(*vi*) It will take $\frac{222}{6}$° = 37 minutes to turn through 222 degrees.

2. (*i*) at 12 o'clock both the hands are exactly at XII; but while the minute hand completes one revolution the hour hand moves from XII. to I. *i. e.* only 5 parts out of 60; and

thus hour hand makes an angle of 30° in one hour and therefore it makes an $\angle$ of $112\frac{1}{2}^\circ$ in 3 hours 45 minutes.

(ii) in 5 hours 10 minutes it makes an $\angle$ of 210°

(iii) The hour hand passes through $172\frac{1}{2}^\circ$ in $\frac{345}{2} \times \frac{1}{30} = 5$ hours 45 minutes

3 The earth revolves 360 degrees in 24 hours or $\frac{360}{24} = 15^\circ$ in one hour It will turn in 3 hours 20 minutes $= 3\frac{20}{60}$ or $\frac{10}{3}$ hours, about $\frac{10}{3} \times 15 = 50$ degrees and it will pass through 130° in $\frac{130}{15} =$ 8 hours 40 minutes

Prop No. 15.

4 (i) The $\angle$ AOC $= 35^\circ$

$\angle$COB $= 180^\circ - 35^\circ = 145^\circ$, i e, the $\angle$ COB is supplementary to $\angle$ AOC.

The $\angle$ BOD is vertically opposite to $\angle$ AOC and $=$ to 35°

The $\angle$ DOA being vertically opposite to $\angle$ BOC is equal to 145°

(ii) all the $\angle$s at O taken together are equal to 4 rt $\angle$s But the two angles COB and AOD are equal to 250° . the remaining $\angle$s AOC and BOD $= 360^\circ - 250^\circ = 110^\circ$ As the $\angle$COA $=$ $\angle$BOD each of the $\angle$s COA and BOD $= \frac{110}{2}$ or 55°

(iii) all the four $\angle$s at O $= 4$ rt $\angle$s or 360° the $\angle$ AOD $=$ $360^\circ - 274^\circ = 86^\circ$ and the $\angle$ COB $=$ $\angle$AOD $\angle$COB $=$ 86° But the $\angle$s AOC + COB + BOD $= 274^\circ$ (hyp) the angles AOC + BOD $= 4$ $\angle$s at O $-$ ($\angle$s COB + AOD)

$$= 360^\circ - 2 \times 86$$
$$= 360^\circ - 172^\circ$$
$$= 188^\circ$$

But $\angle$ AOC $=$ $\angle$BOD each of the $\angle$s AOC and BOD $= 94^\circ$

Prop No 16

5 AB is the given st line , OC and OD two st. lines coming from opposite directions meet in AB at O, and make $\angle$COB $=$ $\angle$AOD, then OD and OC shall be in one st line. $\angle$AOD $=$ $\angle$COB [hyp.] add $\angle$AOC to each

. $\angle$s AOD + AOC $=$ $\angle$s COB + COA.

But the $\angle$s COB + AOC = 2 rt $\angle$s [Theo 1.]
∴ the $\angle$s AOD + AOC = 2 rt $\angle$s
∴ OC and OD are in the same st line [Theo 2]

Prop No. 17.

6 As OX bisects the $\angle$BOD. ∴ $\angle$BOX = $\angle$DOX OX is produced to Y, and the $\angle$BOX = vertical opposite $\angle$AOY, and the $\angle$DOX = the $\angle$COY ∴ the $\angle$AOY = the $\angle$COY Hense the $\angle$AOC is bisected by the st line XY

Prop No 18

7. In the figure as given above. The st line OX bisects the $\angle$BOD, and OY bisects the $\angle$AOC. the $\angle$BOX = $\angle$DOX and the $\angle$AOY = $\angle$COY But the $\angle$s AOC, BOD are = for they are vertical opposite $\angle$s ∴ their halves are equal, *i e*, $\angle$BOX = $\angle$s AOY and DOX = $\angle$COY. As DC is one st line and the $\angle$s DOX, XOD, and COB are together = 2 rt $\angle$s But $\angle$DOX has been proved to be = $\angle$COY [Theo 1] ∴ the $\angle$s XOB, BOC, COY = 2 rt $\angle$s. consequently, the lines OX and OY are in the same line [Theo 2]

Prop No. 19

8 As OX is the bisector of the $\angle$AOB.. $\angle$AOX = $\angle$BOX Now folding the figure about OX, the st line OB will fall on OA for the $\angle$BOX = $\angle$AOX ∴ OA and OB must coincide

(*i*) If the $\angle$AOX be > $\angle$BOX then OA will fall beyond OB as OA'

(*ii*) In case the $\angle$AOX be less than the $\angle$ BOX, OA will fall within the $\angle$BOX as OA''

Prop No 20

9. As the $\angle$BOC = $\angle$BOD and the $\angle$AOC = $\angle$AOD Now by folding the figure about AB, the line OD must coincide with the line OC, since the $\angle$AOD is a rt $\angle$ and = $\angle$AOC a rt $\angle$ and $\angle$BOD = $\angle$BOC, for the equal angles occupy equal space.

Prop. No 21.

10 As the $\angle$ made by a st line is equal to 2 rt $\angle$s.

∴ the $\angle$ at O in AB = 2 rt $\angle$s, now by folding the st. line AB about O and making the st line OB fall on OA, the crease made by

the fold and left on the paper as marked OX in the figure will bisect the ∟ AOB, i e, 2 rt ∟s . the crease OX will make an ∟ of 90° with OA and OB, i. e, OX will be perp to AB

PART I

Page 19

Theor 4

No of Exercise.

1 Let ABC be an isos △ of which side AB = side AC, and BC the base AD bisects the vertical ∟BAC.

Prop No. 22

Now in two △s BAD and CAD, AB = AC, and AD is common, and the included ∟BAD = included ∟CAD BD = CD [Theo. 4]

(ii) and the △s are = in all respects. the ∟ADB = ∟ADC and they are adjacent ∟s

∴ each of them is a rt ∟, and consequently AD is perpendicular to BC.

Therefore the bisector of the vertical ∟ BAC of the isos. △ ABC bisects the base BC at rt ∟s, i e, BD = DC & AD is perpendicular to BC.

2 Prop No 23.

Let O be the middle point of AB and OC perpendicular to AB A point P is taken in OC If straight lines PA & PB be drawn, PA shall be equal to PB

In the two △s PAO & PBO, the side OA = the side OB, and PO common to both and the included ∟ AOP = the included ∟ BOP ∴ both the △s are equal in all respects and the base AP = the base BP.

Prop No 24

3 Suppose ABCD is a square of which side AB = BC = CD = DA and the ∟s ABC, BCD, CDA, & DAB all rt ∟s Then the diagonal AC shall be = BD Now in two △s ABC & DCB, the side AB = DC, and BC is common, and the included ∟ ABC a rt. ∟ = the included ∟DCB also a rt ∟

∴ the △ ABC = the △ DCB in all respects [Theo 4]

∴ AC = DB.

Prop No 25

4 Let ABCD be a square L, M, and N middle points in AB, BC and CD respectively

(i) Join LM and MN Then LM shall be equal to MN Now taking the two △s LBM and NCM, LB half of AB = NC half of DC, and BM = MC because M bisects BC . sides LB and BM = sides NC and MC, and the included angle LBM = the included angle NCM. . the base LM = MN [Theo 4]

(ii) Join AM and DM In the two △s ABM and DCM, AB = DC, and BM = CM, and the angle ABM = ∟DCM ∴ two △s ABM and DCM are equal, and the base AM = DM.

(iii) Join AM and AN. In two △s ABM and ADN, AB = AD being sides of a □ and BM = DN being halves of equal sides BC and DC and the ∟ B = ∟D ∴ the △ ABM = the △ ADN . the base AM = AN. [Theo 4]

(iv) Join BN and DM. In two △s BCN and DCM, BC = DC, and CN = CM, and the ∟ C being common

∴ the △s BCN and DCM are equal and the base BN = DM [Theo 4]

Prop No. 26

5 Let ABC be an isosc △ of which AB = AC From AB and AC, AX and AY equal parts are respectively cut off from AB and AC Join BY and CX Then BY shall be = CX. In the two △s ABY and ACX the two sides AB and AY are respectively = two sides AC and CX, and the ∟ at A is common to two △s ..the bose BY = CX [Theor. 4]

PART I.

Page 21.

(*Theor 6*)

Prop. No. 27.

No. of Exercise.

1. The figure ABCD is four-sided, its side AB = BC = CD = DA and BD is its diagonal.

(i) In the $\triangle ABD$, sides AB and AD are equal [hyp]. $\therefore$ the $\angle$s ABD and ADB at the base BD are equal [Theor 5]

(ii) Similarly $BC = CD$ (hyp) and the $\angle CBD = \angle CDB$ [Theo. 5].

(iii) In (i) part of this exercise it is proved that $\angle ABD = \angle ADB$, and in (ii) part it is shown that $\angle CBD = \angle CDB$.

$\therefore$ The whole $\angle ABC =$ whole $\angle CDA$

Prop No 28

2. ABC is an isosc $\triangle$, the angles ABC and ACB at the base BC are equal Similarly in the isosc $\triangle DBC$, the $\angle$s DBC and DCB at the base BC are equal, the whole $\angle ABD =$ the whole $\angle ACD$

Prop. No 29

3. Two isosc. $\triangle$s ABC and DBC are on the same base BC and on the same side of it. In the $\triangle ABC$ the $\angle$s ABC and ACB are equal [Theor 5]

And similarly in the $\triangle DBC$, the $\angle$s DBC and DCB are equal. [Theor 5.]

Now from the equal $\angle$s DBC and DCB take away the equal $\angle$s. ABC and ACB respectively the remaining $\angle ABD =$ remaining $\angle ACD$.

Prop No 30.

4. AB and AC equal sides of an isosc $\triangle$, are bisected at L and N respectively, and the base BC is bisected at M

$\therefore AL = LB$, $AN = NC$ and $BM = CM$

(i) In the two $\triangle$s LBM and NCM, the sides LB and BM of the one = the sides NC and CM of the other respectively, and included $\angle LBM = \angle NCM$ because they are $\angle$s at the base of an isosc $\triangle$ [Theor. 5]

$\therefore$ the base $LM =$ the base MN [Theor 4]

(ii) Join BN and CL

Now there are two $\triangle$s LCB and NBC of which the two sides LB and $BC = NC$ and CB and the included $\angle LBC = \angle NCB$ and the $\triangle LBC = \triangle NCB$ in all respects. $\therefore$ $LC = BN$. [Theor. 4]

(iii) Now because AB = AC (hyp), and they are bisected at L and N respectively in the △ALN, the side AL = AN. Hence the ∠ALN = ∠ANL.

Again because LM has been proved = MN ∴ ∠MLN = ∠MNL [Theor 5]

Hence the whole ∠ALM = whole ∠ANL.

PART I.

PAGE 26

(Theor 4 & 7)

Prop No 31.

No of Exercise

1 Let ABC be an isosc △ and D the middle point of the base BC. Join AD.

Then (i) AD shall bisect the angle BAC, (ii) AD shall be perpendicular to the base BC.

(i) In the two △s ABD and ACD, the side AB = AC because they are the sides of an isosc △.

AD is common to both, and the base BD = CD for the point D bisects the base BC.

the vertical ∠BAD = vertical ∠CAD

i e, the ∠BAC is bisected by AD [Theo 7]

(ii) The two triangles ABD and ACD being equal in all respects, the ∠ADB = angle ADC but these are the adjacent angles.

∴ the angles ADB and ADC are rt. angles.

i. e, AD is perpendicular to BC.

Prop No. 32.

2 ABCD is an equilateral four-sided figure, and AC is its diagonal

Since AB = AD, and BC = DC and the base AC is common ∴ the △ABC = △ADC in all respects, i. e., (i) the angle ABC = angle ADC and the angle BAC = angle DAC, and angle BCA = angle DCA (ii) ∴ the whole angle BAD = whole angle DCB.

Prop No 33.

3 ABCD is a four-sided figure of which opposite sides are equal, namely AB = DC and AD = BC, join AC.

Then in the two $\triangle$s ABC and ADC, two sides AB and BC = two sides CD and AD each to each, and the base AC is common $\therefore$ the $\triangle$ABC = $\triangle$ADC in all respects [Theor. 7.] and $\therefore$ the $\angle$ABC = CDA.

4. This exercise has already been proved in exercises No. 2 and 3 under Theorem 5 or this can be proved thus.

Prop No. 34.

(i) In the first case both the isosc. $\triangle$s are on the same base and on the same side of the base BC, (ii) second case both $\triangle$s ABC and DBC are on the opposite side of BC.

Prop No 35

(ii) In both these cases join AD Then because in the two $\triangle$s ABD and ACD, the two sides AB and BD in the one are equal to two sides AC and CD in the other and AD is common to both $\therefore$ the two $\triangle$s ABD and ACD are equal in all respects [Theor. 7] $\therefore$ the angle ABD = angle ACD.

5 The figure for this exercise is the same as that of the (ii) case of the last preceding exercise In the two $\triangle$s BAD and CAD, BA = CA, and AD is common, and the third side BD of the one = the third side CD of the other $\therefore$ the angle BAD = angle CAD, *i e*, the angle BAC is divided into two equal parts by AD.

Similarly the angle BDA = angle CDA, *i. e*, the angle BDC is bisected by AD

6. This exercise has already been solved in (ii) case of the exercise No 4 under Theorem 5

Prop No. 36

7. ABC is an isosc. $\triangle$, D and E are the two points in the base BC, equidistant from B and C, *i e*, BD = CE Join AD and AE

In the two $\triangle$s ABD and ACE, the two sides AB and BD = two sides AC and CE, and the angle ABD = angle ACE. [Theor 5] $\therefore$ the base AD = the base AE. [Theor. 4]

Prop No 37.

8 ABC is an equilateral △, and D, E, F are the middle points of the sides AB, BC, and AC respectively Join DE, DF and EF. as AB = BC = AC, and points D, E and F bisect them ∴ AD = DB = BE = EC = FC = AF Now in the three triangles DAF, DBE and ECF two sides of the one = two sides of the other, namely, AD and AF = DB and BE = EC and CF, and the included ∠s A, B, and C are equal [Cor. 2, Theor 5]

The bases of the three △s DAF, DBE and ECF are equal [Theor 4] namely DF = DE = EF.

∴ the △ DEF is equilateral.

Prop No 38

9 The angles ABC ACB and at the base BC of an isosc △ ABC are bisected by BO and CO respectively.

(i) The angle OBC = angle OCB because each of them is half of the angles at the base of the isosc △, ∴ the side BO = side OC [Theor 6]

(ii) Join AO In the two △s BAO and CAO, the two sides BA and AO of one = two sides CA and AC of the other, and the base BO = OC, ∴ the angle BAO = angle CAO [Theor 7]

Prop No 39.

10. ABCD is a rhombus, AC and BD are its diagonals intersecting each other at E In the △s BAC and DAC, BA and AC = DA and AC, and the base BC = base CD, ∴ angle BAC = angle DAC

Now in the △s BAE and DAE, BA = DA, and AE common to both, and the angle BAE = angle DAE, ∴ the base BE = base DE, and the angle AEB = angle AED, but these angles are adjacent, ∴ each of them is a rt. angle.

Similarly taking △s ABE and CBE it can be proved that AE = EC, and that each of the angles AEB and CEB is a rt angle

∴ the diagonals of a rhombus bisect each other at rt. angle.

Prop No 40.

11. In the $\triangle$s BFA and CEA, the two sides BA and AF = CA and AE respectively, and the angles BAF = angles CAE for they are vertical opposite angles ∴ the $\triangle$BFA = $\triangle$CEA in all respects [Theor. 4] ∴ base BF = base CE.

PART I.

PAGE 27.

Exercises on Triangles.

Prop No 41

No of Exercise

1 Make a line AB = 2″ With the centre A and at a distance = 2 1″ draw an arc, and from the centre B at a distance = 1 3″ draw another arc cutting former at C, join AC and BC, ∴ ABC is the repuired $\triangle$.

With the help of the protractor measure the angle A which = 37°, and the angle B = 77°, the third angle C = 66° The sum of three angles = 37 + 77 + 66 = 180°

2 In the figure ABC, a = 7 5 *cm*, b = 7 *cm*, and c = 6·5 *cm* From the point B draw a perpendicular BD on AC and measure out BD with the help of a *cm* scale which in this case = 6 *cm*.

Prop No 42

3 Draw a line AC = b = 7 *cm* at the point C in AC make an angle = 65° with the help of a protractor, and from the second arm CB cut off a part a = 7 *cm* and join it with A; then ACB is the required $\triangle$ of which side AC = 6 *cm*, CB = 7 *cm*, and included angle ACB = 65°

When the two $\triangle$s have the above data they are equal in all respects, according to theor 4 and they are said to be alike in size and shape

To prove the result by experiment, draw two triangles of the same parts, and cut out the one and apply it to the other so that equal sides of the one cover the corresponding sides of the other and the equal angle the equal angle, then the two $\triangle$s will be congruent

Prop. No 43.

4 Describe the triangle in the manner explained above

With the help of the same scale and protractor measure side BC, and the angles B, and C BC = 2 2", angle B = 49°, angle C = 74°.

The triangle drawn with the data just found namely angles B and C, and side BC, and it will be equal to the former in all respects.

Prop No 44.

5 In the accompanying figure AB is the height of the window = 35 feet above the ground, the foot C of the ladder AC is 12 ft. from the wall, i e, BC = 12 ft and the angle at B is rt angle. The figure is drawn with the help of a scale, 1' in = 10' ft

By measuring the ladder AC with the help of the same scale, it is found to be 3 7" of the scale or 37 ft

Prop No 45

Prop No 46

6. I start from A and go North to N a distance = 99 metres, and from N turn East 20 metres to E In plotting the course a scale 1 *cm* = 10 metres is used

The angle at N = a rt angle.

Join EA.

By measuring EA with the above scale, EA is found 10 2 *cm.* nearly, ∴ the distance between E and A is nearly 102 metres.

Prop. No 47

7 The observer is C, AC is the horizon, AD is the direction of sun's rays AB the height of the pole. AC shadow of the pole. angle ACB is the elevation of the sun above the horizon = 42°. angle A = rt angle

angle B = 48°

By measuring AB with the help of the same scale (1" = 10 ft) it is found = 2 7" or 27' feet

Prop. No 48

8 This figure shows the course the surveyor took The distance AD was found by measurement with scale to be 4·25 inches or 425

ft The angle DAB = 135°, and point D bears from A 135° − 90° = 45° the bearing of the point D is 45° towards west, i e, due N W

Prop No 49

9 In the figure B and C are the points on the shore S is a ship.

The bearing of S from B is angle CBS = 33° and from C is angle BCS = 81°

Complete the △ by joining BS and CS On measurement BS = 2 8″ inches on the scale or 280 yds and CS = 1 61″ or 161 yds From S draw SA perpendicular to BC, then SA is the shortest line from S to BC which when measured is found 1 6″ in the scale or 160 yds on the ground

Prop No 50

10 From the accompanying plan draw in scale 1″ = 100 ft.
AB = 2 2″ or 220 ft.

PART I

PAGE 29

Theor 8

Prop No 51

No of Exercise

1 ABC is a △, any two angles of it = less than 2 rt angles.

Take a point D in BC Join AD

The ext angle ADC > the inter angle ABC, again the ext. angle ADB > the int angle ACD [Theor 4]

The angles ADC + ADB > the angles ABC + ACB But the angles ADC + ADB = 2 rt angles [Theor 1]

∴ the angles ABC + ACB < 2 rt angles

Prop No 52

2. (i) Produce BD to meet AC at E

The ext angle BEC is > the int angle BAE [Theor 8]

Again the ext angle BDC is > the int angle DEC [Theor 8]

∴ much more the angle BDC is > the angle BAC.

(ii) Join AD, and produce it to F

Then because in the △ ADB, the ext. angle BDF is > the int. angle BAD. [Theor. 8.]

Again in the △ ADC, the ext. angle CDF is > the int angle CAD. [Theor 4.]

∴ the whole angle BDC is > the whole angle BAC

Prop No 53.

3. The side BC of a △ ABC be produced both ways to D and E the ext. angles ACD and ABE are > two rt angles.

The ext angle ACD and the int angle ACB are = two rt. angles.
Similarly ext angle ABE + int angle ABC = two rt angles.
But the two int. angles ABC and ACB are < 2 rt angles.
∴ the ext. angles ACD and ABE are > two rt. angles.

Prop. No. 54.

4 A is a point outside the line BC Take any point O on the other side of BC From the centre A at the distance AO, describe an arc cutting BC at D and E. Join AD and AE. AD and AE are the only two equal st. lines that can be drawn from A to BC.

If possible draw another st. line AP equal to AD or AE. Then AD = AE

The angle ADE = angle AED [Theor. 5]

But AP = AE, ∴ angle APE = AEP [Theor. 5]

But the angle AEP = ADE. ∴ the angle APE is also = the angle ADE.

But the angle APE is the ext angle of the △ADP ∴ the ext. angle APE = the int. angle ADE which is absurd. [Theor. 8.]

∴ There cannot be drawn more than two equal st. lines from a given point outside a given line to it.

Prop. No 55.

5. The two equal sides AB and AC of an isosc. △ ABC are produced to D and E.

The ext angles CBD and BCE must be obtuse

The int angle ABC together with the adjacent ext angle CBD = two rt angles

Similarly the two angles ACB and BCE = two rt angles

But both the interior angles ABC and ACB are < two rt. angles the ext angles CBD and BCE are > two rt angles

As the int angles ABC and ACB are equal, the angles CBD and BCE are equal, and hence each of the angles BCE and CBD is greater than one rt angle, *i e*, is obtuse

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PART I

PAGE 34

(*Theor 9—12*)

Prop No 56

1 ABC is a rt angled △, the angle C being a rt angle AB is the hypotenuse which is the greatest side

Since the angle ACB = a rt angle, the angles ABC and BAC are also = one rt angle . each of the angles ABC and BAC is < a rt angle

the angle ACB is > each of the angles ABC and BAC But greater angle is subtended by the greater side . AB is > either of the sides AC and BC

Prop No 57

2 The side BC of the △ ABC is the greatest, *i e*, BC is greater than either of the two sides AB and AC

But by Theor 9 the angle opposite to the greater side is greater than the angle opposite to the less the angle BAC opposite to the greatest side BC is greatest of the remaining angles ABC and ACB which are opposite to the smaller sides AB and AC, and these angles ABC and ACB are adjacent to BC the greatest side

· BC the greatest side of the △ ABC makes acute angles ABC and ACB with the smaller sides AB and AC of the △ ABC

3 Take the figure given in exercise 2. (*i*) under Theor 8, page 29.

In the △ ABE, the two sides BA and AE are > the side BE [Theor. 11] Add EC.

∴ AB+AE+EC are together > BE+EC, i. e., BA+AC > BE+EC

Again in the △ DEC, the sides DE and EC are > DC [Theor. 17] Add BD

∴ EC+ED+BD are together > CD+BD, i e., BE+EC > BD+DC

But BA+AC has been proved > BE+EC.

Much more BA+AC > BD+DC.

Prop No 58.

4. In the △ ACD, the ext angle ACB is > the int angle ADC [Theor. 8]

But the angle ACB = the angle ABC, being equal sides of an isosc △

∴ the angle ABC is > the angle ADC.

But greater angle is subtended by the greater side. ∴ AD is > AB. [Theor 10]

But AB = AC.

∴ AD is > either of AB or AC.

Prop No 59.

5. Let ABCD be the quadrilateral figure of which AD is the least side and BC the greatest Each of the angles BAD and CDA shall be greater than their opposite angles, namely BCD and ABC respectively

Join AC.

Then because DC > AD, the angle DAC opposite to the greater side DC is > the angle DCA opposite to the least side AD. [Theor 9]

Again in the △ ABC, BC > AB, the angle BAC is > the angle BCA. [Theor 9]

But the angle DAC has been proved > the angle DCA. ∴ the whole angle DAB is greater than the whole angle DCB.

Similarly, by joining DB it can be proved that the angle ADC is > the angle ABC.

Prop. No. 60.

6. In the $\triangle ABC$, if AC is not $>$ AB, it must be either $=$ AB or angle AB.

From A draw AD meeting BC at D.

AD shall be $<$ AB.

If AC $=$ AB. The angle ACB $=$ the angle ABC.

But the angle ADB is $>$ the angle ACD $\therefore$ the angle ADB $>$ the angle ABD.

The greater angle has the greater side opposite to it. $\therefore$ AB $>$ AD. (Theor. 9.)

Again if AC is $<$ AB. Then the angle ACB is $>$ the angle ABC.

But the angle ADB is $>$ the angle ACD.

Much more the angle ADB is $>$ the angle ABC.

$\therefore$ the side AB is $>$ the side AD.

Prop. No. 61.

7. If the side AB is $>$ the side AC, the angle ACB is $>$ the angle ABC.

But the angle ABC is bisected by BO, $\therefore$ the angle OBC is half of the angle ABC.

In the same manner the angle OCB is half of the angle ACB.

$\therefore$ the angle OCB is $>$ the angle OBC, and the side BO is $>$ the side OC. (Theor. 9.)

Prop. No. 62.

8. In the $\triangle ABC$, the difference of AB and AC is less than BC.

(i) If AB $=$ AC, their difference is $=$ O which is less than BC.

Prop. No. 63.

(ii) If AB is $>$ AC, cut off AD $=$ AC, and join DC. Produce AC to E.

Prop. No. 64.

The $\angle$ ADC $=$ the $\angle$ ACD.

$\therefore$ the suppl. $\angle$ BDC $=$ the suppl. $\angle$ DCE [Cor. 3, Theor. 1]

But the $\angle$ DCE is $>$ the $\angle$ DCB.

$\therefore$ the $\angle$ BDC is $>$ the $\angle$ DCB, and hence the side BC is $>$ the side BD which the difference of AB and AC.

(*iii*) If AB is < AC, from AC cut of AD = AB, join BD, produce AB to E

By the same method of reasoning it can be proved that the ∟ BDC is > the ∟ CBD, and ∴ the side BC is > the side DC, the difference between AC and AB.

Prop. No. 65.

9. O is a point in the △ABC. Join OA, OB, and OC. Then OA + OB + OC shall be greater than half the perimeter of the △ ABC.

OA + OB is > AB, OB + OC is > BC, and OA + OC is > AC. Then the sum of these, *i e*, twice OA + OB + OC > the sum of AB, BC, and AC, *i. e.*, the perimeter of ABC.

∴ the sum of OA, OB and OC is > half the perimeter of the △ ABC.

Prop. No. 66.

10. ABCD is a four-sided figure, its perimeter is greater than the sum of the diagonals. Join AC.

The two sides AB, BC are > AC, and the two sides CD, DA are also > AC ∴ the sum of the four sides is > twice the diagonal AC.

Similarly by joining BD, it can be proved that the sum of the four sides is > twice the diagonal BD.

∴ Twice the sum of the four sides is > twice the sum of the diagonals AC and BD.

∴ the perimeter of the quadrilateral figure is > the sum of the diagonals.

Prop. No. 67.

11. Produce AX to Y.

Then because the ext. angle BXY is > the int angle BAX. [Theor. 8]

But the angle BXY = angle AXC. [Theor. 3.]

∴ the angle AXC is > the angle BAX or CAX, for the angle BAX = the angle CAX.

∴ the side AC is > the side XC.

Similarly the angle AXB is > the angle BAX.

∴ AB is > BX.

Hence the sum of the two sides AB and AC is > the sum of BX and XC, *i. e.*, BC the third side.

This is the alternate method of proving the Theorem 11, without producing the side BA.

Prop No. 68.

12. O is a point within the $\triangle$ ABC, join OA. OB and OC The sum of OA, OB and OC shall be less than the perimeter.

By the application of Exercise 3 under this head it can be proved that $BA+AC>BO-OC$, $AB+BC>OC+OA$, and $AC+BC>OA-OB$.

$\therefore$ Twice the sum of BA. BC and AC is > twice the sum of OA, OB and OC

$\therefore AB+BC+AC>OA+OB-OC$

Prop No 69.

13 AC and BD are the diagonals of a quadrilateral ABCD, and O is a point in it.

Join OA, OB, OC, and OD

Then $OA-OB-OC-OD>AC+AD$

Because in the $\triangle$ BDO the two sides BO and OD are > BD [Theor. 11] Similarly $AO-OC$ are > AC

$\therefore$ the sum of OA, OB, OC, and OD is > the sum of AC and BD.

The exception to the above is when the point O coincides with X the intersection of the two diagonals

Prop No 70

14. In the $\triangle$ ABC AD is the median from A to BC.

The two sides BA and AC are > twice AD.

Produce AD to E make $DE=AD$ and join CE

Then in the two $\triangle$s. ABD and CDE, the two sides AD and BD are = two sides DE and DC, respectively, and the included angle ADB = angle CDE. $\therefore$ the angle $ADC=DEC$. $AB=CE$ Now in the $\triangle$ ACE, the two sides AC and CE are together greater than AE, but $CE=AB$ and $AD=DE$ $\therefore$ AC and AB are > twice AD

Prop No. 71.

15 As proved in the last preceding Exercise 14. AB and AC are >2 AD AB and BC are >2 BE, and AC and BC are >2 CF

$\therefore$ Twice AB, BC and AC are > 2 AD, 2 BE and 2 CF

$\therefore AB-BC+AC > AD-BE-CF$.

PART I.

Page 41.

Parallels.

(Theor 13—15.)

1. In the figure of Theor. 15, the ext. angle EGB = 55°, but the angle EGB = the angle GHD = the angle HKQ.

∴ each of these angles is = 55°

The angle QKF is the supplementary of HKQ

∴ angle QKF = 180—55 = 125°.

Prop. No. 72.

2 AB, CD, and EF are the st. lines perpendicular to the st. line GH

Then AB, CD, and EF are ∥ to one another.

The st line GH meets two st lines AB and CD, and makes int angles BAG, ACD together = 2 rt. angles for each of them is a rt angle {Hyp]

∴ AB is ∥ to CD [Theor 13]

In the same manner CD is ∥ to EF ∴ AB is ∥ to EF. [Theor. 15]

Hence AB, CD and EF are ∥ to one another.

Prop No 73

3. The st line GH meets three ∥ st. lines AB, CD and EF and it is perpendicular to AB one of the ∥ lines, then it is also perpendicular to others AB is ∥ to CD, and GH meets them then the ext. angle GXB = int angle XYD [Theor. 14.]

But the angle GXB = a rt angle, for GH is perpendicular to AB

∴ the angle GYD is also a rt angle and GH is perpendicular to CD also

In the same way it can also be proved that GH is perpendicular to EF also

Prop No 74.

4 ABC and DEF are two angles of which side AB is ∥ DE and BC ∥ to EF.

The angle ABC shall be = or supplimentary.

(i) Supposing the angles face towards the same direction as in (i).

Produce DE to X meeting BC at X

Now AB is ∥ DX, BC meets them. The ext. angle DXC = int. angle ABC [Theor. 14]

For the same reason angle DXC = angle DEF ∴ the angle ABC = the angle DEF.

Prop No 75

(ii) Suppose the angle ABC and the angle DEF oppose each other as in figure (ii).

Produce ED or BC to X meeting BC or ED if produced in X. AB is ∥ to XE, and BC meets them ∴ the alter angles ABC and BXE are equal (Theor 14)

Again BC is ∥ to FE, XE meets them, then the two int. angles BXE and BXF = 2 rt. angles. [Theor 14]

∴ the angle XEF is supplementary to the angle BXE or ABC.

Prop No 76

5 In the two △s AOC and BOD two sides CO and AO are = two sides DO and BO respectively [Hyp] and the angle AOC = angle BOD ∴ the △ AOC = △ BOD ∴ the angle CAO = angle OBD and the angle ACO = DBO. But these angles are altr angles ∴ AC is ∥ to BD [Theor. 13.]

Prop. No 77

6. ABC is an isosc. △, a st line DE is drawn ∥ to the base BC, meetting AB and AC at D and E.

Since DE is ∥ to BC and AB and AC fall on them. Then the ext angle ADE is = to the int opposite angle ABC, and the ext. angle AED is = the int. oppt angle ACB

Prop No 78

7 ABC is an angle and BD its bisector From any point O in BD, a st. line XOY is drawn ∥ to BC, meeting AB at X.

Then the △ BXO is an isosceles △

Since XY is ∥ to BC, and BD falls on them, the ext. angle DOY = the int oppt. angle OBC. But the angle OBC = angle ABO, for BD bisects it. ∴ angle DOY = angle ABO But the angle DOY = angle XOB. [Theor 3.] ∴ XB = OX.

Prop No 79.

8 The angle ABC = angle ACB and the angle YXB = angle ZXC of the △s YBX and ZCX

∴ the remaining angle BYX is = to the remaining angle CZX. But the angle BYX = angle AYZ [Theor. 3]

∴ the angle AYZ = angle AZY, and hence AZ = AY, *i. e.*, the △ZAY is an isosceles,

(ii) From A draw AD [illegible]

Prop. No. 80.

9. ABC is a $\triangle$ of which side BA is produced to D, and the st. line AE bisects the ext. $\angle$CAD. If AE be $\parallel$ to BC and BD meets them, then the $\angle ABC = \angle DAE$ [Theor. 14.]

Similarly AE is $\parallel$ to BC and A meets them the alternate $\angle$s. EAC and ACB are equal [Theo. 14]

But $\angle$ DAE $=$ $\angle$EAC $\therefore$ $\angle$ ABC $=$ $\angle$ACB. Hence the $\triangle$ABC is isosceles

10. ABC is an $\angle$, and BD its bisector, O a point in BD, from O two $\parallel$ st. lines OX and OY are drawn $\parallel$ to BC and AB respectively. Then the figure XBYO shall be a rhombus.

It has already been proved in Ex. 7. under this head that OX $=$ BX, and the $\angle$OBX $=$ $\angle$XOB.

On the same analogy it can also be proved that OY $=$ BY and the $\angle$ OBY $=$ $\angle$ BOY.

Prop. No. 81.

$\therefore$ The whole $\angle$ XOY $=$ $\angle$ XBY. Now Join ~~from~~ XY, then in the two $\triangle$s XBY and XOY, the two sides X~~O~~B and BY are respectively $=$ to two sides XO and OY, and the included $\angle$ XBY $=$ $\angle$ XOY $\therefore$ $\angle$ BXY $=$ $\angle$ OXY, and $\angle$ BYX $=$ $\angle$ OYX. Again BX is $\parallel$ to OY, BY meets them, $\therefore$ the $\angle$s. BYO and XBY are $=$ two rt. $\angle$s. In the same manner OXB and XBY are $=$ two rt. $\angle$s.

From these take away common $\angle$ XBY.

$\therefore$ the remainder OYB is $=$ OXB.

But it has already been shewn that $\angle$ OXY $=$ $\angle$ BXY, and $\angle$ BYX $=$ $\angle$ OYX $\therefore$ the st. line XY bisects the equal and opposite $\angle$s BXO and BYO.

$\therefore$ the $\angle$ BXY $=$ $\angle$ BYX, and hence side XB $=$ YB. But XB $=$ XO $\therefore$ XO $=$ BY $=$ BX $=$ OY. Hence the figure XBYO is a rhombus.

Prop. No. 82.

11. The st. line DZ is the bisector of the $\angle$ CDB and from a point Z in DZ a st. line ZX is drawn parallel to AB $\therefore$ XZ $=$ DX, as proved in Ex. 7. under this head.

In the same manner XY $=$ XD $\therefore$ XY $=$ XZ.

Prop No 83 Prop No. 84

12 ∴ PA makes 12 revolutions in a minute; i e, one revolution in 5 seconds, or in other words it moves 72° in one second

QB makes 10 revolutions in a minute, i e, one revolution in 6 seconds or it moves only 60° in one second

(i) When PA and QB point opposite ways they are 180° apart

∴ the fastest pivot is found 180° in advance in $\frac{180°}{72-60}$ = 15 sec after their start from the same position

(ii) These will take 5 × 6 seconds, the L C M. of the time they take to make one revolution, to point towards the same direction Since the rods PA and QB start parallel
inting the same way, and PA revolves more rapidly than QB, ∴
ll be again parallel (i) pointing opposite ways, when PA
de half a revolution more than QB, and (ii) pointing
ne way, when PA has made one complete revolution
than QB.
se PA makes 12, and QB makes 10
lutions per min.

PART I

PAGE 43, THEO 16

Prop No 85

1. ABC is an equi △ Every equi △ is equiangular ∴ ABC △ is equiangular [Cor 2, Theo 5]

All the three ∠s of a △ are = two rt ∠s or 180°

∴ each of the angles of ABC = $\frac{180}{3}$ = 60°

Prop No 86

2. ABC is a rt ∠d isosc △, having a rt. ∠ at B. Since in a rt ∠d △ hypotenuse is the greatest side

∴ the sides AB and BC are equal and they contain the rt ∠ B.

As the three ∠s of a △ are = two rt ∠s So the ∠s BAC and BCA are = one rt ∠, for the ∠ at B = one rt. ∠

But the ∠ BAC = ∠ BCA [Theor 5]

∴ each of the ∠s BAC and BCA = half a rt ∠ or 45°

Prop No 87

3 In the △ ABC, the ∠ ABC = 36° and the ∠ ACB 123°

∴ the remaining third ∠ BAC = 180° − (36° + 123°) = 21°

Prop No. 88

4 ABC is a △ of which the angle ABC = 111° and the angle ACB = 42°

∴ the angle BAC = 180° − (42° + 111°) = 27°.

Prop. No. 89.

5 The angle $ACB = 180^\circ - 134^\circ = 46^\circ$ and the angle $ABC = 180 - (42^\circ + 46^\circ) = 92^\circ$.

Prop No. 90.

6 The angle $ACD = 180^\circ - 118^\circ = 62^\circ$ and the $\angle$ $BAC = 180^\circ - (51^\circ + 62^\circ) = 67^\circ$

Prop No 91.

7 ABC is a $\triangle$, and XAY is drawn $\parallel$ to BC.

The $\angle$ ABC = the angle XAB for they are the alternate $\angle$s [Theor. 14]

Again the angle ACB = angle YAC [Theor. 14]

Add to these the $\angle$ BAC $\therefore$ the three $\angle$s ABC, ACB and BAC = three $\angle$s XAB, YAC and BAC But all the three $\angle$s at A = two rt $\angle$s

$\therefore$ the three $\angle$s of $\triangle$ ABC = two rt. $\angle$s

Prop No 92

8. Let the pair of st lines AB and CD be perpendicular to another pair of st lines FE and GH respectively. Now produce BA and DC to meet at X, and FE and GH to meet at Y. [illegible]Y cutting DC produced at Z The $\angle$ BXZ shall be equal to the angle DYZ

Now in the two $\triangle$s, the $\angle$ XBZ = $\angle$ YDZ for they are rt. $\angle$s; and the $\angle$XZB = the angle YZD. [Theor. 3.]

$\therefore$ the remaining angle BXZ = the remaining angle DYZ.

(Many figures can be drawn to prove the above. The figure here drawn is one of them

PART I.

Page 44, Theor 16, Cor 1.

Prop. No 93.

(i) By applying the formula $nD + 360 = n \times 180$, when n = No. of sides and D = No of degrees in an angle.

Then for hexagon $6D + 360 = 6 \times 180$ $\therefore$ $D = \dfrac{6 \times 180 - 360}{6} = 720^\circ$ in one angle

Prop. No. 94.

(ii) For octagon $8D + 360 = 8 \times 180$ $\therefore D = \dfrac{8 \times 180 - 360}{8} = 135^\circ$ in one $\angle$.

Prop No. 95.

(iii) For a decagon 10 D + 360 = 10 × 180 D = $\frac{1800 - 360}{10}$ = 144° in an ∟.

PART I.

PAGE 45, THEOR. 16.

Prop No. 96.

1. All the ∟s. of the △ ABC = two rt. ∟s = 180° suppose ∟ A = x, ∟ B = $2x$, and ∟C = 3C ∵ A + B + C = $x + 2x + 3x$ = $6x$ ∴ $6x$ = 180° ∵ x = 30° ∴ angle A = 30° angle B = 60° and ∟C = 90°.

Prop No. 97.

2. (i) ∟ A = x, each of B and C = $2x$. ∟s. A + B + C = $5x$, $5x$ = 180° ∴ x = 36° ∵ ∟A = 36° and each of the ∟sB and C = 72°.

(ii) A + B + C = $x + 4x + 4x = 9x$ ∴ $9x$ = 180° ∴ x = 20 ∟ A = 20°, each of the ∟s B and C = 80°.

Prop No. 98

3. The ext. ∟ ACD = 126° ∴ int. adj. ∟ ACB = 180 − 126 = 54°.

Similarly ∟ ABC = 180 − 94 = 86°.

Lastly ∟ BAC = 180 − 54 − 86 = 40°.

Prop No. 99.

4. Let x and y be the ∟s at the base BC.

Then $x + y$ = 162°, and $x - y$ = 60° add $\frac{x - y = 60}{2x = 222}$ ∴ x = 111 and y = 51

∴ the remaining ∟ BAC = 180° − 51° − 111° = 18°.

Prop No. 100.

5. ∟ ABC = 84° ; ∟ ACB = 62°

(i) ∟ BAC = 180° − (84° + 62°) = 34°

(ii) ∟ DBC = 42°, ∟ DCB = 31°

∴ ∟ BDC = 180 − (42 + 31) = 107°

Prop. No. 101.

6. ∟ CBE = 180° − 74° = 106° ∴ ∟ CBD = 53°

Again angle BCF = 180° − 62° = 118° ∴ ∟ BCD = 59°

Prop No. 102.

7. ∟ BCD = 114½°, ∟ ABC = 50, ∟ BAD = 75½

$\therefore \angle s\ A+B+C=240^\circ$

But all the $\angle$s of the figure $=360^\circ$

$\therefore \angle\ D=360-(240^\circ)=120^\circ$

Prop. No. 103.

8. Let $\angle A=x$, $\angle B=2x$, $\angle C=3x$, $\angle D=4x$.

$\therefore x+2x+3x+4x=360^\circ$ or $10x=360^\circ$ $\therefore x=36^\circ$

$\therefore \angle A=36^\circ$, $\angle B=72^\circ$,

$\angle C=108^\circ$, $\angle D=144^\circ$.

Prop. No. 104.

9 In the accompanying five-sided figure angle $B=40^\circ$, $\angle C=78^\circ$ $\angle D=122^\circ$, $\angle E=135^\circ$

All the $\angle s=5\times 180-360=540^\circ$

The given 4 $\angle s=375^\circ$ $\therefore \angle A=540^\circ-375=165^\circ$

Prop. No 105.

10. According to the cor. [Theor. 16]

(i) All the $\angle$s of a figure of n sides $+$ 4 rt. $\angle$s $=$ twice as many rt. $\angle$s as there are sides.

$\therefore n \angle s+4$ rt $\angle s=2n$ rt. $\angle$s $n \angle s=2n$ rt. $\angle$s-4 rt. $\angle$s or one $\angle=\dfrac{2n-4}{n}$ rt. $\angle$s or $\dfrac{2(n-2)}{n}$ rt $\angle$s.

(ii) In the figure vertex A is joined to each of the other $\angle$s, except the two immediately adjacent to A, the whole figure is divided into as many $\triangle$s as there are sides minus two, i. e., $(n-2)$ $\triangle$s.

The three angles of a $\triangle=$ 2 rt. angles.

$\therefore$ all the angles of $(n-2)$ $\triangle s=2$ rt. angles $\times$ $(n-2)$ $\triangle$s or $2(n-2)$ rt angles

But there are n sides.

$\therefore$ one $\angle=\dfrac{2(n-2)}{n}$ rt. $\angle$s.

11. One $\angle$ of a regular polygon $=\dfrac{2(n-2)}{n}$ rt. $\angle$s.

Therefore in (i) case

$$108^\circ=\frac{2n \text{ rt } \angle s-4 \text{ rt } \angle s}{n}$$

or $108n=2n\times 90-360$

$108x - 180n = -360^\circ, 72x = 360^\circ$

$n = 5$ The figure is 5 sided

(ii) $156^\circ n = 180n - 360^\circ, 24n = 360^\circ$

$\therefore n = 15.$ The figure is 15 sided.

12. Prop. No. 106 (i) (ii) (iii)

As all the angles at a point taken together are four rt ∟s, i e, 360°

In order to know which of the regular figures can be so fitted together round a point as to form a plane surface, the 4 rt ∟s or 360° be divided by the number of degrees contained in one angle of the figure In case an ∟ of a regular figure is contained an exact number of times in 4 rt ∟s, that very figure can be so fitted as to form a plane surface

(i) An ∟ of equi $\Delta = 60^\circ$

$\therefore \frac{360}{60} = 6$ if six equi Δs are so fitted as shown in figure (i) they form a plane surface

(ii) So with a square whose one $\angle = 90^\circ$, $\frac{360}{90} = 4$ four squares can be placed side by side as in figure (ii) to form a plane surface

(iii) An ∟ of a hexagon $= 120^\circ$.

$\frac{360}{120} = 3$ ∴ Three hexagons can be so arranged, as in figure (iii)

For other regular figures this rule cannot be applied Suppose octagons are so arranged One ∟ of an octagon $= 135, \frac{360}{135} = 2 + \frac{90}{135}$ i e, after placing the ∟s of two octagons there remains a gap between $= 90^\circ$, and if 3 octagons are so placed they overlap

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Cor 2, Theor 16

Ex 1 In the figure to example (i) under Cor I, Theor 16, produce DE to X As the ∟ DEF one of the ∟s of a regular hexagon $= 120^\circ$, and the two ∟s DEF and FEX are $= 180^\circ$.

$\therefore$ the ext $\angle$ FEX $= 180° - 120° = 60°$, *i. e.*, $\frac{2}{3}$ of a rt $\angle$ which is the value of an $\angle$ of an equi. $\triangle$.

2 Just in the manner given above produce DE in the figures (*ii*) and (*iii*) example to cor 1, Theor. 16, then in the figure (*ii*) the int $\angle$ DEF $= 135°$ $\therefore$ the ext $\angle$ FEX $= 180° - 135° = 45°$.

Figure (*iii*) the int. angle DEF $= 144°$.

$\therefore$ the ext. $\angle$ FEX $= 180° - 144° = 36°$.

3. As all the exterior $\angle$s. of a regular polygon are $= 4$ rt. $\angle$s.

$\therefore$ (*i*) the sides of the polygon having an ext $\angle = 30$ are $= \frac{360}{30} =$ 12, *i e*, the polygon is 12 sided.

(*ii*) The polygon is $\frac{360}{24} = 15$ sided AB is $\|$ to CD and EF meets them at E and F.

EO and FO bisect the $\angle$s *B*EF and EFD

Then the $\angle$ EOF is a rt. $\angle$

Prop No. 107.

4. The int $\angle$s BEF and EFD are $= 2$ rt $\angle$s $\therefore$ the $\angle$s. OEF and OFE half the two int. $\angle$s. are $=$ one rt $\angle$. But the three $\angle$s OEF, OFE and EOF are $= 2$ rt $\angle$s and the $\angle$s OEF and OFE are $=$ one rt $\angle$ $\therefore$ The remaining $\angle$ EOF $=$ one rt. $\angle$.

Prop. No 108.

5 ABC is a $\triangle$, base BC is produced both ways to X and Y

Now ext $\angle$s ABX and ACY together with the int. $\angle$s ABC and AC*B* are $=$ four rt $\angle$s.

But the three $\angle$s of $\triangle$ A*B*C $= 2$ rt. $\angle$s.

Now taking away the $\angle$s ABC, AC*B* and *B*AC $= 2$ rt $\angle$s.

The remainder ext $\angle$s A*B*X and ACY — $\angle$ *B*AC $= 2$ rt. $\angle$s.

Prop No. 109 P. 20

6 A*B*C is a $\triangle$ the $\angle$s A*B*C and AC*B* at the base *B*C are bisected by D*B* and DC.

The three $\angle$s A*B*C, AC*B* and *B*AC are $=$ two rt. $\angle$s.

The half of equal things are equal.

$\therefore$ half of the $\angle$s A*B*C, ACB and BAC $=$ one rt angle.

Again the angles DBC, DCB and BDC $= 2$ rt angles.

Now by taking away the equals, the remainders are equal.

$\therefore$ BDC $- \frac{1}{2}$ BAC $=$ one rt angle, *i. e.*, the angle BOC $= \frac{1}{2}$ angle BAC $+ 90°$. 2nd Proof. $\angle BOC = 180° - \angle OBC - \angle OCB = 180° - \frac{1}{2}B -$

Hall $= 180° - \frac{1}{2}(B + C)$

Prop No. 110. P. 20

7. The ext $\angle$s BCE and CBD together with adj int. $\angle$s ACB and ABC are $=$ 4 rt. $\angle$s and the three $\angle$s of the $\triangle$ ABC $=$ 2 rt $\angle$s Take away the equals.

The remainder ext $\angle$s BCE and CBD minus $\angle$ BAC are $=$ 2 rt $\angle$s or half of these, i e, OCB, OBC and minus $\frac{1}{2}$ of $\angle$ BAC $=$ one rt $\angle$ or 90°.

But the $\angle$s BCO, CBO and BOC are $=$ 2 rt. angles.

Again take away the equals.

Then the remainder the angle BOC $+ \frac{1}{2}$ angle BAC $=$ one rt. angle or 90° . the angle BOC $= 90° - \frac{\angle BAC}{2}$

Prop. No 111

8. ABCD is four-sided and all the int. angles $\angle$A, $\angle$B, $\angle$C and $\angle$D are $= 2 \times 4$ rt angles $-$ 4 rt angles or $=$ four rt. angles

$\therefore$ their halves or the $\angle\frac{A}{2} + \angle\frac{B}{2} + \angle\frac{C}{2} + \angle\frac{D}{2} = 2$ rt. $\angle$s.

But in the $\triangle$ OBC the angles $\frac{B}{2}$, $\frac{C}{2}$ and BOC are $=$ two rt. angles Subtracting the latter from the former we get angle $\frac{A}{2}$ $+$ angle $\frac{D}{2}$ $-$ BOC $= 0$

i e, angles $\frac{A}{2}$ and $\frac{D}{2}$ $=$ angle BOC.

Prop No. 112.

9 The $\angle$ ABC $=$ $\angle$ ACB, AB $=$ AD and AB $=$ AC

$\therefore$ AC $=$ AD and consequently the $\angle$ ACD $=$ ADC

$\therefore$ the $\angle$s ABC and ADC are $=$ $\angle$s ACB and ACD.

or $=$ $\angle$ DCB.

But the three $\angle$s DBC, BDC and BCD are together $=$ 2 rt $\angle$s.

$\therefore$ the $\angle$s DBC and BDC are $=$ one rt. $\angle$ and the $\angle$ DCB is also $=$ one rt $\angle$

Prop No 113.

10 ABC is a rt angled $\triangle$ having $\angle$ B a rt. $\angle$ and D is the middle point in AC. Join BD

Produce BD to E making DE $=$ BD and join AE.

Now in the two $\triangle$s ADE and BDC, AD = DC and BD = DE, and the $\llcorner$ ADE = BDC $\therefore$ the $\triangle$ ADE = $\triangle$ BDC in all respects, $\therefore$ the angle DAE = angle DCB To each of these add the angle BAC. Then the whole angle BAE = angles BAC and ACB which are = one rt. angle, $\therefore$ angle BAE = one rt. angle.

Now in the two $\llcorner$s ABC and BAE, BC = AE and AB is common, and the angle ABC = angle EAB. $\therefore$ the base AC = the base BE. But AD is $\frac{1}{2}$ AC and BD is $\frac{1}{2}$ BE $\therefore$ BD = AD = CD.

PART I.

PAGE 49, [THEOR 17.]

On the identical equality of triangles.

Prop. No. 114.

1. ABC is an isosc. $\triangle$ and AB = AC the angle ABC = angle ACB CO is perp. on AB, and BP on AC.

Then in the two $\triangle$s OBC and PCB, the angles OBC and BOC of the one are = angles PCB and CPB of the other and the side BC opposite to the = sides

$\therefore$ the $\triangle$ OBC = $\triangle$PCB in all respects.

$\therefore$ BP = OC.

Prop. No. 115.

2. BO is the bisector of the angle ABC, and O a point in the bisector from which OP and OQ perpendiculars are drawn to AB and BC respectively. In the two $\triangle$s BPO and BQO, the angle BPO = BQO, and the angle PBO = QBO, and one side BO common, $\therefore$ two $\triangle$s BPO and BQO are = and OP = OQ.

Prop No. 116.

3. In the two angles AOX and BOY, AO = BO, the angle AXO = angle BYO and the angle AOX = angle BOY.

$\therefore$ the $\triangle$ AOX = $\triangle$ BOY.

$\therefore$ AX = BY.

Prop. No. 117.

4. In the $\triangle$ ABC, AD bisects the angle A and is at rt. angle to BC the side AB shall be equal to AC.

Now in the two $\triangle$s ABD and ACD, the angle BAD = angle CAD, and the angle ADB = angle ADC, and AD is common

. the $\triangle$ ABC = $\triangle$ ACD and AB = AC – i. e, the $\triangle$ ABC is isosceles

Prop No 118

5. ABC is a $\triangle$, if AD bisects BC at rt angle then AB = AC In the two $\triangle$s ABD and ACD, the angle ADB = angle ADC, and the side BD = side CD and AD is common, then $\triangle$ ABD is = $\triangle$ ACD in all respects

. AB = AC

Prop No 119 P. 21

6 ABC is a $\triangle$, in which AD bisects the angle BAC, and the base BC AB shall be = AC Produce AD to E, make ED = AD. Join CE

In the two $\triangle$s ABD and ECD, AD = DE and BD = CD and the angle ADB = angle EDC.

.. AB = EC [Theor 4] the angle BAD = angle CED. But the angle BAD is = angle CAD [Hyp]

$\angle$ CEA = angle CAD

.. AC = EC [Theor 6]

But AB is proved = EC.

.. AB = AC

Prop No 120

7. AB || to CD, the st. line EF meets them at E and F.

The point O is the middle point in EF, and OP, and OQ, are drawn perpendicular to AB and CD respectively Now in the two $\triangle$s OPE and OQF, the angle OPE = angle OQF being rt. angles, and the angle POE = angle QOF, and the side OE = OF. [Hyp]

$\therefore$ $\triangle$ OPE = $\triangle$OQE in all respects and OP = OQ.

Prop. No 121

8 The st line EF is terminated by two || st lines AB and CD and bisected at O, another st line PQ passes through O and terminates at P and Q

In the two $\triangle$s EOP and FOP, the angle EOP = angle FOQ, [Theor 3] and angle PEO = angle QFO [Theor. 14] and EO = FO [Hyp.] $\because$ the $\triangle$ EOP = $\triangle$ FOP.

.. PO = QO.

Prop No 122. P. 22

9 O is a point equidistant from two ∥ st lines AB and CD, and through it two st lines PQ and XY pass and terminate by the ∥ st lines AB and CD

In the two △s EOP and FOQ, the angle EOP = *angle* FOQ [Theor 3] and the angle OEP = OFQ and the side adj. to them. OE = OF. ∵ the △ EOP = △ FOQ, and OP = OQ, and EP = FQ

Similarly in two △s EOX and FOY, it can be proved that OX = OY, and XE = FY

whole XP = whole QY

Prop No 123. P. 22

10 In the two △s ABC and ACD, AB = AD, and BC = CD, and the base AC is common.

∴ the △ ACB = △ ACD, each to each . the angle BAC = angle DAC and the angle BCA = angle DCA, *i. e*, AC bisects the angles BAD and BCD

Since because the △ BAD is an isosc △, for AB = AD, and the angle ABD = angle ADB and the angle BAO has been proved = angle DAO the angle AOB = angle AOD but they are adj. angles each of them is a rt angle.

∴ AC is perpendicular to BD

Prop No 124

11 There are two △s BAO and DCO, in which the ∟ BAO = angle DCO being rt. angles, and the angle AOB = COD. [Theor. 3.] and one side AO = CO

∴ the △ BAO = △DCO and ∴ BA = CD. Hence by measuring CD the breadth of the river is known.

PART I

PAGE 54 REVISION LESSON ON △s.

1. (i) The property of interior ∟s of a △ is that all the interior ∟s = two rt. ∟s

   (ii) That all the exterior ∟s = four rt. ∟s, all the inter angles together with 4 rt. angles are = twice as many rt. angles as there are sides, correspond to a polygon of n sides also where all the inter. angles = 2 n rt. angles − 4 rt. angles or 2 $(n-2)$ rt. ∟s.

The property enumerated in (ii) is shared by a $\triangle$ with all other polygons

2 The $\triangle$s can be classified with regard to their angles into three kinds, i. e, rt angled $\triangle$, obtuse angled $\triangle$, and acute angled $\triangle$.

Theor. 16. The three angles of a $\triangle$ are together = two rt angles.

And cor 1 of Theor 8. Any two angles of a $\triangle$ are together less than two rt. angles. And cor. 2 of Theor. 8. Every triangle must have at least two acute angles

Prop. No 125.

3 Theor 5, 7, 9.

Since in the $\triangle$ ABC, the sides AB and BC or A and C are equal, and each of them is > AC or B b

$\therefore$ the angle B is < either of angles A and C. [Theor 9.]

Now the two angles A and C are together less than two rt. anles [Cor. 1. Theor. 8] But the angle A = angle C [hyp]

$\therefore$ each of the A and C = less than a rt. angle.

And the angle B has been shewn to be < either of the angles A and C. $\therefore$ the angle B is also less than a rt. angle Hence the $\triangle$ ABC is acute angled.

Prop. No. 126.

4. Theorems, Theor. 6, 10.

(i) the Third angle C = $180° - 48° - 51° = 81°$

$\therefore$ the greatest side is AB [Theor. 10]

Prop. No. 127.

(ii) The third angle C = $180° - 2 \times 62\frac{1}{2} = 55°$

The side AC = side BC while the side AB is less than AC or BC.

5. Identically equal $\triangle$s. are—

(i) Theor. 17. Prop. No. 128.

(ii) Theor. 4. Prop. No. 129.

(iv) Theor. 7. Prop. No. 130

(vi) Theor. 18. Prop. No. 131.

(iii) Triangles need not be equal. see (i) 1

Prop. No. 132. id

(v) Ambiguous case, see (ii), P. 51

Prop. No. 133.

6. (i) Triangles are equal in all respects when

1 Two sides and included angles are equal. Theor. 4.

2 Three sides are respectively equal. Theor 7.

3. Two ∠s and one side either opposite to or adjacent to the equal angles (Theor. 17)

4. The △s are rt. angled, and Hypotenuses and one side of each equal. (Theor. 18.)

(ii) The triangles in the following cases may or may not be equal; when

1. The three angles are equal.

2 Two sides and one angle are equal, the equal angles being not included between the equal sides

7.(i) In two triangles which have their respective ∠s equal, the ∠s are not dependent on the arms; the arms may be longer or shorter but the angles remain the same, hence the equality of the two △s in all respects remains doubtful. (ii)

Prop No 134. P. 24.

8 (i) AB is the given st. line and C a point without it. CD is the perpendicular to AB CE and CF are oblique lines one on each side of CD making ∠s DCE and DCF equal.

In the △ CDF, since the angle CDF is a rt. angle ∴ the ∠ CFD is less than a rt ∠ [Theor 8, Cor. 1]

i. e, the angle CFD is < the angle CDF.

∴ CD is less than CF. [Theor. 10]

In the same manner it can be shown that CD is less than any other obliques CE, CP or any other obliques that may be drawn from C to AB

(ii) The obliques CE and CF make ECD and FCD ∠s equal to each other; and the angle CDE is = anglo CDF, and the side CD is common ∴ The △s CDE and CDF are equal. ∴ CE = CF.

(iii) From C draw another oblique CP making with CD an ∠ DCP greater than the ∠ DCF.

The ext angle CFD of the △ CFP is greater than the int and oppt ∟ CPF [Theor 8]

But the angle CFD has been shown less than a rt ∟,
the ∟ CFP is greater than a rt ∟ [Theor 1]
. much more the ∟ CFP greater than the angle CPF

CP is > than CF [Theor 10]

9 The solution of this is given on page 28 which consult.

Prop No 135 P.25

10 The angle QPA = 15°, PA = 4 2 cm by measurement

„	QPB = 30,	PB = 4 6	„	„
„	QPC = 45,	PC = 5 6	„	„
„	QPD = 60,	PD = 8 1	„	„
„	QPE = 75	PE = 15 2	„	„

Prop No 136 P.24

11 In the △ BAP, AB = 4 cm is fixed while AP = 3 cm rotates about the point A, tracing the changes of its position along the arc P_1, P_3, P_6 as the angle BAP increases from O to 180

When the angle BAP_1 is 0°, AP_1 coincides with AB and BP = 0 cm.
„ „ becomes 30° as BAP, BP increases to 2 cm
„ „ BAP_2 is 60° as BP_2 increases to 3·5 cm
„ „ BAP_3 is 90° as BP_3 „ 5 cm
„ „ BAP_4 is 120° as BP_4 „ 6 1 cm
„ „ BAP_5 is 150 as BP_5 „ 6 7 cm nearly
„ „ BAP_6 is 180° as BP_6 and BA become in one and the same line and thus BP_6 becomes equal to 7 cm

Prop No 137 P.24

12 The approximate height of AB = 40 cm

Prop No 138 P.25

13 The approximate distance AB = 110 ft

Prop No 139 P.25

14 The distance of the ship A from the Light-house ∟ is 342 yds nearly, and that of the ship B is 692 approximately by measurement

PART I

PAGE 39, Theor 20-21.

Prop No 140.

1 ABCD is a four-sided figure of which opposite sides AB and DC, and AD and BC are equal Join AC

Then in the two ∠s ABC and CDA, the two sides AB and BC of the one = two sides CD and AD of the other, and the base AC common to both, the △ ABC = △ CDA, [Theor. 4] and the ∠ BAC = ∠ DCA, and the ∠ BCA = ∠ DAC but these are alternate ∠s.
∴ AD is ∥ to BC

In the same manner it can be proved that AB is ∥ to DC [Theor 13.]

∴ ABCD is a parallelogram.

2 In the above figure the ∠ ABC is = the ∠ ADC, and the ∠ BAD = ∠ CDA.

But the four angles of a quadrilateral are = 4 rt angles

∴ the angles BAD and ABC are = two rt angles. [Inf 5, Theor 16.] and these are two intr angles on the same side of AB, which meets two other st. lines AD and BC, ∴ AD is ∥ to BC. [Theor 13]

Similarly AB is ∥ to DC.

∴ ABCD is a parallelogram

Prop No 141.

3 In the figure ABCD the diagonals AC and BD bisect each other. In the two △s ADE and CBE, the side AE = side EC and the side DE = the side BE, and the included ∠ AED = included ∠ BEC ∴ the △ ADE = △ CBE, and the base AD = base BC, and the ∠ DAE = the ∠ BCE [Theor. 4.]

But these are the alternate angles, ∴ AD is ∥ to BC. [Theor. 13]

Similarly by taking two △s AEB and DEC it can be demonstrated that AB is ∥ to DC

∴ the figure ABCD is a parallelogram.

Prop No 142

4 ABCD is a rhombus, of which AC and BD are diagonals, intersecting each other at O

∵ Diagonals of a parallelogram bisect each other.
∴ AO = OC and BO = DO [Cor. 3, Theor. 21]

Now in the two $\triangle$s AOB and AOD, the two sides AO and OB in the one = two sides AO and OD in the other, and the base AB = the base AD $\therefore$ the $\angle$ AOB = $\angle$ AOD [Theor 7]

But these being adjacent $\angle$s and equal to two rt. $\angle$s $\therefore$ each of the $\angle$s AOB and AOD is a rt $\angle$ Hence AO or AC is at rt. $\angle$ to BD and bisects it

Prop No 143.

5 ABCD is a parallelogram, and the diagonals AC and BD are equal.

Then the $\triangle$s ABC and DCB, the two sides AB and BC of the one are = to two sides DC and BC, and the base AC is common, $\therefore$ the $\triangle$s ABC and DCB are equal and the angle ABC = angle DCB. [Theor 4] But these are the int angles on the same side of BC, and equal to 2 rt. angles Therefore each of them is a rt. angle. In the same manner each of the angles BAD, and CDA is also a rt angle

Prop No 144.

6. ABCD is a parallelogram, AC and BD are diagonals, If the angle BAD be not equal to the angle CDA, then AC and BD are not equal. Let the angle BAD be less than the CDA.

Now in the two $\triangle$s BAD and CDA, AB and AD = CD and DA, each to each, and the angle BAD less than the angle CDA.

$\therefore$ the base BD is less than the base AC [Theor 19]

PART I.

PAGE 60.—EX ON PARALLELS AND PARALLELOGRAMS.

Prop. No. 145

1. As all the sides of a rhombus are equal, and its opposite angles are also equal, and the diagonal bisects the opposite angles. Now if the rhombus ABCD is folded round the diagonal BD, the angle ADB will coincide with the angle CDB for these are equal, and the line AD will fall on DC for AD = DC, and A will fall on C, similarly the angle ABD will coincide with the angle CBD, and AB will cover BC, and the angle BAD will coincide with the angle BCD for the angle BAD = angle BCD

the $\triangle$s BAD and BCD are symmetrical about BD

In the same manner it can also be shown that the $\triangle$s ABC and ADC are symmetrical about AC

Prop No 146.

2. ABCD is a square, and AC and BD are the diagonals As proved in the last preceding exercise 1, the $\triangle$s BAD and BCD are symmetrical about BD, and the $\triangle$s ADC and ABC are symmetrical about AC.

Prop No. 147.

(ii) The lines EF and GH which join the middle points in the opposite sides of the square ABCD, are the other lines of symmetry.

Prop No 148.

3. ABCD is a rectangle, and BD is a diagonal.

The sides AB and AD in the $\triangle$ ABD are = to the sides CD and BC in the $\triangle$ DCB respectively, and the base BD is common to both, $\therefore$ the $\triangle$ ABD is = $\triangle$ DCB in all respects. [Theor 7.]

The diagonal of rectangle is not an axis of symmetry.

A rectangle is symmetrical about the lines that join the middle points in the opposite sides.

Prop No 149.

4 There is no axis about which a rhomboid can be symmetrical.

For neither the diagonal bisects the opposite angles nor a line joining the middle point of the opposite sides make equal angles with the sides, and hence if one of the $\triangle$s. BAD and DCB be applied to the other it will not cover the other, nor does the figure AF cover the figure EC

Prop No. 150.

5. The diagonal AC, which bisects the angles BAD and BCD, is an axis of symmetry in the figure ABCD.

Prop. No 151. Prop. No. 152.

6. (i) ABCD and EFGH are the two parallelograms, having the two adj. sides AB and AD in one = two adj sides EF and EH of the other, and the angle BAD = angle FEH,

$\therefore$ the $\triangle$ BAD = $\triangle$ FEH. [Theor. 4.]

$\therefore$ by applying the $\triangle$ BAD upon the $\triangle$ FEH, the side AD will fall on EH, and the points A and D will coincide with the points E and H, for AD = EH.

AB coinciding with EH, AB will fall on and coincide with EF for the angle BAC = angle FED and AB = EF ∴ BD will coincide with FH.

Similarly the △BCD will coincide with △FGH.

Prop No 153 Prop No. 154

(ii) ABCD and EFGH are two rectangles of which adj sides BA and AD are = adj sides FE and EH and the included ∠s BAD and FEH are equal for each of them is a rt angle ∴ The △BAD = △FEH.

Similarly the △DCD = the △FGH

the rectangle ABCD = rectangle EFGH.

Prop No 155. Prop No. 156

7 Join DB and HF

In the two △s ABD, EFH, the two sides AB and AD in one are = two sides EF and EH in the other, and the included angle BAD = included angle FED

∴ △ABD = △EFH [Theor 4]

Prop No 157 Prop No 158

∴ the △ABD if applied to the △EFH, the both will coincide

Similarly in other two △s BCD and FGH, the two sides BC and CD = two sides FG and GH respectively, and the base BD = base FH.

∴ the △ BCD = △ FGH and

∴ they coincide when one is applied to the other

Theoretical

Prop No 159

8 ABCD is a parallelogram, BD its diagonal, O the middle-point in BD, a st line PQ is drawn through O meeting AD in P and BC in Q. Then the two △s POD and QOB the ∠s POD and QOB are equal [Theor 3], and the angle PDO = the angle OBQ [Theor 14] and one side BO = side OD.

∴ the △ POD = △QOB in all respects OP = OQ [Theor 17]

∴ PQ is bisected at O

Prop No 160

9. BD is a diagonal in a parallelogram ABCD, and AE and CF are two perpendiculars on BD from two opposite ∠s A and C.

Now in the two $\triangle$s AED and CFB, the rt angle AED = rt angle CFB, and the angle ADE = alternate angle CBF, and one side AD = one side BC.

$\therefore$ the $\triangle$ AED = $\triangle$CFB, and AE = CF. [Theor. 17]

Prop No 161

10 The opposite sides of a parallelogram are equal.

$\therefore$ AD = BC [Theor 21]

Half of equal things are equal $\therefore$ AX = CY

Now AX and CY are equal and parallel, and the two st lines AY and CX join them towards the same parts

$\therefore$ AY and CX are also equal and parallel. [Theor. 20.]

$\therefore$ AYCX is a parallelogram.

Prop No 162

11. Place the two $\triangle$s ABC and DEF so that the base BC when produced be in the one and the same st. line with its equal and parallel side EF

Then because AB is $\parallel$ DE and BF meets them.

$\therefore$ the ext $\angle$ DEF = int. opposite $\angle$ ABC. [Theor. 14]

Again in the $\triangle$s ABC and DEF, two sides AB and BC of one are = two sides DE and EF of the other, and the included $\angle$ ABC = $\angle$ DEF. AC = DF, and the $\angle$ACB = $\angle$DFE [Theor. 4]

Now the st line BF cuts the two st. lines AC and DF, and make the ext $\angle$ ACB = to the int and opposite $\angle$DFE.

$\therefore$ AC is $\parallel$ DF [Theor 13]

It has also been proved equal to it

Prop No 163 fig. P.28

12. (1) Produce DC to E and make DE = AB and join BE Then because DE is $\parallel$ and equal to AB $\therefore$ BE is = and $\parallel$ AD But AD = BC $\therefore$ BE = BC $\therefore$ the $\angle$BCE = $\angle$BEC. [Theor. 5.]

Now DE is $\parallel$ AB, and CB meets them

$\angle$ABC = $\angle$ BCE which is = BEC. [Theor. 14]

Again the $\angle$ ADE = $\angle$ ABE [Theor. 21]

$\therefore$ $\angle$ ADE = $\angle$ ABC + $\angle$ CBE = $\angle$ BEC + $\angle$ CBE.

To each of these add equal angles ABC and BCE respectively.

$\therefore$ the $\angle$ ADE + $\angle$ ABC = $\angle$ BCE + $\angle$ BEC + $\angle$ CBE. But the $\angle$ BCE + $\angle$ BEC + $\angle$ CBE = two rt.

$\angle$s = 180°. ∴ the $\angle$ ADE + $\angle$ ABC = 180°

All the $\angle$s of the quadrilateral ABCD are = 4 rt $\angle$s. and the $\angle$s ADE and ABE are = two right $\angle$s, ∴ the remaining two angles DAB and BCD are = 2 rt. $\angle$s

∴ $\angle$ ADC + $\angle$ ABC = 180° = the $\angle$ DAB + $\angle$ BCD.

(ii) Join AC and BD. Then in the two $\triangle$s DAB and CBA, AD and AB are = BC and AB and the $\angle$ DAB = the $\angle$ ABC.

∴ the $\triangle$ DAB = the $\triangle$ CBA [Theor. 4]

And ∵ the diagonal AC = the diagonal BD

(iii) Bisect AB at O, and CD at P, and join PQ. Then because AO = OB, and DP = CP, and the side AD = side BC, and the $\angle$s ADP and DAO are = the $\angle$s BCP and CBO respectively. ∴ The whole figure AOPD = the whole figure BOPC in all respects, and hence the quadrilateral ABCD is symmetrical about PO.

P. 61 Prop. No. 164. P. 28 fig.

13. (i) AP and BQ are two equal rods which turn round two pivots A and B at equal rates clockwise, *i e*, they make equal $\angle$s at A and B respectively in the same time The rods start parallel but in opposite sense, *i. e*, at the time of their start they point towards diametrically opposite directions, namely AP begins its start while pointing towards the North, at the same time BQ begins its move while pointing towards South.

AP and BQ as shown in the diagram represent the position of both the rods at the time of their start to move.

Join AB and PQ cutting at O.

In the two $\triangle$s PAO and QBO, the angle PAO = the angle QBO, being rt. $\angle$s and the angle AOP = the angle BOQ and PA = BQ. ∴ PO = QO, and AO = BO.

If AP moves and in a certain time describes an angle PAP′ then BQ also describes an $\angle$ QBR′ = an $\angle$ PAP′ in the same time, for AP and BQ turn at equal rates.

Then AP′ shall be ∥ BQ′.

Now AP is parallel to BQ, and BQ' makes an angle QBQ' with BQ, the st. line BQ when produced will meet PA, produced if necessary, let them be produced and meet at C

Then because BQ is ∥ PC and BQ' produced meets them.

∴ the $\angle$ QBC = the alternate $\angle$ PCB [Theor 14] But the $\angle$ QBC = the $\angle$ PAP', ∴ the $\angle$ PAP' = the $\angle$ PCB But the st line PC meets two other st. lines AP' and BC, and makes the exterior $\angle$ PAP' = int. and oppt $\angle$ PCB.

∴ AP' is ∥ BC or BQ' [Theor. 13]

(ii) Join P'Q', cutting AB at O.

Then because the $\angle$ PAB = the $\angle$ QBA, for they are rt. $\angle$s, and their parts the $\angle$s PAP' and QBQ are also equal, ∴ the remainders the $\angle$ P'AO = the $\angle$Q'BO.

Now in the two $\triangle$s P'AO and QBO' the $\angle$ P'AO = Q'BO the $\angle$ P'AO = Q'BO, and the $\angle$ P'OA = the $\angle$ QOB and P'A = Q'B [Hyp.] (Theor 3) ∴ AO = BO.

This result was also obtained by joining P to Q, ∴ O is the point through which the line PQ will pass whatever parallel position the two rods AP and BQ occupy in their rotation round A and B

P.61 Prop. No 165.

Numerical and Graphical.

14 CAD = a is the ext $\angle$ of the $\triangle$ ABC, int $\angle$ $a = \frac{3}{7}$ of ext. $\angle$ a or ext. $a = \frac{7}{3}$ of int. $\angle$ a. But the int. $\angle$ + ext. $\angle$ = two rt angles = 180°

$$\therefore a + \tfrac{3}{7} a = 180° \text{ or } \tfrac{10}{7} a = 180°$$

$$\therefore a = 180 \times \tfrac{7}{10} = 126° \quad \therefore \text{int. angle } a = 180 - 126 = 54°$$

Now 3B = 4C, or B = $\frac{4}{3}$C

But B + C = ext. $\angle$ a = 126°

$$\text{or } \tfrac{4}{3}\text{C} + \text{C} = 126 \;\therefore\; \text{C} = \frac{126 \times 3}{7} = 54°$$

$$\therefore \text{B} = 126 - 54 = 72°$$

Prop. No. 166.

The yatch sails from West to due E, but finding to hinder her eastward course, she turns round and sails towards B making an $\angle$ of 63° out-ward, A and B she again turns 78°, at C 119°, at D

about 64° and finally at F she again resumes her course due east. As all the ext $\angle$s of this five-sided figure $= 4$ rt. $\angle$s $= 360°$. But the sum of all the ext $\angle$s given is $63° + 78° + 119° + 64° = 324°$.

∴ the last turn in her course of $360° - 324° = 36°$ brings her to proceed due east.

16. All the ext $\angle$s of a figure are $= 4$ rt. $\angle$s. And the int. $\angle$s $=$ twice as many rt $\angle$s as there are sides minus 4 rt. $\angle$s, suppose n be the sides.

Then int. n $\angle$s $= 2n$ rt $\angle$s $- 4$ rt $\angle$s

$$= 2(n-2) \text{ rt } \angle\text{s}$$

But by Hyp int $\angle$s $= 2(n-2) \times 90°$

$$\text{i e, } 360° = 180n - 360.$$

$$n = \tfrac{720}{180} = 4 \text{ sides.}$$

∴ the figure is four sided.

P.41 Prop No 167. fig. P.29

17. In this figure ABCDE

All the int. $\angle$s $= 2(n-2) \times 90 = 900° - 360° = 540°$

But the sum of the four given $\angle$s $= 110° + 115° + 93° + 152° = 470°$.

∴ the remaining angle $A = 540 - 470 = 70°$. The st line AB meets two others BC and AE and makes two angles ABC and BAE $= 110 + 70 = 180$ or two rt. angles.

∴ BC is ∥ AE [Theor. 13.]

With the ruler and pen join EC, and then with the help of compasses measure out first AB, and then by placing the two ends of the compasses so extended on the points E and C, it is found that AB is $=$ EC. And in the same manner by measuring BC and then applying the compasses to AE, it is also found that they are equal, hence the figure ABCE is a parallelogram.

Prop. No 168. fig. P.29

18 (i) AP moves in the direction of P P′, and BQ in that of Q Q′, at the time of their start the sum of the $\angle$s they make with AB $= 0$, and when they become parallel the sum of the $\angle$s they make with AB is two rt $\angle$s or 180°

AP makes an $\angle$ of $7\frac{1}{2}°$ per second of time and BQ makes an $\angle$ of $3\frac{3}{4}°$ per second of time so they together make an $\angle$ of $7\frac{1}{2}° + 3\frac{3}{4}° = 11\frac{1}{4}°$ in one second

$\therefore$ they will make an $\angle$ of 180° in $180 \times \frac{4}{45} = 16$ seconds, so they will become parallel in 16 seconds after the start

(ii) AP moves at the rate of $7\frac{1}{2}$° per second and so it makes an $\angle$ of $\frac{15}{2} \times 12 = 90°$ in 12 seconds, and thus assume the position as AP' at rt. $\angle$ to AB.

BQ moves at the rate of $3\frac{3}{4}$° per second and so in 12 seconds the $\angle$ described by BQ will be $= 12 \times \frac{15}{4} = 45°$, and BQ will assume the position BQ' making an $\angle$ of 45° with the line AB

As the two rods AP and BQ are of unlimited length AP' and BQ' if produced will join at O.

Now in the $\triangle$ OAB, the $\angle$ OAB = a rt $\angle$, and the $\angle$ ABO $= \frac{1}{2}$ a rt $\angle = 45°$.

$\therefore$ the remaining $\angle$ BOA $= \frac{1}{2}$ a rt $\angle = 45°$.

(iii) At the moment of the start of AP, and BQ the $\angle$ between them was 180°, and as they began to move onwards, the $\angle$s between them and AB began to increase, while the $\angle$ made by the conjunction of AP and BQ diminished by the rate of $7\frac{1}{2}° + 3\frac{3}{4} = 11\frac{1}{4}$° per second, and this diminution continues till they become parallel

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PART I.

PAGE 64, [THEOR 22.]

On parallels and parallelograms.

Ex 1, and 2 Solved in the book which see.

Prop No 169

3 Z and Y are the middle points of the two sides AB and AC of the $\triangle$ ABC Join ZY and produce it to V making ZY = YV, join VC.

In $\triangle$s AYZ and CYV, AY = CY, ZY = YV and the $\angle$ AYZ $= \angle$ CYV. $\therefore$ the $\triangle$s AYZ and CYV are congruent

$\therefore$ AZ = CV, and ZY = YV, and the $\angle$ AZY $= \angle$ CVY and they are alternate about ZY $\therefore$ AB is $\parallel$ CV.

But CV is proved = AZ = BZ

$\therefore$ ZV is also = and $\parallel$ BC. [Theor 20]

But ZV is double of ZY, because ZY = YV.

$\therefore$ ZY is half of BC.

Prop No 170

4. In the ex 3 above it has been proved that ZY is = half BC = BX and parallel to BC The st lines BZ and YX join the extremities of two = and ∥ st lines ZY and BX, are themselves = and ∥ [Theor 20]

∴ ZYXB is a parallelogram and ZX is its diagonal.

∴ the △ ZBX = △ ZYX

In the same manner the st line ZX which joins the middle points of AB and BC, is also = and ∥ CY, and ZY has been proved = and ∥ XC, ∴ ZXCY is also a parallelogram, and XY its diagonal, ∴ △ CXY = the △ ZXY [Theor 21] ∴ the three △s ZBY, ZXY and YXC, are = to one another, and the △s ZAY and XYC are proved congruent [Ex 1 above] ∴ all the four △s ZAY, ZBY, ZXY and YXC are equal in all respects.

Prop No 171

5 In △ ABC, ZY is the st line joins the middle points of AB and AC ∴ ZY is ∥ BC

From A the vertex draw a st. line AX to the base cutting ZY at O

From O draw OV ∥ AB meeting BC at V.

Because ZO is ∥ BC and AX meets them

∴ the ext ∠ AOZ is = int ∠ OXV [Theor 14]

Again AB is ∥ OV, and AX meet them, ∴ the ext angle XOV is = int. ∠ BAX

∴ in the two △s AZO and OVX, the two ∠s AOZ and ZAO of the one are = the two ∠s OXV and VOX of the other, and the side AZ = the side OV, for AZ = ZB = OV, ∴ the two △s AZO and VOX are congruent, ∴ AO = OX, i e, AX is bisected at O

Prop. No 172.

6. In the two △s BAX and DCY the side AX = CY, and the side AB = DC, and the included ∠ BAX = the included ∠ DCY, for they are the opposite ∠s of the parallelogram ABCD.

∴ The △s BAX and DCY are congruent, and BX = DY [Theor 4] but BX and DY join the extremities of two = and ∥ st. lines XD and BY, they are therefore = and ∥ [Theor. 20]

Now in the $\triangle$ ADP, OX is drawn $\parallel$ the base DP from the middle point X of AD, $\therefore$ OX bisects AP at O, *i. e.*, AO = OP. [Ex. 1 above]

Similarly in the $\triangle$ CBO, PY is $\parallel$ BO from the middle point Y, $\therefore$ OP = PC.

But AO = OP, $\therefore$ AO = OP = PC, *i. e*, AC is divided into three equal parts.

Prop No 173.

7. ABCD is a quadrilateral figure, and E, F, G, H are the middle points of AB, BC, CD and AD respectively. Join EH, EF, FG, and GH.

Then EFGH is a parallelogram.

Join AC. In the $\triangle$ ABC, E and F are the middle points of AB and BC, $\therefore$ EF is $\parallel$ AC the base so also in the $\triangle$ ADC, GH is parallel to AC.

$\therefore$ EF is parallel to GH.

In the similar manner it can also be proved that EH is $\parallel$ to FG.

$\therefore$ the figure EFGH is a parallelogram.

Prop. No 174.

8. Since EFGH is a parallelogram as proved in the last preceding exercise 7. EG and FH are the diagonals of the parallelogram.

$\therefore$ they, *i e*, EG and FH bisect each other. [Cor. 3, Theor. 21]

Prop No. 175.

9. There can be two cases of this exercise (*i*) in which A and B points lie on the same side of CD, and (*ii*) where A and B points are on opposite sides of CD.

Cons —From A draw AX′Q′ $\parallel$ CD meeting OX and BQ at X′ and Q′ in (*i*) and OX and BQ produced in (*ii*).

~~Prop No. 176~~

(*i*) In the $\triangle$ ABQ′, O is the middle point in AB, and OX′ is $\parallel$ BQ, $\therefore$ OX′ = $\frac{1}{2}$ BQ. And XX′ = $\frac{1}{2}$ (AP + QQ′).

$\therefore$ OX′ + XX′ = $\frac{1}{2}$ (AP + QQ′ + BQ). OX = $\frac{1}{2}$ (AP + BQ) = $\frac{1}{2}$ (4 2 + 5 8) = 5· cm.

Prop No 177.

(*ii*) OX′ = $\frac{1}{2}$ (QQ′ + BQ) and XX′ = $\frac{1}{2}$ (AP + QQ′).

OX′ − XX′ = $\frac{1}{2}$ (BQ′ − AP − QQ′) or OX = $\frac{1}{2}$ (BQ − AP) = $\frac{1}{2}$ (5 8 − 4 2) = OD 8 cm.

Prop No 178

10 Let AB, CD and EF be three ∥ st. lines, and OPR, and GHK two transversals, cutting the parallels at B, P, R, G, H and K respectively, PH shall be the arithmetic mean of OG and RK

From O draw a st line OXY ∥ GHK, cutting CD and EF at X and Y respectively

Then each of the figures OH and XK is a parallelogram In the $\triangle$ ORY, PX is drawn parallel to RY from the middle point P in OR, for the intercept OP and PR are = by hyp.

$PX = \frac{1}{2}$ of RY and $XH = \frac{1}{2}(OG + YK)$

Hence by adding $PX + XH = \frac{1}{2}(RY + OG + YK)$.

or $PH = \frac{1}{2}(OG + RK)$

$\therefore$ PH is the arithmetic mean of OG + RK.

Prop No 179

11 ABCD is a trapezium of which the sides AD and DC are parallel, and the st line EF is drawn joining the middle points E and F in AB and DC respectively

Then EF shall be ∥ to AD and BC, and EF shall be equal to $\frac{1}{2}(AD + BC)$, AD = a cm , and BC = b cm.

EF shall be $= \frac{1}{2}(a + b)$

From D draw DG ∥ AB meeting EF at H, and BC at G Then the figure ABGD is a parallelogram. Because in the $\triangle$ DCG, from the middle points F and H in DC, and DG, the st line FH is drawn, $\therefore$ FH is ∥ and $= \frac{1}{2}$ CG and $EH = \frac{1}{2}(AD + BG)$ Adding these together $FH + EH = \frac{1}{2}(CG + AD + BG) = \frac{1}{2}(AD + BC)$.

$\therefore$ EF is ∥ AD and BC and is also $= \frac{1}{2}(a + b)$.

Prop No 180.

12 $1a$, $2b$, $3c$, $4d$, and $5e$ are parallels from the points, 1, 2, 3, 4 and 5 in OX meeting OY at a, b, c, d, and e respectively, by measuring the lengths of these parallels with a cm scale they are found as follows —

$1a = 1$ cm , $2b = 1\ 9$ cm $3c = 2\ 8$ cm , $4d = 3\ 8$, and $5e = 4\ 7$ cm

. By adding all these = 14 2 cm , dividing by 5 we get 2 8 nearly which is the length of the $3c$ line

In the trapezium $1a\,e5$, the lines $1a$ and $5e$ are parallels, and the line $3c$ divides the oblique lines 1, 5, and ae into two equal parts, hence as proved in the last preceding exercise

$3c = \frac{1}{2}(1a + 5e)$ or $\frac{1}{2}(1 + 4 \cdot 7) = 2 \cdot 8$ cm If one of the two st. lines OX. OY be divided into any number of equal parts, say, 1, 2, 3, 4, n, $n+1$, . $(2n+1)$, and parallels be drawn from these points to meet the other

∴ the mean ‖ is $= \frac{1}{2}\{1 + (2n+1)\} = \frac{1}{2} \times 2\,(n+1)$ or $(n+1.)$

∴ $(n+1)$th line is the mean

Prop No 181

13. ABCD is a parallelogram, and EF any st line, without the parallelogram, AP, BQ, CR and DS are the perpendiculars drawn from the angular points A, B, C, D to the st. line EF

O is the point where diagonals AC and BD bisect each other, and OX is the perpendicular from O on EF. Since all these perpendiculars are at rt ∟s to EF, ∴ they are parallel to one another.

Now in the trapezium BQSD, a st line OX is drawn from the middle point of one oblique BD ‖ BQ and DS, ∴ $OX = \frac{1}{2}(BQ + DS)$ as has been proved in a previous exercise. Similarly in the trapezium APRC, the middle st. line $OX = \frac{1}{2}(AP + CR)$.

∴ $\frac{1}{2}(BQ + DS) = \frac{1}{2}(AP + CR)$.

∴ $BQ + DS = AP + CR$

Prop. No 182.

14 Let ABC be an isosc △, having $AB = AC$; in the base BC a point D is taken from it DE and DF perpendiculars are drawn on AB and AC respectively, and BG is drawn perpendicular from the ∟ B to AC. Then $DE + DF = BG$.

~~Prop No 183~~

(i) Let the point D be in the base BC

From D draw DH ‖ AC meeting BG at H.

. Then DFGH is a parallelogram, $DF = GH$.

DH is ‖ FG, and BG falls on them ∴ the ext ∟ BHD = the int oppt angle FGH which is a rt. angle.

∴ the angle BHD is also a rt angle

Now in the two △s BHD and BED, the angle BHD = the angle BED, for they are rt angles, and the angle BDH = the angle EBD,

because the angle BDH = the int oppt angle ACB. [Theor 14] And the side BD is common, ∴ the △ BHD = the △ BED, and the side HB = ED [Theor. 17] But DF has been proved = HG

∴ ED + DF = BG.

No-183 (ii) If the point D be taken in the CB produced, and perpendiculars be drawn from it to the sides AB and CA produced as shown in figure (ii) BG = DF − DE. The same construction being made as in figure (i) and GB be produced

Then the two △s BHD and BED are equal. [Theor. 17].

∴ BH = DE But DF = GH. [Theor. 21]

∴ DF = GB + BH or GB + DE.

∴ DF − DE = BG.

Prop. No. 184.

15 ABC is an equilateral △, and D a point within it from which DE, DF and DG perpendiculars are drawn on AB, AC, and BC respectively.

Then the sum of DE, DF and DG is = AP.

Through D draw XDY ∥ BC, cutting AP at O.

Now the △ AXY is an equiangular. [Theor. 14.]

Hence equilateral [F. cor., Theor. 6]

The perpendiculars from the angular points af an equi. △ to the oppt sides are equal.

Now as proved in the last preceding ex. 14, DE + DF = the perpendicular drawn from X on AY = AO, adding DG which is = OP. DE + DF + DG = AO + DG or AO + OP = AP.

Prop. No. 185.

16. AB and CD are two equal and parallel st. lines; EF is another st. line.

From A, B, C and D points AP, BQ, CR and DS perpendiculars are drawn to EF, then the projection PQ shall be = RS.

From A and C draw AG and CH ∥ EF, meeting BQ and DS at G and H respectively.

Because the ∟ BGA = the ∟ DHC, and the ∟ BAG = the ∟ DCH, and the side AB = DC; Ex. 4. Theor. 15. P. 41

∴ the △ ABG = the △ CDH, and side AG = the side CH. [Theor. 17]

But AG = PQ and CH = RS [Theor. 14.] ∴ PQ = RS.

PART I.

PAGE 68, ON LINEAR MEASUREMENTS.

1.

1 25″ in

2 72″ in

3 08″ in

2.

2 68″ in

6 cm 8 mm.

When 1 cm $=0{\cdot}3937''$ in.

Then $\frac{2\ 68''}{0\ 3937}=6\ 75$ cm.

3.

5 7 cm.

or 2 25″ in by measure.

By calculation $5\ 7\times0{\cdot}3937$

$=2\ 244$ inches

4. The line AB represents 3 15″ in

A —————————— B

by measuring it is found 7·93 cm.

or 7 cm 9 3 mm.

∴ by calculation 1 cm $=0\ 39''$ in.

5.

A —— 2 9 cm —— B

6 2 cm

C —————————— D

(i) By measure AB $=1\ 15''$ in.

(ii) „ CD $=2\ 47''$ in

From the (i) case $1''$ in. $=2\ 52$ cm.

„ (ii) „ „ $=2\ 57$ cm.

$2\,\overline{)\,5\ 09}$

average 2 54 cm.

6

3 36″ in represents 336 miles

4 08″ in. represents 408 miles

7. When $1''=$ one kilometre $=1000$ metre

∴ 850 mitres will be represented by $0\ 85''$

2980 mitres will be represented by $2\ 98''$

1010 mitres will be represented by $1\ 01''$

$0\ 85''$

$2''\ 98$

$1\ 01''$

8 When $1'' = 100$ links . 417 links $= 4.17''$
as $0.3937'' = 1$ cm . $4.17'' = \frac{4.17}{.394} = 10.6$ cm
10 cm 6 mm

---

9 1 cm $= 5$ km then 8.5 cm $= 42.5$ km but 1 cm $= 0.3937''$.
8.5 cm $= 3.35''$
$3.35'' = 8.5$ cm

---

10 55 miles are represented by $2.75''$ then $1''$ represents $\frac{55}{2.75} =$ 20 miles the scale is $1'' = 20$ miles
or when $1'' = 2.54$ cms and 20 miles $= 32$ kms 1 cm. represents 12 kms

11 $1'' = 35$ miles, $4.2'' \times 35 = 147$ miles
the distance between Paris and Calais is 147 miles

This distance if expressed in kilometres would be $147 \times \frac{8}{5}$ $= 235.2$ kms
and $4.2'' = 10.668$ cm the scale of the map in metric measure is 1 cm $= \frac{235.2}{10.668} = 22$ kms nearly

12 The distance between Exeter and Plymouth is $37\frac{1}{2}$ miles, represented on the map by $2\frac{1}{2}''$ the scale of the map $=$ $\frac{75}{2} \times \frac{2}{5} = 15$ miles or $1'' = 15$ miles

Distance between Lincoln and York is 88 km or $88 \times \frac{5}{8}$ $= 55$ miles, and 7 cm $= 7 \times 0.3937 = 2.7559''$. $1'' =$ $\frac{55}{2.7559} =$ 19.95 miles or 20 nearly

13 Diagonal scale showing yards, feet and inches

Prop No 186

PART I

PAGE 79, PROBLEMS 1 – 7

Lines and Angles

Prop No 187 fig. P. 34

1 Prop No 188.

2 The angle ABC is a rt angle which is divided into three equal parts by the st lines BO and BP Dividing again the CBP and PBO into two equal parts, the angle $CBX = 45°$, which in turn is trisected by the st lines BX and BY

Prop No. 194

8. AB is a given st line, and P a given point, it is required to draw a line PQ making with AB an ∠ equal to a given ∠

From P draw PS ∥ AB At the point P in PS make an ∠ SPQ = the given ∠, PQ meeting AB or AB produced if necessary at Q Then because PS is ∥ AB and PQ meets them.

the ∠ SPQ = the alternate ∠ PQA [Theor 14]

∴ PQ is drawn inclining to AB at an ∠ equal to the given ∠

Prop No 195.

9 In the two △s PHK, and P'HK, the side PH = HP' (cons) and HK is common, and the ∠ PHK = the ∠ P'HK, for they are rt ∠s the △ PHK = the △ P'HK in all respects ∴ the ∠ PKH = P'KH But the ∠ P'KH = QKB [Theor 3]

∴ the ∠ PKH = the ∠ QKB

i e, the st. lines PK and QK make equal ∠s with AB

Prop No 196

10 P is a given point, and A and B two other points It is required to draw a st line from P so that the perpendiculars drawn from A and B on that line may be equal

(i) Join PB and AB, and at the point P in the st line PB make an ∠ BPQ = the ∠ PBA [Prob 5]. Then PQ shall be the required line From A and B draw AO and BR perpendiculars to PQ Then because AB ∥ PQ and AO and BR are at rt ∠s to PQ, making the angles AOR and BRO = two rt ∠s. [Theor 13]

∴ AO is ∥ BR And the figure ABRO is a parallelogram.

∴ AO = BR [Theor 21]

(ii)(a)

PART I

PAGE 84, PROB 8—10.

Graphical Exercises.

Prop. No 197

1 ABC is the required △

AD is the perpendicular from A on BC = 4 3 cm nearly.

BE " " " B on AC = 6·1 cm "

CF " " " C on AB = 5 2cm. "

2 Prop No 198

$BX = 1{\cdot}57''$ nearly $\therefore \frac{BX}{CX} = \frac{1\ 57''}{1\ 44''} = 1\ 09''$

$CX = 1{\cdot}44$ " and $\frac{c}{b} = \frac{2\ 75''}{2\ 5''} = 1{\cdot}1$.

3. Prop. No 199.

AC = 210 yds.

4 Prop. No 200.

The $A = 180° - (47° + 68°) = 65°$

By measurement the approximate size of $AB = 77m$, and $AC = 62m$. $AD = 58m$

5 The yatch steers 9 knots in 1 hr. or 60 mts. fig. P. 38

$\therefore$ its motion in 20 mts $= \frac{1}{3}$ of $9 = 3$ kts.

" 35 mts $= \frac{35 \times 9}{60} = 5\ 25$ kts

Her distance from A the harbour is 6 5 knots, and in order to run home she must steer $45° + 30° = 75°$ South of East or $15°$ Eastward from the South

Prop. No. 201.

6 The third side $b = 9{\cdot}05$ cm.

$\sqrt{c^2 - a^2} = \sqrt{(c-a)(c+a)} = \sqrt{5 \times 16{\cdot}2} = \sqrt{81} = 9$ cm.

Prop No 202.

7. The third side has got two values as given below with corresponding values of the $\angle$ C

(i) $a = 4\ 4$ cm. $\angle c = 118°$	(ii) $a = 9{\cdot}5$ cm. $\angle c = 62°$

$\therefore$ The two values of the $\angle$ C are supplementary.

8. Prop. No. 203. Prop No. 204 Prop. No 205. Prop. No. 206.

(iv) This case is impossible for a is less than the perpendicular from C on AB, which measures about 4 8 cm.

Prop No. 207 (Scale $1'' = 100$ yds)

9 The distance between the rods at A and bridge at C is 380 yds. by measurement

Prop. No 208

10. BC is the base = 4 cm. Bisect BC at D, from D draw DA at rt. $\angle$s to BC make DA = 6·2 cm. Join AB and AC.

Then ABC is the required $\triangle$. Because BD = DC, and AD is common, and the included $\angle$s ADB and ADC are equal.

$\therefore$ AB = AC. [Theor 4]

Prop No 209

11 Let A be the given st line and O the given vertical ∟, it is required to draw an isosc △ having its vertical ∟ = ∟ O, and the altitude = st line A

Take any st line CD, and a point P in it From the point P draw RP a st line at rt ∟s to CD, and make PR = st line A

Bisect the ∟ O (Prob 1) A the point R in PR make an ∟ PRX = ½ the ∟ O, the arm RX meeting CD in X [Prob 5]

Similarly make the angle PRY = ½ the angle O, on the other side of PR

The figure XRY is the required △

The ∟ PRX = the ∟ PRY, and the ∟ RPX = the ∟ RPY, and PR is common, ∴ RX = RY [Theor 17]

Prop No 210

Follow the same construction with the exception that the altitude is 6 cm. and the vertical ∟ = 60°. Each of the sides of the equi △ = 7cm.

12 Prop No 211

13 Prop No 212

Let P be the given altitude from the ∟ A on BC, and L and M the given ∟s, it is required to draw a △, having ∟ B = the ∟ M, and the ∟ C = the ∟ L, and altitude = st line P.

Take any line EF, and a point D in it

At D in EF draw DA at rt ∟s to EF, making AD = the st. line P At the point P make an ∟ DAB = to the complimentary ∟ of M (or 90° - ∟ M), the arm AB meeting EF at B

Similarly make the ∟ DAC = the ∟ (90° - L) [Prob 5] Then ABC is the required △ In the △ ADB, the ∟ at D is a rt ∟. [Const] the ∟s DAB and ABD are = to one rt ∟. [Theor 16]

But the ∟ DAB = the ∟ L (const)

the ∟ ABD = the ∟ M

Similarly it can be proved that the ∟ ACD = the ∟ L

Prop No. 213 Prop No 214

14. B and C are the given ∟s, and b one side Take a st line EF, and a point C′ in it At C′ make an ∟ B′C′A = the given ∟ C, and make C′A = the given st line b Now at the point A in

C'A make an ∟ CA'B' = the ∟ (180° − B − C) or the ∟D supplementary to ∟s B and C The arm AB meets EF at B.

Then AB'C' is the required △ of which the side AC' = b given side, and the ∟ C' = the ∟ C, and the ∟ B' = 180° − the ∟C = the ∟ C'AB' or the ∟ B

Prop No 215 Prop. No. 216

15 Produce one arm of the ∟ L, the ext ∟ thus formed is, the supplementary ∟ M = 180° − L Bisect the ∟ M

AC is the base of an isosc △ and the ∟ L is the vertical ∟ of that △ It is required to describe that △ at the point C in AC make an ∟ ACB = $\frac{1}{2}$ the ∟ M or half the supp ∟ of L [Prob 5.]

In the same manner make the ∟ CAB = $\frac{1}{2}$ ∟ M and let the two arms AB and CB meet at B Then the △ ABC is the required one, and the vertical ∟ ABC = the given ∟ L. For the three angles of the △ ABC = two rt. angles

But by construction the angles BAC and BCA = the angle M = 180° − L ∴ the remaining angle ABC = 180° − M = angle L

Prop No 217

16 Take a st line BD = 7 3 cm = $a+b$ At D make an angle BDA = 45°, and from the centre B at a distance BA = 5 3 cm, draw an arc cutting AD in A, and from A draw AC at rt. angles to BD Then ABC is the required △ Since in the △ ACD, the angle ACD is a rt angle (Cons.) and the angle ADC = 45°, ∴ the angle DAC = 45° ∴ AC + CD [Theor 6]

By measuring CD or AC is found = 2 8 cm and BC = 4 5 cm., i e, BD = $a+b$ = 4 5 + 2 8 = 7 3 cm

$\sqrt{a^2+b^2} = \sqrt{4\ 5^2+2\ 8^2} = \sqrt{28\ 09} = 5\ 3$ cm. = AB.

Prop No 218

17 Draw a st. line EF = $a+b+c$ the perimeter = 12 cm. At the point E in EF make an angle FEG = the angle B = 70°, [Problem 5] Similarly make the ∟ EFH = 80° or ∟ C at F. Now bisect the angles FEG and EFH by the straight lines EA and FA which meet when produced at A [Prob 1] From the point A draw AB ∥ EG and AC ∥ FH and meeting EF at B, and C respectively Then ABC is the required △. The ∟ AEG = the ∟ EAB and the ∟ AFH = the ∟ FAC, for EG ∥ AB, and FH ∥ AC.

[Theor 14] But the $\angle$ AEG = the $\angle$ AEF, and the $\angle$ AFH = the $\angle$ AFE (Const) $\therefore$ the angle EAB = the angle AEF or AEB and angle FAC = the angle AFC, and therefore EB = AB and FC = AC. (Theor. 6) $\therefore$ the three sides AB, BC, and CA are = EB, BC and CF or $a+b+c$ the perimeter. AB being $\parallel$ EG, and AC $\parallel$ FH, the ext angle ABC = the int oppt angle BEG, and the ext angle ACB = the int and oppt angle HFC But these angles at E and F are = 70° and 80° respectively

$\therefore$ the angle ABC = 70° and the angle ACB = 80°.

By measuring AB = c = 4 8 cm.

BC = a = 2 6 cm

AC = b = 4 6 cm.

Prop No 219. Prop No 220.

Draw a st line CD = $b+c$ = 10 cm, and at the point C make an angle DCB = 60° and make the arm CB = 6·5 cm Join BD. At the point B in BD, make an angle DBA = the angle BDC, the arm BA meeting CD at A. [Prob 5] Then ABC shall be the required $\triangle$. Since the angle DBA = the angle BDA, BA = AB (Theor 6) CD = $b+c$ $\therefore$ CA + AB = $b+c$, and CB = 6 5 cm and the angle C = 60. Hence ABC is the required $\triangle$.

Prop No 221.

BD = $c-b$ = 1 cm, at the point B in BD make the angle DBC = 55°, and make BC = A = 7 cm Join CD, and at the point C in CD make an angle DCA = to the ext $\angle$ CDA of the $\triangle$ CBD, and let CA and BD produced meet at A. Then ABC is the required $\triangle$. As the $\angle$ ACD = the $\angle$ ADC [Const]

$\therefore$ AC = AD [Theor 6]

But $c-b$ = 1 cm. Add AC = AD = b to both $c-b+b=1+b$

$\therefore$ $c=1+b$ or AB.

By measuring AC or b = 7 cm

$\therefore$ C = 7 + 1 = 8 cm.

PART I.

PAGE 89

Construction of Quadrilaterals.

Prop No 222.

1. PQ is a given st. line. It is required to describe a rhombus

each of whose sides is = PQ Take a st. line BC = PQ

Describe on BC an equi. $\triangle$ ABC [Prob. 8]

From the point A draw AD ∥ BC, and from C draw CD ∥ AB, meeting AD at D [Prob 6] AD is ∥ BC and AC meets them. ∴ the angle DAC = the angle ACD In the same manner the angle BAC = the angle ACD. [Theor 14] and the angle ADC = the angle ABC [Theor 21.] But each of the angles ABC, BAC, and ACB, being an angle of the equi. $\triangle$. is = 60°.

∴ each of the angles CAD, ADC, and ACD is also = 60°.

Hence the angles ABC and ADC of the rhombus ABCD are equal and each of them is 60° while the remaining two equal angles are = 360° − 120° = 240°, or each of them is = 120°.

Prop No 223

AB is the given st line of 2 5" inches The construction is the same as given in Prob 13

Join AC and BD In the two $\triangle$s DAB and CBA, the sides AD and AB are = sides CB and AB and the angle DAB = the angle CBA for they are rt angles. ∴ DB = AC [Theor. 4.]

By measurement AC = BD = 3 54" nearly

Prop No 224.

AB = 3" is the diagonal. Bisect AB at O. From O draw CD at rt angles to AB. and make CD = OD = AO or OB. Join AD, AC, BC and BD. Then because in the $\triangle$AOC, AO = OC ∴ the angle ACO = the angle CAO. [Theor. 5] and the angle AOC is a rt. angle, therefore each of the angles ACO and CAO is half a rt. ∟.

In the same manner it can be proved that each of the angles ADO, DAO, DBO, BDO. CBO. BCO is half a rt. angle.

∴ each of the four angles A, D, B, and C is a rt angle.

Now in the two $\triangle$s ACO and BCO. the sides OA, OC and OB are = one another. and the angle AOC = the angle BOC, for they are rt. angles

∴ AC = BC [Theor. 4.] In the same manner it can be proved that AC or BC is equal to each of the sides BD and DA. Hence the figure is equilateral, it is also proved rectangular.

∴ ACBD is a square and it is described on AB a diagonal

By measurement each of the sides AC, CB, BD, and AD = 2 13" nearly

4 Make the side AB = 5 5 cm Bisect both the diagonals BD and AC. From the centres A and B and with radius equal to half AC = 3cm and BD – 4cm respectively, draw arcs cutting each other at O Join AO and OB Produce AO to C, making OC = AO , BO to D making OD = BO Thus AC = 6cm and BD = 8cm Join CD. Then CD is = and ∥ AB

Prop No 225

In the two △s OBA and ODC, the two sides OB and OA of the one are = two sides OD and OC of the other, and the included angle BO = the included angle DOC

∴ the △ OBA = the △ ODC in all respects and side AB = side DC, and the angle OBA = the angle ODC, and the angle OAB = the angle OCD and they are the alternate angles

∴ AB is also ∥ CD [Theor 13]

Now join CB and DA Then because AB is proved = and ∥ CD, ∴ CB is also = and ∥ DA [Theor 21] ∴ ABCD is a parallelogram having the diagonals AC = 6 cm , and BD = 8 cm

By measurement AD = 5 cm nearly

Prop No 226 fig P46

5 Place the equal diagonals AC and BD in such a way that they bisect each other at O, and make vertically opposite angles AOB and COD = 60° Join AB and CD, as AO = BO = OC = OD for they are the halves of equal diagonals

∴ each of them is = 3 cm and the angle AOB = the angle COD = 60° and the angle OAB = the angle OBA, and each of them is therefore = AOB the △ AOB is equilateral In the same manner the △ COD is also equilateral As the sides of these two are equal, DC is = and ∥ AB

Join now AD and BC The sides AD and BC join the two = and ∥ st lines, they are also = and ∥ [Theor 20]

The angles COD + DOA are = two rt angles, but the angle COD = 60 (hyp) the angle DOA = 180 – 60 = 120°

Again OD = OA, angle ODA = angle OAD = $\frac{1}{2}$ (180° – 120°) = 30°

But the angle OAB $= 60°$. ∴ the angle DAB $= 90°$ or a rt. angle. ∴ the parallelogram ABCD is a rectangle [Cor 1, Theor. 21]

Perimeter $= 2\,(AB + AD) = 2\,(5.2 + 3) = 16.4$ cm

If the angle between the diagonals be increased from 60° to 90° the diagonals would bisect each other at right angles, and the parallelogram will assume the form of a square, whose perimeter will be $= 4 \times \sqrt{\frac{36}{2}} = 4 \times 4.24 = 16.96$ cm.

The excess above the former $= 16.96 - 16.4 = 0.56$ cm

∴ Percentage of excess $= 3.4$ cm

Prop No 227.

6 Only the four sides of a quadrilateral do not determine the exact shape of it With the value of the four sides given in the exercise a series of figures can be drawn two of which ABCD and ABC′D′ are given in the accompanying diagram. In order to determine the exact shape of a quadrilateral it is therefore necessary that either one of the angles or the diagonal be given

At the point A in the given st line AB $= 5.6$ cm make an angle BAD $= 60°$, and cut off AD $= 3.3$ cm Then from the points D and B and at the radius 4 cm. and 2.5 cm respectively draw arcs cutting each other at C, then join DC and CB Then ABCD is the required figure having the angle A $= 60°$. In the same mannar the figure ABC′D′ can be described with the angle A $= 30°$.

By increasing the angle A to 100° the position of the line AD will be given in the figure by AD‴, and then the distance between D‴ and B would become greater than the sum of the two sides BC + CD $= 6.5$ cm, and the construction fails.

In the same manner if the value of the angle A continues to decrease the two lines AD and CD at one position become a st. line as shown by the dotted line AC in figure The value of the angle A at this position is 17° nearly, and the construction fails Similarly when AD becomes at rt angles to AB, the two sides BC and CD become a st line as shewn by BD'', the construction fails.

∴ The construction of this figure is only possible so long as the value of the angle A remains between 17° to 90°

Prop No 228

7. Draw the diagonal $BD = 2.6''$, and from the points B and D

with the radius 3" and 2 8" respectively draw two arcs on the same side of BD cutting each other at A Join BA and DA In the manner with radius equal to 1 7" and 2 5" respectively draw two arcs on the opposite side cutting each other at C. Join BC and DC

ABCD is the required figure with BD as diagonal

The condition necessary to make the construction possible, is that the diagonal must be < the sum of the two sides on each side of it, otherwise the construction must fail.

The diagonal AC = 4 2" nearly by measure

(ii) Prop No 229.

Describe the figure ABCD, about the diagonal AC in the manner given above in (i)

By measuring with protractor

the angle ABC = 90°

and the angle ADC = 90°

PART I.

Page 94

On Loci.

Prop No 230 fig. P48

1 Let ABC be a circle, it is required to find the locus of a moving point P so that its radial distance from the circumference ABC be constant Find O the centre of the circle ABC, and join OP.

Now from the centre O and radius = OP discribe a circle PQR

Then because every point in the circumference PQR is equidistant from O, and so every point in the circumference ABC is also equidistant from O

∴ Every point on the circumference PQR is equidistant from the circumference ABC, i e, to whatever position the point P may move on the circumference PQR, it is always at a constant distance from the circumference ABC

The circumference PQR is the locus of the moving point P.

Prop No 231

2. For construction and proof see ex. 6 on Problems 1–7, p. 79.

Prop No 232

3. A and B are the two points within the circle PQR. Join AB, and bisect AB at O From O in AB draw another line ROP at rt angles to AB and meeting the circle PQR at R and P. Join AR, BR, AP and BP.

Then because AO = OB and OR is common and the angle AOR = the angle BOR ∴ AR = BR [Theor 4]

Similarly AP = BP.

There are only two points

4 (i) Prop No. 233 (ii) Prop. No 234

This exercise can have two form —

(i) When AB is ∥ CD. Take any transversal EF, meeting AB and CD at E and F. Bisect EF at O, and from O draw a st line ∥ AB and CD meeting RQ, produced if necessary, at P. Then P is the position equidistant from AB and CD.

From P draw PG and PH perpendiculars to AB and CD

Then PG = PH (For proof see solution of exer. 9 under Theor 17, page 49)

(ii) When AB and CD are not ∥, let them meet at O when produced Bisect the angle AOC by OP [Prob 1.] meeting RQ, produced if necessary at P. From P draw PG and PH perpendiculars to AB and CD produced Then the two △s OPG and OPH being equal in all respects [Theor 17.] PG = PH. Hence P is the position required

Prop No 235.

5. A and B are the two fixed points From the centre A with a radius 4 cm. describe an arc POR, and from the centre B with a radius = 5 cm describe another arc PSR, cutting the former at P and R Because any point P or R moving along the arc POR is 4 cm. from A. In the same way any point P or R moving along the arc PSR is 5 cm from B

∴ The two points P and R where two arcs cut each other are 4 cm. and 5 cm. from A and B respectively.

Prop No 236

(i) Let AB and CD be not parallel Draw two st lines EF and GH ∥ AB each on one side of it at a distance of 3 cm In like manner draw EG and FH ∥ CD on each side at a distance of 4 cm, and let these four st lines when produced meet at E, F, G, and H. These four points are at the distance of 3 cm from AB or AB produced, and of 4 cm from CD or CD produced Let fall perpendiculars EQ, FP, GR and HO from E, F, G, H on AB produced if necessary. Then EQ = FP = GR = OH = 3 cm Similarly perpendiculars ES, FZ, HY and GX on CD produced are equal to one another, ∴ ES = FZ = HY = GX = 4 cm

(ii) When AB is ∥ CD the construction fails

Prop No 237

7 AB and AC are two rulers placed at rt angles at A, and a rod AX slides on the pivot A, between AB and AC Bisect AX at P. By sliding AX from AB to AC, the point P discribes an arc OPR Then this arc is the locus of P, as all the angles at A = 4 rt angles, and the angle BAC = one rt angle

∴ the arc OPR is one fourth of the circle that can be drawn from the centre A and any radius AP

Prop No 238

8 Let AB be the hypotenuse of the rt angled △s ACB, ADB and AEB on AB as their common base Bisect AB at O Join OC, OD and OE Then AO = CO = OD = OE = OB. Ex. 10 - P 47

∴ a circle described from O as centre with the radius = AO will pass through C, D, E and B ∴ the locus of the vertices of the rt angled △s ACB, ADB and AEB is the semicircle ACDE.

Prop No 239

9 Let X, X′ and X″ be the three positions of moving point X on the fixed st line BC P is the middle point in AX, and P′ and P″ are the middle one in AX′, and AX″ Join PP′ and P′ P″

Then the line PP′P″ is the locus of the middle point P. In the △ AXX′, PP′ is the line joining the middle points of the sides AX and AX′, ∴ PP′ is ∥ XX′ [Ex. 2, Theor. 22, p 64]

In like manner $P'P''$ is $\parallel X'X''$ $\therefore PP''$ is a line $\parallel XX''$, and hence the locus of the middle point P is the line $\parallel$ BC.

Prop No 240. (i), (ii), (iii)

10 There are three cases, (i) the fixed pt. A is on the circ of the given circle, (ii) the said point A is within the circle and (iii) the pt A is out of the circle. C is the centre of the circle, and X, X′, X″ and X‴ are points in the circumference where the pt X comes by moving In the (i) case A and X coincide Join A with these points, C the centre lies in the line joining A and X″. Now bisect these lines AX, AX′ AX″ and AX‴, at P, P′, P″, P‴ respectively. Bisect the line PP″ at O If from the pt O as centre and with the distance OP or OP″ a circle is drawn the circumference of it passes through P, P′, P″ and P‴ the middle points of the st lines AX, AX′, AX″, and AX‴. The circumference of this circle will touch the given circle in case (i), and in case (ii) it will pass between A and X, and will remain within the given circle, while in the (iii) it will pass between the pt. A and the given circle cutting the latter in two points.

Prop. No 241 Prop. No. 242.

11 Bisect AB at O, and AX at P Join OP. Then OP is $\parallel$ BX. BX revolves about B, and so traces out the circle X, X′, X″. At whatever points X′ or X″ the moving point X reaches in the revolution AX always remains at rt angles to BX. The middle point P in AX always remains at a distance = PX, and consequently traces out a circular course PP′P′ $\parallel$ the course of X round B.

Hence the locus of the middle point P in AX is a circle $\parallel$ the circle XX′X″.

12 Prop. No. 243 (v) Prop No 244 (ii)

(i) P is the given point from which PM and PN perpendiculars are drawn on OX & OY respectively. From OX & OY cut off $OS = OS' = PM + PN = 6$ cm Join SS′, which is the locus of the point so that PM + PN is always constant.

SOS′ is an isosc $\triangle$, $\therefore$ the angle $OSS' =$ the $\angle OS'S$ and each of them is = half a rt. angle.

$\therefore SM = PM$ and $PN = NS'$. But $PN = OM$ and $PM = ON$ for they are the opposite sides of a rectangle.

$\therefore PM + PN = SM + OM = NS' + ON = 6$ cm.

Similarly, by taking a point P' in SS', and drawing $P'M'$ and $P'N'$ perpendiculars to OS, and OS', it can be shewn that $P'M' + P'N' = OM' + SM' = ON + N'S' = OS$ or $OS' = 6$ cm

$\therefore$ SS' is the locus of the point P so that $PM + PN = OS$ or $OS' = 6$ cm. Constant

(ii) In this case the constant $PM - PN = 3$ cm From the side OX cut off $OS = PM - PN = 3$ cm At the point S in SX make an angle $XSP = 45°$ SP is the locus of the point P. From P draw PM and PN perpendiculars to OS and OS' respectively. Then $SO = OM - MS = PN - PM$ or $PM - PN = -3$ cm Constant

Similarly we take another point P' in SP and let perpendiculars $P'M'$ and $P'N'$ fall on OX and OY respectively Then $OS = OM' - M'S = PN - PM = 3$ cm

13 Prop No 245.

(i) Take any point M in OX, and cut off $ON = 2$ OM From the points M and N draw perpendiculars MP and NP meeting each other at P. Join OP, then OP is the locus. As MN is a rectangle, $OM = PN$ and $ON = PM$. But $ON = 2$ OM. $\therefore$ $PM = 2$ PN.

Prop No 246.

(ii) Similarly to the above case (i), make $ON = 3$ OM', and draw perpendiculars PN and PM, meeting at P Join OP.

Then OP is locus of the point P so that $PM = 3$ PN.

14. Prop No 247.

Let BC and DE be the two given ∥ st. lines and A a given point, and F the given distance, it is required to find point or points at a given distance from the given point A and at an equal distance from the two ∥ st. lines

Prop. No 248.

The position of the point A admits three cases, (i) when A is out of the ∥ sides, (ii) when A lies between the ∥ lines, and (iii) when A is on one of the lines.

Prop No 249

From A draw AG perpendicular, if the position of A so admits, to *BC*, and produce AG or GA, as the case may be, to meet DE at

H Bisect GH at O, and from O draw POQ ∥ BC or DE [Prob 6.]

From A as centre with a radius = F draw an arc MXN cutting PQ at M and N Then the points M and N are at a given distance F from the point A, and at an equidistance from BC and DE.

In the case when the given distance F is greater than the perpendicular AO from A on PQ, there are always two such points. But when F is = AO, there is only one point and that is O which at a given distance from A and midway between the two parallels BC and DE When F is less than AO this problem becomes impossible.

15. Prop No 251.

Let S be the given point and MX the given st. line and the perpendicular SO from S on MX = 2″.

Produce OS to P and make OP = $2\frac{3}{4}$″.

From P draw a st line QPR ∥ MX [Prob 6]

From the centre S with a radius = $2\frac{3}{4}$ draw arcs to cut QPR at P and R. Join SQ and SR Then Q and R are the two points which are at a distance of $2\frac{3}{4}$″ from S and also a distance OP = $2\frac{3}{4}$″ from MX the given st. line

16 Prop No 252.

MX is the given st line and S the given point From S draw SO perpendicular to MX and produce OS to Y Bisect OS at P. Then the point P is the vertex of the curve

Below this point P draw a series of st. lines all ∥ MX from points 1, 2, 3, 4, 5, 6, &c on PY. Now from the centre S with the radius = 01, 02, 03, 04, 05, 06, &c, draw arcs, cutting parallels drawn from the points 1, 2, 3, 4, 5, 6, &c, respectively on both sides of OY, at P^1, P^2, P^3, P^4, P^5, P^6, &c These points P, P^1, P^2, P^3, &c, &c, are equidistant from the point S and the st line MX. Join these points and there will be a curve which is called a parabola having MX for its axis and the point S for its focus

Prop No 253.

17. Let AB be the base, C the altitude and DE the given st line. At B in AB draw BF at rt angles to AB, making BF = C. From F draw FG ∥ AB [Prob. 6] meeting DE, and DE produced at G. Join AG and BG. Then AGB is the required △, of which

AB is the base and FB = C the altitude, and the $\llcorner$ AGB on the st line DE

Prop No 254.

18. ABC is a triangle Bisect the $\llcorner$s B, A, C by st lines BO, AO and CO All these bisecting lines meet at the point O [Ex II, page 96]. From this point O, draw OP, OR, and OQ, perpendiculars to BC, AC, and AB respectively Then OP = OR = OQ

In the two $\triangle$s OBP and OBQ, the $\llcorner$ OBP = the $\llcorner$ OBQ and the $\llcorner$ OPB = the $\llcorner$ OQB, and OB being common, then the $\triangle$ OBP = the $\triangle$ OBQ, and OP = OQ

In the same manner it can be shewn that OP = OR.

OP = OR = OQ

Prop No. 255 (i) Prop No 256 (ii)

(i) Take points Q′ and R′ in OX and OY respectively so that OQ′ = OR′ = ½ (OQ + OR) Join Q′R′ Then Q′ R′ is the locus of the middle point P of QR Draw PS and PT perpendiculars to OX and OY respectively The $\triangle$s QSP and RTP are cougruent and QP = PR

Similarly by taking Q″ and R″ points in OX and OY, it can also be proved that Q″P″ = P″ R″ when OQ″ + OR″ = OQ + OR = constant

(ii) From OQ cut off OQ′ = OR − OR Bisect OQ′ at S at S in QS make an angle QSP = 45° Then the st line SP is the locus of the middle point of QR

Prop No 257 (i) Prop No 258 (i)

Let S and S′ be the two points in PP°, so that PS = S′P′ or SP + SP° or SP + S′P′ = constant 3 5″ Bisect SS′ at 6 or O Take any number of points between SO, and number them 1, 2, 3, 4, 5 They should be close together near S, and the spaces should gradually widen as they approach O Take the distance P 1 in the compasses, and with centres S and S′ describe arcs at P′, P¹ and P², P² on both sides of S & S′ respectively Take the distance P°1 in the compasses, and with centre S′ cross the arcs at P′ and P′, and with the centre S cross the arcs at P², P²,

Take the distance P2 in the compasses, and with centres S and S¹ describe arcs at P³ P³ on both sides of S, and P⁴ and P⁴ on

that of S^1. Take $P'2$ in the compasses, and with centre S cross the arcs at P^4, P^4, and with centre S′ cross the arcs P^3, P^3

Proceed in the manner described above with each of the points 3, 4, 5, and 6 in SO, and then join the intersecting points of arcs. The curve thus sketched is the ellipse.

The intersecting points of the arcs at P′ P^3, P^2, P^4, &c, &c., are the successive places of the point P in its progress round the foci S and S′ so that $SP+S'P=S'P^2+SP^2=SP'+S'P'=S'P^4+SP^4=SP^5+S'P^4= \qquad =3\ 5''$ constant

(ii) Prop No 259

Join SS′ and in the st line SS′ take two points P and P′ such that $SP=S'P'$, and the distance between P and $P=1\ 5=SP-S'P$ Produce SS′ both ways and take any number of points J, L, M, and N in PS produced, and points J′, L′, M′, and N′ in P′S′ produced, so that $SJ=S'J'$, $JL=J'L'$, $LM=L'M'$, $MN=M'N'$

Now with centre S′ or S with a radius $=P'J'$ or PJ, P′L′ or PL, P′M′ or PM, P′N′ or PN describe arcs on both sides of SS′ Again with SS′ as centres with radius $=P'J$ or PJ', P′L or PL′, P′M or PM′, P′N or PN′ describe arcs cutting the former arcs P^2, P^3, P^4 and P^5 on both sides of P′S′, and at P_0, P_1, P_2, P_3 on both sides of PS′ Now join P′ with the points of intersection P^2, P^3, P^4, P^5 on both sides of S′, and similarly join P with points of intersection P_0, P_1, P_2, P_3 on both sides of S Two curves of a peculiar shape will be formed as shown in the diagram one round the point S′ or the other round the point S This kind of curves are called Hyperbolas with S′, S for their foci The property of such curves is that the difference of the distance of any point on the curve from the two foci is constant For example $SP^3-S'P^3=PL'-P'L'=PP'=1{\cdot}5''$ for P′L′ is common

PART I

PAGE 98.

Miscellaneous problems.

Prop No 260 *also Hall*

1 Through A draw DAE ∥ BC [Prob 6] at A in AE make an ∟ $FAE=X$ [Prob 5] Produce FA to meet BC at G.

Then AGC is the required ∟, DE is ∥ BC, and FAG meets them, then the ∟ FAE = the ∟ AGC [Theor 14]

But the ∟ FAE = the ∟ X (const)

∴ the ∟ AGC = the ∟ X

Prop No 261 also Hall

2 From OB the greater arm of the ∟ AOB cut off OC = OA. Join AC From the centres A and C with any radius describe two arcs cutting each other at D Join D with E the middle point in AC Then DE produced will bisect the ∟ AOB

In the two △s AOE and COE, OA = OC (Const) and OE is common, while the base AE = the base CE.

∴ the ∟ AOE = the ∟ COE. [Theor 4]

Prop No 262 also Hall

3 Join OP, and produce it to R making PR = OP From R draw RC ∥ OB meeting AO at C (Prob 6) Join CP and produce it to D meeting OB Then CPD is the required line For CR is ∥ OB, and OR meets them, the ∟ CRP = the ∟ POD. [Theor. 14,] and the ∟ CPR = the ∟ DPO [Theor. 3,] and OP = PR (Const)

∴ CP = PD.

Prop No 263 also Hall

4 Bisect OB at D, and from B draw BE ∥ OC, meeting OA at E. Join ED and produce it to F to meet OC Then EDF is the required transversal Prove in the same way as given in the last preceding Exercise 3

Prop No 264

5 Let A be the given point, BC and DE two ∥ st lines It is required to draw lines from A to DE so that the intercepted parts of them between BC and DE be = the given line F.

From A draw AO perpendicular to BC, and produce it to meet DE at P. O as centre with a radius = F, draw two arcs cutting DE at Q and R Join OQ and OR

From the point A draw AGH and AKM ∥ OQ and OR respectively, meeting or terminate with DE at H and M Then GH = KM = F Because GHQO and KMRO are parallelograms, of which the opposite sides are =, viz, GH = OQ and KM = OR But OQ = OR = F ∴ GH = KM = F

There will be only one solution of this exer if OP = F only touches DE; and when the distance between the parallels or OP is greater than F, there will be no solution.

Prop No 265

6(i) Bisect the angle A by AD, meeting BC at D. Through D draw DE ‖ AB meeting AC at E, and DF ‖ AC meeting AB at F.

Then AEDF is the required rhombus. The side AE = DF and DE = AF, for they are the opposite sides of a parallelogram [Theor. 21] and the angle EAF = EDF. But AD bisects the angles EAF and EDF, ∴ the angle EAD = the angle EDA.

∴ ED = EA. But ED = AF and AE = DF.

∴ AE = ED = DF = AF Hence the figure EAFD is a rhombus.

Prop No 266.

7. AB is the given st line, it is required to trisect it. On AB describe an equil △ ABC Bisect the ∟s A and B by AO and BO. From the point O draw OD and OE ‖ AC and BC respectively, meeting AB at D and E Then the st. line AB is trisected at D and E, OD is ‖ AC, OE is ‖ BC and AB meets them, ∴ the ∟CAD = the ∟ ODE, and the ∟ CBE = the ∟ OED. [Theor. 14] But the ∟ CAB = the ∟ CBA for they are the ∟s of equi △, ∴ the ∟ ODE = the ∟ OED = 60°, ∴ the ∟ DOE = 60°.

∴ OD = OE = DE, again OD is ‖ AC and AO meets them.

∴ the ∟ CAO = the ∟ AOD. [Theor 14]

But the ∟ CAO = the ∟ OAD ∴ the ∟ OAD = the ∟ AOD, and so AD is = DO. In the same manner OE = EB. But OD = OE = DE ∴ AD = EB = DE. ∴ AB is trisected at D and E.

Prop No 267.

8. (i) Let O, P, Q, be the middle points of the △.
Join OP, OQ, PQ.

From the point O draw a st. line AOB ‖ PQ, and from P draw BPC ‖ OQ. Similarly from the point Q draw AQC ‖ OP, meeting AB and BC at A and C respectively

Then the figure ABC is the required △.

Prop No 268 Prop No 269

(ii) Let X and Y be the two sides, and AD the median on the third side From the centre A with a radius $= \frac{1}{2}$ Y describe an arc on one side of AD and from the other point D as centre with a radius $= \frac{1}{2}$ describe an arc cutting the former at E. Join AE and DE, produce AE to C making EC = AE or AC = Y. Join CD and produce it to B From the centre A with radius = X draw an arc cutting CD produced at B Join AB Then ABC is the △ required Bisect AB at F Join EF Then EF which joins the middle points E and F is ∥ BC, and also half of BC [Ex 2 and 3, p 64]

Prop No 270

(iii) P and Q are the two medians and AB is the third side From the centre A with radius $= \frac{2}{3}$ of Q draw an arc, and from B as centre with radius $= \frac{2}{3}$ of P draw an arc cutting the former at O Join OA and OB, and produce AO to D making AD = Q Produce BD to E making BE = P Join AE and BD and produce them to meet at C

Then ABC is the required △

Prop No 271

(iv) P, Q, R are the three medians Draw a △ ODC, with the $\frac{2}{3}$ of the three medians as sides of which OD $= \frac{2}{3}$ of Q, OC $= \frac{2}{3}$ of R, and CD $= \frac{2}{3}$ of P Produce CO to G and make CG = R Produce DO to A making OA = DO Join AG and produce it to B making AG = GB Join BC and AC. Bisect AC at F Join FO and BO By (Theor III, page 96) CG and BF are concurrent, and AE also joins them at O from the ∟ A AE bisects BC.

∴ ABC is the required △

PART II.

Page 101

On Tables of length and area.

Prop No 272

1. (i) Suppose AB = one yard, then ABCD is the sq on AB = one sq yard But AB and AD are divided into 3 equal

parts at a, and b, and 1 and 2. From these points draw ||s to the adjacent sides. As $Aa = \frac{1}{3}$ of AB or one yard = one ft ∴ the figure $1a$ = sq. on Aa = sq on 1 ft. There are such 9 squares within ABCD.

∴ 1 sq yd $= 3 \times 3$ sq. ft.

Prop No 273.

(ii) AB represents one ft, and it is divided into 12 parts. ∴ each of the parts on AB = one inch, and 1, 1 is = one sq inch ABCD = sq on AB = 1 sq ft. There are 12 sq inches in the first row, but there are 12 such rows, ∴ 1 sq ft $= 12 \times 12$ sq. inches or 12^2 sq. inches.

Prop. No. 274.

(iii) Suppose AB represents one cm, then ABCD is one sq. cm. AB is divided into 10 equal parts. So there are 10 rows of 10 sq cm, i. e, 10^2 cm.

Hence 1 sq. cm. $= 10^2$ sq *mm*.

Prop. No. 275.

2 AB is a given st. line, and the figure ABCD a sq on it. Bisect AB at a and AD at b. From a and b draw st. lines || the adjacent sides AD and AB respectively Thus the whole figure ABCD is divided into four minor sqs which are on sides = Aa or Ba, *i e*, half of AB ∴ the sq ABCD on AB = four times the sqs. on Aa, *i e*, half of AB.

Prop No 276

3 ABCD is a sq. described on AB = 1″ AB is sub-divided into 10 parts and so is AD. Hence there are 10×10 small sqrs. within ABCD. But every one of these small sqrs. has for its side one of the parts into which AB is divided, *i e*, $\frac{1}{10}$ of 1″ ∴ the sq on 1″ $= 10 \times 10$ times the sqr. on $\frac{1}{10}$″ or 0 1″.

4. 1″ = 5 miles Hence 1 sq inch = 25 sq. miles. ∴ 6 sq. inches represent 150 sqr miles.

PART II.

Page 102

On area of rectangles.

Prop No 277.

1. $a = 2''$ and $b = 3''$. ABCD is the required figure AB = 3″ inches.

and AD $= a = 2''$ Divide AB into three equal parts, each $= 1$ inches and AD into two parts, each $= 1''$. Now there are two rows each containing 3 sqrs $\therefore$ the rectangle AC contains 2×3 sqrs $= 6$ sqr. inches.

$a \times b =$ area $\therefore 2 \times 3 = 6$ sqr. inches area.

Prop No 278

2. ABCD is the rectangle $AB = 4''$ and $AD = 1.5''$

$\therefore$ the area $= a \times b = 1.5'' \times 4'' = 6''$ sqr inches

AB is divided into 4 parts each $= 1''$, and AD contains one such part and a half $\therefore$ there are 4 sqrs. in the first row along AB, while in the second row there are half squares. Or in other words each part on AB is divided into 10 equal parts, hence there are $10 \times 4 = 40$ equal parts on AB In the same manner AD is divided into $10 + 5 = 15$ equal parts Now there are 15 rows of 40 sqrs in each $\therefore$ the rectangle contains $40 \times 15 = 600$ small sqrs. But a sqr. having its one side $= 1''$ contains 100 such small sqrs

$\therefore$ the rectangle ABCD contains $\frac{600}{100} = 6$ sqr. inches.

Prop No 279.

3. ABCD is a rectangle $AB = 3.5''$ and $AD = .8''$ or $\frac{8}{10}''$

One inch is divided into 10 parts $\therefore$ AB is divided into $10 \times 3.5 = 35$ parts and AD is divided into $.8$ or $\frac{8}{10} \times 10 = 8$ parts.

$\therefore$ the first row along AB contains 35 small sqrs. each side of which $= \frac{1}{10}''$, and there are such 8 rows.

$\therefore$ the figure ABCD contains $35 \times 8 = 280$ small sqrs.

But one small sqr $= (\frac{1}{10})^2$ or $\frac{1}{100}''$ $\therefore$ the whole figure or 280 small sqrs. $= \frac{280}{100} = 2.8''$ sqr. inches

The area $= a \times b = .8 \times 3.5 = 2.8$ sqr inches.

Prop No. 280.

4. ABCD is a rectangle such that $AB = a = 2.5''$, and $AD = b = 1.4''$. Every $1''$ of the squared paper is divided into 10 parts. $\therefore$ AB contains $2.5 \times 10 = 25$ divisions and AD contains $1.4'' \times 10 = 14$ divisions. $\therefore$ the rect. ABCD is divided into $25 \times 14 = 350$ compartments each of which represents $\frac{1}{10}$ square inch. There are 25 rows containing 14 such squares, $\therefore$ the rectangle contains $25 \times 14 = 350$ sqrs. But 100 sqrs make up 1 sqr inch $\therefore$ $\frac{350}{100} = 3.5$ sqr. inches is the area. In other words area $= 2.5 \times 1.4 = 3.5''$ sqr. inches.

Prop. No. 281.

5 In the rectangle ABCD, $AB = a = 2\ 2''$, $AD = b = 1{\cdot}5''$. The rectangle ABCD is therefore divided into compartments each of which represents $\frac{1}{10}$ sqr inch. Now there are $2\ 2 \times 10 = 22$ rows each containing $1\ 5 \times 10 = 15$ sqrs. ∴ the rectangle contains $22 \times 15 = 330$ sqrs, each of which is $\frac{1}{100}$ sqr inch ∴ the figure contains $\frac{330}{100} = 3\ 3$ sqr inches.

The area $= 2\ 2'' \times 1\ 5'' = 3\ 3''$ sqr inches.

Prop. No 282.

6 The side AB of the rectangle $= a = 1\ 6''$, and the side $AD = b = 2\ 1''$. Each inch is divided into 10 equal parts ∴ AB is divided into $1\ 6 \times 10 = 16$ parts, and AD into $2\ 1 \times 10 = 21$ parts. The rectangle ABCD is divided into compartments each of which is $= \frac{1}{10} \times \frac{1}{10}$ sqr inches.

Now there are 16 rows of such sqrs along AB, and 21 rows along AD. ∴ the whole figure ABCD contains $21 \times 16 = 336$ such squares ∴ ABCD contains $\frac{336}{100} = 3\ 36$ sqr. inches. The area $= 2\ 1'' \times 1\ 6'' = 3 \times 36$ sqr inches.

7 The area of the figure is $= ab$

But $a = 18$ metres, and $b = 11$ metres.

∴ the area $= 18 \times 11 = 128$ sq metres.

8 The area of the rectangle is $= ab$

But $a = 7$ ft, and $b = 72$ in or $\frac{72}{12} = 6$ ft.

∴ the area $= 7 \times 6 = 42$ sqr. ft

9. The area of a rectangle $= a \times b$.

But $a = 2{\cdot}5$ km and $b = 4$ metres, as 1 km. is $= 1000$ metres, hence $a = 2\ 5 \times 1000 = 2500$ metres.

∴ The area $= 4 \times 2500 = 10000$ sqr metres.

10. Area $= a \times b$. But $a = \frac{1}{4}$ mile or $\frac{1760 \times 3\ 6}{4}$ inches and $b =$ 1 inch. ∴ the area $= \frac{1760 \times 369}{4} \times 1 = 15840$ sq. inches or 110 sq ft.

11 The area $= a \times b = 30$ sq cm., but $a = 6$ cm. ∴ $b = \frac{30}{6} = 5$ cm.

Below is given the figure

Prop. No. 283.

ABCD is the rectangle of which length $AB = 6$ cm. or divided into 6 parts, 6 if multiplied by 5 produces 30. ∴ there are 5 rows

each containing 6 sqrs there are 30 sqrs each of which is = 1 sq cm.

Prop. No 284. Prop No 285

12 Area = $a \times b$ But area = 3 9 sq in. and breadth $b = 1\ 5$.

$\therefore$ 3 9 sq in = $a \times 1\ 5$ $\therefore$ $a = \frac{3\ 9}{1\ 5} = 2\ 6$ cm.

ABCD is the required rectangle of which length $a = 2\ 6$ cm and breadth $b = 1\ 5$.

But each inch contains 10 parts, so length a contains 26 parts, and breadth b contains 15 parts $\therefore$ there are $26 \times 15 = 390$ compartments in the figure each of which is $\frac{1}{10} \times \frac{1}{10}$ sq in in area

the area $= \frac{390}{100}$ sq inches or 3 9 sq in

13 (i) When the length is tripled without altering the breadth, the area becomes threefold, for the area is repeated three times.

(ii) But when length and breadth both are tripled the area becomes nine times, for we multiply both the dimensions of the figure by 3, which means three rows of squares in the length three times

ABCD is the original figure, but when tripled in one direction it assumes the form and size of AEFD, which contains only three such figures as ABCD. But when this tripled form tripled again in the other dimensions it assumes the form and size of AEGH Thus there are three rows each containing three squares or 9 squares.

Prop No 286. Prop. No. 287

14. ABCD is a plan of a rectangular garden of which AB = a = 3 6″, and AD = b = 2 5″, but each inch = 10 yards AB = 36 yds and AD = 25 yds . The area = $36 \times 25 = 900$ sq yds

Now the area is made = 900 sq yds + 300 sq. yds = 1200 sq. yards, but the breadth remains 25 yds Then $a = \frac{\text{area}}{b} = \frac{1200}{25 \text{ yds}}$ sq. yds. = 48 yds as 10 yds = 1″ $\therefore$ 48 yds. = 4·8″ in our plan.

$\therefore$ The length of the new plan would be represented by 4 8″.

15 The length of the rectangular enclosure = 6 5 cm and the breadth = 4 5 cm But 1 cm. represents 20 metres 6·5 cm. = 6 5 × 20 = 130 metres and 4 5 cm. = 4·5 × 20 = 90 metres.

Hence the area $= a \times b = 130 \times 90 = 11700$ sq. metres.

16 The length, and breadth of a plan are 4·5 cms and 3 2 cm. $\therefore$ the area $= 4\ 5 \times 3{\cdot}2 = 14{\cdot}40$ sq. cms. Thus a plan of 14 4 sq cm, represents an area of 1440 sq yds.

$\because$ 1 sq cm represents $\frac{1440}{14\ 4}$ sq. yds. or 100 sq yds

. 1 cm represents 10 yds. and consequently the scale is 1 cm. = 10 yards.

17. The scale being $1'' = 100$ ft and $1''^2$ sq in $= 100^2$ sq ft

$\therefore$ the area 52000 sq ft. can be represented by 5·2 sq. inches. $\therefore$ area $= a \times b = 5\ 2$ sq in.

But $a = 3\ 25''$ $\quad b = \frac{5\ 2}{3\ 25} = 1{\cdot}6''$

Then the breadth of the plan is $= 1\ 6''$.

18 First neglecting the gap on the upper side of the figure and the lap on the right side, the area of the figure would be $= 20 \times 30 = 600$ sq ft

Now taking the gap the area of which $= 5 \times 10 = 50$ sq ft. and that of the lap being $5 \times 10 = 50$ sq ft $\because$ By subtracting the area of the gap and adding that of the over lap we get the same result, for these areas are equal $\because$ the area of the figure is 600 sq. ft.

19. The area of the gap on the upper side being $24 \times 12 = 288$ sq ft, and that of the extended part on the upper right hand corner being also the same, i. e., $24 \times 12 = 288$ sq ft.

$\because$ by neglecting these equal addition and subtraction the area of the whole figure $= 48 \times 24 = 1152$ sq ft.

20 Area of the whole figure $= 15 \times 10 = 150$ sq ft The area of the rectangular white space $= (10 - 5)\ (15 - 5) = 50$ sq ft This being subtracted from the above area of the figure 150 sq. ft. $-$ 50 sq. ft. leaves 100 sq ft for the area of the shaded part of the figure.

21 The length of the whole figure $= 7 + 4 + 4 = 15$ and the breadth $= 4{\cdot}5 + 4 + 4 = 12\ 5$.

$\because$ the area of the figure $15 \times 12\ 5 = 187{\cdot}5$ sq ft The area of the white space $= 4\ 5 \times 7 = 31\ 5$ sq. ft. which when subtracted from the area of the whole figure 187 5 sq. ft leaves the area of the shaded part $= 156$ sq. yards.

22 The whole length of the figure being 15 ft., from which subtracting the breadth of the shaded part 5 ft. we get the length of the two white parts $= 10$ ft. In the similar way by subtracting the breadth 5 ft from the breadth of the whole figure 12 ft we get the breadth of the white parts $= 7$ ft.

$\therefore$ the area of the four white corners in the figure $= 7 \times 70 = 70$ sq ft

But the area of the whole figure $= 12 \times 15 = 180$ sq ft.

$\therefore$ the area of the shaded cross $= 180 - 70 = 110$ sq ft

23. The length of each of the shaded corners $= \frac{30 - 18}{2} = 6$ feet and the breadth is $= \frac{20 - 8}{2} = 6$ ft.

$\therefore$ the area of the 4 corner squares $= 4 \times 6 \times 6 = 144$ sq ft and the area of the middle shaded portion $= 18 \times 8 = 144$ sq ft which when added to the area of the four shaded corners 144 sq ft gives the area of all the shaded parts $= 144 + 144 = 288$ sq ft

24 The whole figure is a square, its area $= 12 \times 12 = 144$ sq ft But the middle shaded square is half of the whole figure.

the area of the shaded part $= \frac{1}{2}$ of $144 = 72$ sq ft

25 The whole figure is a rectangle The diagonals bisect it the shaded parts are equal to the white portion. Hence the area of the shaded parts $= \frac{1}{2}$ of $(10 \times 15) = 75$ sq feet

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PART II.

Page 105, Theor 24

1 The area of a parallelogram $=$ base $\times$ height

(i) Area $= 5.5$ cm $\times 4$ cm $= 22$ sq. cm

(ii) " $= 2.4'' \times 1.5'' = 3.6$ sq inches.

Prop No 287

2 Make $AB = 2.5''$. At the point A make an $\angle BAD = 65°$ Cut off $AD = 1.5'$ From points B & D draw BC & DC parallel to AD and AB respectively Then ABCD is the required parallelogram.

Draw DE a perpendicular from D on AB. Measure DE which is found to be $1.37''$ nearly $\therefore$ area $= 1.37'' \times 2.5'' = 3.425''$ sq in. approximately, because no perpendicular can be exactly found out without the help of trigonometry and logarithms The perpendicular BF from B on AD is $2.28''$, and $\therefore$ area $= 1.5'' \times 2.28'' = 3.42$ sq in.

$\therefore$ The average of the two areas being $\frac{3 \cdot 425'' + 3 \cdot 42''}{2} = \frac{6 \cdot 845''}{2} =$ 3·4225" sq. in.

Prop No 288.

3 5 metres are represented by 1 cm $\therefore$ by scale 30 mtrs. = 6 cm, and 25 mtrs = 5 cm. ABCD is the parallelogram, AB = 6 cm. and AD = 5 cm while $\angle$A = 50° DE and BF are perpendiculars from D and B on AB and AD respectively. DE = 3 8 cm and BF = 4 6 cm in the plan or 19 metres and 23 metres respectively. $\therefore$ areas are 570 sq metres, and 575 sq. metres Hence average of these two areas = $\frac{570 + 575}{2} = \frac{1145}{2} = 572 \cdot 5$ sq. metres

Prop No. 289

4 Area of a parallelogram = base × height

$\therefore$ height = $\frac{\text{area}}{\text{base}} = \frac{4 \cdot 2 \text{ sq in}}{2 \cdot 8''} = 1 \cdot 5''$

If AD = 2" the parallelogram would be as given in the figure.

Prop No 290

5 Area of a rhombus = base × height $\therefore$ Altitude = $\frac{\text{area}}{\text{base}} =$ $\frac{3 \cdot 86 \text{ sq in}}{2} = 1 \cdot 93''$. Now the adjacent sides and altitude being given, a rhombus can be drawn which is given in the figure ABCD

The acute $\angle$s at A and C = 70°.

PART II.

Page 107, Theop. 25

1. Area of a $\triangle$ $\frac{1}{2}$ × base × height. $\therefore$ area in

(i) $= \frac{1}{2} \times 24$ ft. $\times 15$ ft $= 180'$ sq. ft.

(ii) $= \frac{1}{2} \times 4 \cdot 8'' \times 3 \cdot 5'' = 8 \cdot 40''$ sq in.

(iii) $= \frac{1}{2} \times 160$ mtr. $\times 125$ mtrs $= 10000$ sq metres

Prop No 291.

2 (i) In the $\triangle$ ABC, AD the perpendicular = 4·5 cm.

$\therefore$ area $= \frac{1}{2} \times 3 \cdot 4 \times 8 \cdot 4 = 14 \cdot 28$ sq cm.

Prop No 292.

(ii) The perpendicular on AC = b = 6 1.

$\therefore$ area $= \frac{1}{2} \times 6 \cdot 1 \times 5 = 15 \cdot 25$ sq cm.

Prop No 293

(iii) The perpendicular AD on BC or $a = 6\ 5$ cm.

$\therefore$ area $= \frac{1}{2} \times 6\ 5 \times 6\ 5 = 21\ 12$ sq cm

Prop No 294

3 ABC is a rt $\angle$ed $\triangle$ having C as rt $\angle$ The area of a $\triangle$ $= \frac{1}{2} \times$ base $\times$ height In this $\triangle$ AC is the perpendicular on BC, and $\therefore$ it is the height

$\therefore$ The area of the $\triangle$ ABC $= \frac{1}{2} \times$ BC $\times$ AC now substituting their values The area $= \frac{1}{2} \times 6 \times 5 = 15$ sq cm The hypotenuse AB $=$ C $= 7\ 8$ cm and the perpendicular CD from C on AB or $c = 3\ 8$ cm The area $= \frac{1}{2} \times 3\ 8 \times 7\ 8 = 14\ 82$ sq cm nearly The error is $=$ $15 - 14{\cdot}82 = {\cdot}1\ 8$ sq cm

$\therefore$ The P C of error is $= \frac{100 \times\ 18}{15} = 1\ 2$ sq cm

Prop No 295

4 ABC is a rt $\angle$ed $\triangle$ having a rt $\angle$ at C

The area $= \frac{1}{2} \times 4\ 5'' \times 2\ 8'' = 6'\ 30''$ sq in. The hypt AB $= 5\ 3''$, and the perpendicular CD from C on AB $= 2\ 37''$.

$\therefore$ Now area $= \frac{1}{2} \times 2\ 37'' \times 5\ 3'' = 6\ 28''$ sq in $\therefore$ The error is $=$ $6\ 3 - 6\ 28 =\ 02$ sq in $\therefore$ P. C of error is $=\ 31$ sq in

5 In a $\triangle$, altitude $= \frac{\text{area}}{\text{base}}$ or base $= \frac{\text{area}}{\text{altitude}}$.

$\therefore$ (i) altitude $= \frac{80 \text{ sq in}}{20''} = 4''$ inches

(ii) base $= \frac{10\ 4 \text{ sq cm}}{1\ 6 \text{ cm}} = 6\ 5$ cm.

Prop No. 296.

6 ABC is the required $\triangle$, in which BC $= a = 3''$, AC $= b = 2\ 8''$, and AB $= c = 2\ 6''$ The perpendicular from A on BC $= 2\ 23''$

$\therefore$ the approximate area $= \frac{1}{2} \times 3'' \times 2\ 24'' \times 3\ 36'' =$ sq. in.

PART II.

PAGE 109

On area of a Triangle.

Prop No 297.

1. (i) XY is $\parallel$ BC, and $\triangle$s XBC and YBC are on the same base BC, and between the same $\parallel$s XY and BC.

$\therefore$ the $\triangle$ XBC $=$ the $\triangle$ YBC. [Theor 26]

(ii) The $\triangle$s BXY and CXY are on one base XY and between the same ||s XY and BC.

∴ the $\triangle$ BXY = the $\triangle$ CXY. [Theor. 26.]

(iii) The $\triangle$ BXY is proved to be = the $\triangle$ CXY. Add the $\triangle$ AXY to both

∴ the whole $\triangle$ ABY = the whole $\triangle$ ACX.

(iv) Because the $\triangle$ BXY = the $\triangle$ CXY. From these equals take away the common $\triangle$ XKY, then the remainder $\triangle$ BKX = the remainder $\triangle$ CKY.

Prop No 298.

2 ABC is a $\triangle$ and D the middle point in BC Join AD. Then because BD = DC. The two $\triangle$s ABD and ACD are on equal bases BD and DC, and between the same ||s BC and that drawn through A || BC. ∴ the $\triangle$ ABD = the $\triangle$ ACD [Note Theor. 26]

In order to divide the area of a $\triangle$ into 3 equal parts, the base must be divided into three parts, and the points of section be joined with the vertex. Thus the $\triangle$ will be divided into three $\triangle$s of equal areas

Prop. No. 299.

3 ABCD is a parallelogram, AC and BD are its diagonals intersecting each other at E, and they bisect each other at E. [Theor 21, Cor 3]

AE = EC and BE = ED.

Now the $\triangle$ ABC = the $\triangle$ DBC, for they are on one base BC and between the ||s AD and BC. From these take away the common part BEC

∴ the $\triangle$ AEB = the $\triangle$ DEC

In the same manner it can be shewn that the $\triangle$ AED = the $\triangle$ BEC

But the $\triangle$ AEB = the $\triangle$ AED, for they are on equal bases and between the same ||s. ∴ the $\triangle$ AED = the $\triangle$ AEB or DEC, and hence all the four $\triangle$s AEB, AED, DEC and BEC are equal.

Prop No. 300.

4 Because BX = XC The $\triangle$ BXY = the $\triangle$ CXY. [Theor. 26, Note] And for the same reason the $\triangle$ ABX = the $\triangle$ ACX.

[Theor. 26, Note.] Subtracting the former from the latter the remainders are equal, i e, the △ ABY = the △ ACY.

Prop No 301.

5 The △ ABC = the △ADC [Theor 21] as they are on the same base AC, ∵ their altitudes BP and DQ are equal [Conv. Corol, Theor 24] (i) and (ii) since the △s ADX and ABX on the same base AX, and similarly the △s CDX and CBX on the base CX, and these △s have equal altitudes BP and DQ, ∴ the △ ADX = △ABX, and the △ CDX = △CBX [Cor Theor. 24], whether the point X be taken in AX or AC produced

Prop No 302.

6 ABC is a △ D and E are the middle points in AB and AC Join DE DE shall be ∥ BC Join BE and CD As the median BE bisects the △ ABC, the △ ADC = the △ DBC, and the median DC bisects the same △, the △ AEB = the △ ECB. Half of the same thing are equal ∵ the △ DBC = the △ ECB. But they are on the same base, ∴ DE is ∥ BC [Theor. 27.]

Prop. No 303.

ll 7. ABCD is a trapezium of which the side AD is ∥ BC. E and F are the middle points in the oblique sides AB and DC Join EF. EF shall be ∥ AD and BC. From A draw AH ∥ DC, and cutting EF at G.

As proved in the exer. 11 it can be proved that EF is ∥ AD or BC.

Prop No 304.

8 In the parallelogram ABCD, AD = BC, and the point X bisects AD and Y bisects BC. ∵ AX = BY or CY. ∴ the parallelogram AY = the parallelogram DY.

∵ the parallelogram AY is half of ABCD But the diagonal BX divides AY into two equal parts or bisects it. The △XAB is = the △ ZAB or the △ Z′AB, since they are on the same base AB, and between the same parallels AB and XYZ′.

∴ the △ AZB or AZ′B is also half of the parallelogram AY, i. e, one fourth of the whole figure ABCD.

Prop. No 305.

9. Since the $\triangle$ BYC, and the parallelogram ABCD are on the same base BC, and between the same ∥s AD and BC.

∴ the $\triangle$BYC is half of the parallelogram ABCD. [Theor. 25]

In the same manner the $\triangle$ AXB is half of ABCD.

∴ the $\triangle$BYC = the $\triangle$ AXB

Prop No. 306.

10. Through P draw OPQ ∥ AB and DC. Then because the $\triangle$ APB = half of the parallelogram BO. [Theor 25] and the $\triangle$ DPC = half of the parallelogram CO [Theor. 25] But the two figures BO and CO = the whole figure ABCD.

∴ the two $\triangle$s APB and DPC are together = half the whole parallelogram ABCD.

PART II.

Page 110

On area of Triangles.

Prop No 307.

1. ABC is a plan of a triangular field, AB = 1·9″, AC = 2″ and BC = 3 7″

From A draw AD perpendicular on BC. AD = 0·68. ∴ the area of the plan ABC = $\frac{1}{2}$ × 68″ × 3 7″ = 1·258″ sq inches. The area of the field = $\frac{1}{2}$ × 370 × 68 = 12580 sq yds

Prop No 308.

2 ABC is the plan of a triangular enclosure in which AB = 6·2 cm., BC = 7·2 cm. included ∟ ABC = 45°. AD the perpendicular from A upon BC = 4 4 cm.

∴ the area of the plan = $\frac{1}{2}$ × 4·4 × 7·2 = 15 84 sq cm

And the area of the enclosure = $\frac{1}{2}$ × 144 × 88 = 6336 sq. metres.

Prop. No 309.

3. Area of the $\triangle$ ABC = 6 6 sq cm, and the base BC = 5·5 cm. ∴ the altitude = $\frac{6\,6 \times 2}{5\,5}$ = 2·4 cm. The locus of the vertex A of the $\triangle$ ABC is therefore the line drawn through the point A ∥ the base BC, or a line on either side of BC ∥ it and at a distance = 2 4 cm.

Now in the $\triangle$ ABC, BC = 5 5 cm, BA = 2 6 cm., and the altitude AD = 2 4 cm.

∴ AC = 5 2 cm

Prop No 310

4. The area of the $\triangle$ ABC = 3 06 sq in, and BC = a = 3″. Then the altitude AD = $\frac{3\ 06 \times 2}{3}$ = 2 04″ The locus therefore of A is a st line at a distance of 2 04 cm and ∥ BC

Now in the $\triangle$ ABC, BC = 3″, AD = 2 04″ and the $\angle$ C = 68°

∴ By measurement AC or b = 2 22″

Prop No 311 (*i*), (*ii*), (*iii*), (*iv*), (*v*), (*vi*), and (*vii*)

5 (*i*) When the $\angle$ ABC = 0°, AB and BC coincide, and consequently there is no $\triangle$, and hence area = 0

(*ii*) AB makes with BC an $\angle$ = 30° The altitude AD from A on BC = 2·5 cm

∴ the area of $\triangle$ABC = $\frac{1}{2} \times 2\ 5 \times 6 = 7\ 5$ sq cm.

(*iii*) The $\angle$ ABC = 60°, and AD = 4 3 cm

the area of $\triangle$ ABC = $\frac{1}{2} \times 4\ 3 \times 6 = 12\ 9$ sq cm

(*iv*) The $\angle$ ABC = 90°, *i e*, AB becomes altitude ∴ the area = $\frac{1}{2} \times 5 \times 6 = 15$ sq cm

(*v*) The $\angle$ ABC = 120°, the altitude AD on BC produced = 4 3 cm ∴ the area of the $\triangle$ ABC = $\frac{1}{2} \times 4\ 3 \times 6 = 12{\cdot}9$ sq cm.

(*vi*) The $\angle$ ABC = 150°, and AD = 2 5 cm

∴ the area = $\frac{1}{2} \times 2\ 5 \times 6 = 7\ 5$ cm

(*vii*) Here the $\angle$ ABC becomes 180° or = 2 rt $\angle$s ∴ AB and BC are in one st line, hence no $\triangle$ and area = 0

Angle	0° & 180°	30° & 150°	60° & 120°	90°
Base BC	6 cm	6 cm	6 cm	6 cm
Altitude AD	0	2 5 cm	4 3 cm	5 cm
Area of the $\triangle$ABC	0	7 5 sq cm	12·9 sq cm	15 sq cm.

Theoretically.

Prop No 311.

6 ABC and DEF are $\triangle$s having the two sides AB and AC of the one = the two sides DE and DF of the other, and the $\angle$ BAC supplementary to the $\angle$ EDF Produce BA to G, and make AG = AB. Join GC Then because AG and AC = DE and DF respectively, and $\angle$ GAC = the $\angle$ EDF, for each of the $\angle$s GAC and EDF is supplementary to the $\angle$ BAC $\because$ the $\triangle$ GAC = the $\triangle$ EDF [Theor 4] But the $\triangle$ GAC = the $\triangle$ BAC, because they are on equal bases and between the same parallels [Theor. 26.] $\therefore$ the $\triangle$ ABC = the $\triangle$ DEF.

Such $\triangle$s can be identically equal if the $\angle$s contained by equal sides are rt $\angle$s, i e, the $\angle$s BAC and EDF are rt $\angle$s.

Prop No 312.

7 Let ABC be a $\triangle$, it is required to draw an isosc $\triangle$ on the base BC = the $\triangle$ ABC

Bisect BC at D From D in BC draw DE at rt. $\angle$s to BC. Through A draw AEF $\parallel$ BC meeting DE at E Join EB and EC Then EBC is the required isosc $\triangle$.

Since the $\triangle$s ABC and EBC are on the same base BC, and between the same $\parallel$s BC and AF, $\therefore$ the $\triangle$ ABC = the $\triangle$ EBC. [Theor 26]

Prop No 313

8 ABCD is a four-sided figure, and EFGH is a parallelogram formed by joining the middle points E, F, G and H in the four sides AB, BC, CD, and DA Then the area of the parallelogram EFGH is half of the figure ABCD. Join AC, and from D draw DM altitude on AC. Then because in the $\triangle$ ADC the st. line HG joins the middle points G and H in DC and DA. $\therefore$ GH is $\parallel$ and half of the base AC, and it also bisects DM at O The area of triangle ADC $= \frac{1}{2} \times AC \times DM$

And the area of the parallelogram HKLG = KLXOM or $= \frac{1}{2}AC \times \frac{1}{2}DM = \frac{1}{4} \times AC \times DM$ $\because$ the area of the parallelogram HKLG $= \frac{1}{2}$ the area of the $\triangle$ ADC.

In the same manner by drawing BN perpendicular to AC, it can be shewn that the area of the parallelogram EKLF $= \frac{1}{2}$ the area of the $\triangle$ ABC.

$\therefore$ the area of the parallelogram EFGH = $\frac{1}{2}$ the area of the quadrilateral ABCD

Prop No 314.

9 RQ is ∥ BC $\therefore$ the △ RBQ = the △ RCQ [Theor 26] From these equals take away RXQ, then the remainder △ RXB = the remainder △ QXC Again the △ AQB = the △ CQB, for they are on equal bases AQ and CQ and having the same altitude. [Cor. Theor 26] Now from the △ AQB take away RXB, and from the △CQB take away the △ QXC, then the remainder AQXR = the △BXC

Prop No 315.

10 ABC and DCB are two equal △s on the base BC, but on opposite sides of BC, join AD. Then AD shall be bisected by BC, at G From A and D draw AE and or BC produced, DF perpendiculars to BC or BC produced Then because the △s ABC and DCB are equal and on the same base BC, then altitudes AE and DF are also equal, for area of △ = $\frac{1}{2}$ × base × height.

Now in the △s AEG and DFG, the ∟ AEG = the ∟ DFG, being rt ∟s, and the ∟ AGE = the ∟ DGF, [Theor 3,] and one side AE = the one side DG

$\therefore$ AG = DG [Theor 17]

PART II.

Page 111

To be attempted after Theor 29.

Prop No 316

1. $a = 20$ ft, $b = 13$ ft., $c = 11$ ft.

$$p^2 = c^2 - x^2,\ p^2 = b^2 - (a-x)^2$$

$$c^2 - x^2 = b^2 - (a-x)^2$$

$$11^2 - x^2 = 13^2 - (20-x)^2 = 169 - 400 + 40x$$

$$\therefore 40x = 352 \qquad x = \tfrac{352}{40}$$

Now $c^2 - x^2 = p^2$ or $p^2 = 121 - 78{\cdot}3225 = 42{\cdot}6775$ $p = \sqrt{42{\cdot}6775}$ $= 6{\cdot}54$ ft

area $= \frac{1}{2} \times a \times p = \frac{1}{2} \times \frac{20}{10} \times 6{\cdot}54 = 65{\cdot}4$ sq ft

2. $a = 14$, $b = 15$, $c = 13$ yds

$$\therefore 13^2 - x^2 = 15 - (14-x)^2$$

$$\therefore 28x = 169 - 29 - 140 \quad \therefore x = \tfrac{140}{28} = 5 \text{ yds.}$$

Now $p^2 = 169 - 25 = 144 \quad \therefore p = \sqrt{144} = 12$ yds.

$\text{area} = \frac{1}{2} \times 12 \times 14 = 84$ sq yds.

3. $a = 21m, b = 20m, c = 13m$

$c^2 - x^2 = b^2 - (a - x)^2$

$169 - x^2 = 400 - 441 + 42x - x^2$

$\therefore 42x = 210 \therefore x = 5m$

and $p = \sqrt{144} = 12m - \text{area} = \frac{1}{2} \times 21 \times 12 = 126$ sq m.

4 $a = 30cm, b = 25cm, c = 11cm$

$c^2 = 625 - 900 + 60x$

$60x = 121 + 275 = 396 \quad \therefore x = \frac{396}{60} = 6.6cm.$

Now $p = \sqrt{121 - 43.56} = 8.8cm$

$\text{area} = \frac{1}{2} \times 30 \times 8.8 = 132$ sq cm.

5. $a = 37$ ft., $b = 30$ ft, $c = 13$ ft

$c^2 - x^2 = b^2 - (a - x)^2$

$169 = 900 - 1369 + 74x$ or $74x = 638.$

$\therefore x = \frac{638}{74} = 8.62$ ft

Now $p = \sqrt{169 - 74.304} = 9.73$ ft nearly

$\text{area} = \frac{1}{2} \times 37 \times 9.73 = 180.2$ sq. ft. nearly.

6. $a = 51m, b = 37m, c = 20m.$

$c^2 - x^2 = b^2 - (a - x)^2$

$400 = 1369 - 2601 + 102x, \quad 102x = 1632.$

Now $p = \sqrt{400 - 256} = 12m$

$\therefore \text{area} = \frac{1}{2} \times 12 \times 51 = 306$ sq m.

7. (i) $c^2 - x^2 = b^2 - (a - x)^2$ or $c^2 - x^2 = b^2 - a^2 + 2ax - x^2.$

$\therefore 2ax = c^2 + a^2 - b^2.$

$$\therefore x = \frac{c^2 + a^2 - b^2}{2a}.$$

(ii) $p^2 = c^2 - x^2$, by substituting the value of x we get

$$p^2 = c^2 - \left(\frac{c^2 + a^2 - b^2}{2a}\right)^2$$

(iii) $p^2 = c^2 \left(\frac{c^2 + a^2 - b^2}{2a}\right)^2$

Resolving into factors the right hand term becomes.

$$\left(c - \frac{c^2 + a^2 - b^2}{2a}\right) \left(c + \frac{c^2 + a^2 - b^2}{2a}\right)$$

$$=\frac{b^2-c^2-a^2+2ac}{2a}\times\frac{a^2+c^2-b^2+2ac}{2a}$$

$$=\frac{1}{4a^2}(b^2-[a-c]^2)\ ([a+c]^2-b^2)$$

$$=\frac{1}{4a^2}(b-a+c)\ (b+a-c)\ (a+c+b)\ (a+c-b)$$

$$p^2=\frac{(b-a+c)\ (b+a-c)\ (a+c+b)\ (a+c-b)}{4a^2}$$

$$p=\sqrt{\frac{(b-a+c)\ (a+b-c)\ (a+b+c)\ (a+c-b)}{4a^2}}$$

$\therefore$ Area $=\frac{1}{2}\times p\times a$

$$=\frac{a}{2}\times\frac{1}{2a}\times\sqrt{(b+c-a)\ (a+b-c)\ (a+b+c)\ (a+c-b)}$$

$$=\frac{1}{4}\sqrt{(a+b+c)\ (a+c-b)\ (a+b-c)\ (b+c-a)}$$

PART II

Page 113, Theorem 28

1 Area of a trapezium $=\frac{h}{2}(a+b)$

$a=3\ 3''$, $b=4\ 7''$ and $h=1\ 5''$

area $=\frac{1}{2}\times 1\ 5''\times(3\ 3''+4\ 7'')=6''$ sq in.

2 Area of a quadrilateral $=\frac{1}{2}\times$ diagonal $\times$ sum of offsets.

Diagonal AC $=17$ ft and sum of offsets $=11+9=20$ ft

area $=\frac{1}{2}\times 17\times 20=170$ sq ft

3 Diagonal AC $=8\ 2$ cm sum of offsets being $3\ 4+2{\cdot}6$ cm $=6$ cm

area $=\frac{1}{2}\times 8\ 2\times 6=24\ 6$ sq cm

When 1 cm represents 5 metres, then 1 sq cm represents 25 sq metres . the area $=25\times 24\ 6=615\ 0$ sq metres

Prop No 317.

4. Diagonal BD $=4\ 2''$, sum of offsets AE and CF $=2\ 4''\times 1\ 6''=4''$

Area $=\frac{1}{2}\times 4\ 2\times 4=8\ 4''$ sq in.

Prop No 318

5 The $\angle$ DAB $=90^\circ$ or a rt. $\angle$. Hence diagonal BD $=\sqrt{7\ 7^2\times 3\ 6^2}=8\ 5$ cm.

By measurement the $\angle$ at C is also a rt $\angle$, or by calculation from the sides BC and CD find BD $= \sqrt{6 \cdot 8^2 \times 5 \cdot 1^2} = 8 \cdot 5$, hence also the $\angle$ C is a rt $\angle$.

$\therefore$ the area of the whole figure is the sum of the area of two rt. $\angle$ed $\triangle$s

$\triangle$ ABD $= \frac{1}{2} \times 7 \cdot 7 \times 3 \cdot 6 = 12 \ 06$ sq cm.
$\triangle$ BCD $= \frac{1}{2} \times 6 \ 8 \times 5 \ 1 = 17 \ 34$ „

Sum $= 29 \ 40$

By drawing perpendiculars AE and CF on BD and measuring them they are found 3 2 cm and 4·1 cm respectively.

The area of the whole figure $= \frac{1}{2} \times 8 \ 5 \times 7 \ 3 = 31 \ 02$ sq cm.

Prop No 319.

6 ABCD is the required trapezium in full size Measure CD which is $= 2''$, and from C drop a perpendicular CE on AB, on measurement it is found to be 1 75″. Now apply the formulæ for the area of a trapezium, $\frac{1}{2} \times b \times (a + b)$

Here $b = 1 \ 75''$, $a = 2''$, $b = 4''$,
The area $= \frac{1}{2} \times 1 \ 75 \times 6 = 5 \ 25''$ sq in.

Prop No 320.

7 In the trapezium ABCD, let fall DE a perpendicular from D on AB which $= 4$ cm. by measure

$\therefore$ the area $= \frac{1}{2} \times b \times (a + b)$
$= \frac{1}{2} \times 4 \times 12$
$= 24$ sq cm.

8 When the diagonals are at rt $\angle$s, one of the diagonals becomes offsets of the other, $\therefore$ the area of a quadrilateral $= \frac{1}{2} \times$ diagonal^2.

9 When the given diagonals intersect each other at a given $\angle$ the sum of the offsets is constant, wherever they may cut each other and consequently the area of the figure is the same.

PART II.

Page 115.

Prop. No. 321.

1. (i) Area of the $\triangle$ ADE $= \frac{1}{2} \times 3 \times 5 = 7 \ 5$ sq cm.
„ „ „ DAC $= \frac{1}{2} \times 4 \times 6 = 12$ sq cm.
„ „ „ ABC $= \frac{1}{2} \times 2 \times 6 = 6$ sq. cm.

The area of the whole figure ABCDE $=25\ 5$ sq cm

Prop No 322

(ii) The $\triangle$ ABD is equilateral and its area $=\frac{1}{2}\times 5\cdot 2\times 6=$ $15\cdot 6$ sq cm In order to find the area of the figure ABCDE, it is necessary to add the area of the $\triangle$ BDC $=\frac{1}{2}\times 1\times 6=3$ sq cm to the area of the $\triangle$ ABD, and subtract that of the $\triangle$ ADE $=\frac{1}{2}\times 1\times 6=3$ sq cm because the chain line AD falls outside the figure

$\therefore$ the area of the figure $=25\ 5+3-3$ sq cm $=25\ 5$ sq. cm

2. (i) The figure ABDE is a square, $\therefore$ its area $=2\ 5''\times 2\ 5''=$ $6\cdot 25''$ sq in and the area of $\triangle$ DBC $=\frac{1}{2}\times 2\ 16\times 2\cdot 5=2\ 7$ sq. in.

$\therefore$ the area of the figure ABCDE $=8\ 95''$ sq. in

(ii) Area of the $\triangle$ AXD $=\frac{1}{2}\times 2\ 5\times 1\ 25=1\ 5625$ sq in

" " " CYB $=\frac{1}{2}\times 2\times 1\ 75=1\ 75$ sq in

" " trap DXYC $=\frac{1}{2}\times 2\ 75\times 4\ 5=6\cdot 1875$ sq in.

$\therefore$ the area of the whole figure $=9\ 5$ sq cm.

Prop. No 323

3 In the accompanying figure ABCDEF,

area of the triangle	ABC	$=\frac{1}{2}\times 50\times 180=$	4500 sq in.
" " "	AXF	$=\frac{1}{2}\times 50\times 60=$	1500 "
" " "	CZD	$=\frac{1}{2}\times 30\times 80=$	1200 "
" " trap m	FXYE	$=\frac{1}{2}\times 70\times 100=$	3500 "
" " "	EYZD	$=\frac{1}{2}\times 30\times 120=$	1800 "

$\therefore$ the area of the whole figure $=12500$ "

PART II

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Prop No 324

1. (i) P, Q, R, and S are the middle points of AB, BC, CD and DA respectively Join PQ, QR, RS, and PS. Then because AP = PB = CR = DR, and AS = SD = BQ = QC, and the $\angle$s at A, B, C, and D are rt. $\angle$s, the four $\triangle$s ASP, DSR, PQB, and RQC are equal to one another in all respects. [Theor. 4.]

∴ the sides PS, PQ, QR and RS are equal to one another, ∴ the figure PQRS is a rhombus.

(ii) Join PR and QS. Then PR is parallel and equal to AD and BC, and SQ is ∥ and = AB and DC.

But the area of a rectangle = ht × base.

∴ the area of the rectangle ABCD = SQ × PR.

But SQ and PR are the diagonals of the the rhombus PQRS, ∴ the area of the rhombus = $\frac{1}{2}$ × PR × QS.

∴ the area of the rhombus PQRS is half of the rectangle.

Yes. It is true for all quadrilaterals whose diagonals bisect at rt. angles

In the accompanying rhombus the diagonals PR and QS intersect each other at rt. angles

The area of the △ PSR = $\frac{1}{2}$ × SO × PR.

" " " PQR = $\frac{1}{2}$ × OQ × PR.

Adding these the area of the rhombus = $\frac{1}{2}$ (SO + OQ) PR
= $\frac{1}{2}$ × SQ × PR

Prop No. 325.

2. ABCD is a parallelogram and BD is its diagonal and E the middle point in BD Through E draw any line FG meeting AD at F, and BC at G

Now the △ ADB = the △ CBD, and the △ FED = the △ GEB [Theor 17] ∴ the remainder ABEF = CDEG Add the △ GEB to the former and the △ FED to the latter. ∴ the figure ABGF = the figure CDFG, i e, the parallelogram ABCD is divided into two equal parts

Therefore any st line drawn through the middle point in a diagonal bisects the parallelogram

Hence. (i) Join the given point P with the middle point E produce EP both ways to meet AD and BC at X and Y respectively, ∴ the line XEY bisects the parallelogram.

(ii) From the point E draw EL perpendicular to AB and produce LE to meet CD at M, ∴ the st line LEM bisects the figure

(iii) QR is a given st. line Through the point E draw Q′ER′ ∥ QR meeting AD and BC at Q′ and R′ respectively

∴ the st line Q′ER′ bisects the parallelogram.

Prop No 326.

3. (i) By the help of Theor 17 it can be proved that the $\triangle$ PXB = the $\triangle$ QXC Therefore by adding the $\triangle$ QXC to the trapezium and taking away the $\triangle$ PXB from it, the trapezium ABCD becomes the parallelogram APQD.

Hence trapezium ABCD = parm APQD

(ii) The area of the $\triangle$ AXD = $\frac{1}{2}$ the area of the parm, APQD [Theor 25]

the $\triangle$ AXD = $\frac{1}{2}$ of the trapezium ABCD or the trapezium = twice the $\triangle$ AXD.

Prop No 327

4 ABCD is a quadrilateral of which the diagonals AC and BD cut each other at rt $\llcorner$s, AC = 2 5" and BD = 3"

the area of ABCD = $\frac{1}{2} \times 2\ 5 \times 3 = 3\ 3''$ sq in Now in the accompanying two figures of a quadrilateral in (i) the diagonals bisect each other at rt $\llcorner$s, and in (ii) they cut each other at rt $\llcorner$s, but do not bisect each other, but the area remains the same Suppose they cut each other at O

Then the area of $\triangle$ ABD = $\frac{1}{2} \times AO \times BD$.

and „ „ BCD = $\underline{\frac{1}{2} \times CO \times BD}$

sum of these $= \frac{1}{2} \times BD\ (AO + CO)$

$= \frac{1}{2} \times BD \times AC$

Hence the rule that when the diagonals of a quadrilateral cut at rt $\llcorner$s, the area = half of the product of the two diagonals, whether the diagonals bisect each other or not

Prop No 328

5 Draw AB = 8 cm From A draw AE at rt $\llcorner$s to AB, making AE = 3 cm Through the point E draw the st line DEC ∥ AB From the centre A with a radius = 3 2 cm draw an arc cutting DC at D Join AD Then from the point B, draw BC ∥ AD, ABCD is the required parallelogram, area of the parallelogram ABCD = $8 \times 3 = 24$ sq cm

the perpendicular CF on AD from C = $\frac{24}{3\ 2} = 7{\cdot}5$ cm. By measurement also CF = 7 5 cm

Prop No 329

6 Draw a st line AB = 2·5", from the centre A with a radius = 1·7" half of one diagonal draw an arc, and from the centre B with a radius = 1·2" half the other diagonal draw another arc cutting the former at E. Produce AE to C making EC = AE, and also produce BE to D making ED = BE. Join DC, AD and BC. Then ABCD is the required parallelogram.

In order to determine the area of the parallelogram, the perpendicular distance on either of the adjacent sides AB or AD should be known. Draw DF perpendicular to AB and measure it out. In this case DF = 1·44".

∴ area of ABCD = 1·44 × 2·5" = 3·6 sq in.

7 ABCD is a parallelogram on the fixed base AB, and EABF another parallelogram on AB of equal area with ABCD, *i. e*, on the same base AB and between the same parallels BC and DF

Join AC and BD the two diagonals cutting each other at K. Also join EB and AF diagonals cutting each other at M. Join KM as the diagonals of a parallelogram bisect each other AK = KC, DK = KB and EM = MB and AM = MF. Now in the △ DBE, the sides DB is bisected at K and EB is bisected at M.

∴ the st. line KM joining them is ∥ the base DE or DF.

∴ the locus of the intersection the parallelogram's diagonals is the st. line ∥ to the fixed base AB, drawn from the point of the intersection of the diagonals

PART II

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Prop. No 330.

1. By measurement AB is found to be 5 cm.

∴ the area of the square on AB = 5^2 = 25 sq cm.

Prop No. 331.

2. Draw a line BC = 2·4". At C draw CA at rt ∟s to BC, making AC = 1". Join AB Then ABC is the required △. The hypotenuse AB = $\sqrt{1^2 + 2{\cdot}4^2}$ = 2·6" and the area = $\frac{1}{2}$ × 1" × 2·4" = 1·2"

sq. in. By measurement AB = 2 6″, and ∴ the area of the square on AB $= 2{\cdot}6^2 = 6{\cdot}76''$ sq. in.

Prop. No 332.

3. $a = 15$, $b = 8$, and $c = 17$

Now $c^2 = 17 \times 17 = 289$.

$a^2 = 15 \times 15 = 225$

$b^2 = 8 \times 8 = 64$

∴ $a^2 + b^2 = 289$. But $c^2 = 289$

∴ $a^2 + b^2 = c^2 = 289$.

PART II.

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Prop. No 333.

1. (i) $a = 3$ cm, $b = 4$ cm

But $c^2 = a^2 + b^2 = 3^2 + 4^2 = 25$.

∴ $c = \sqrt{25} = 5$ cm

By measurement also, c is found 5 cm.

Prop No 334.

(ii) $a = 2{\cdot}5$ cm, and $b = 6$ cm

But $c^2 = a^2 + b^2$

$= 2{\cdot}5 \times 2{\cdot}5 + 6 \times 6$,

$= 42{\cdot}25$.

∴ $c = \sqrt{42{\cdot}25} = 6{\cdot}5$ cm.

On measuring AB is found just 6·5 cm.

Prop. No. 335

(iii) $a = 1{\cdot}2''$, $b = 3{\cdot}5''$

$c^2 = a^2 + b^2 = 1{\cdot}2^2 + 3{\cdot}5^2$

$= 1{\cdot}44 + 12{\cdot}25$.

$= 13{\cdot}69$

∴ $c = \sqrt{13{\cdot}69} = 3{\cdot}7''$.

On measurement AB is found $= 3{\cdot}7''$

Prop No. 336.

2. (i) Draw AB = C = 3·4″

Upon AB describe a semi circle ACB From the centre B with a radius = 3″ draw an arc cutting the semicircle at C join AC and BC.

Then ABC is the required $\triangle$

$\therefore$ AB or $c = 3.4''$, and BC or $a = 3''$

But $c^2 = a^2 + b^2$

or $c^2 - a^2 = b^2$

$\therefore 3.4^2 - 3^2 = 2.56 = b^2$

$\therefore b = \sqrt{2.56} = 1.6$

This result is also verified by measurement of AC.

Prop. No 337.

(ii) Construct the $\triangle$ ABC by the method explained above $c = 5.3$ cm, $b = 4.5$ cm.

Now $c^2 = a^2 + b^2$ or $c^2 - b^2 = a^2$

$\therefore (c - b)(c + b) = a^2$

Hence $(5.3 - 4.5)(5.3 + 4.5) = a^2$

or $a^2 = .8 \times 9.8 = 7.84$

$\therefore a = \sqrt{7.84} = 2.8$ cm

On measuring BC it is found $= 2.8$ cm.

Prop No 338.

3 AB is a ladder whose one end A reaches the window-sill 40 ft high from the ground BC. B the foot of the ladder is 9 ft from the wall AC

$\therefore$ the ladder $AB = \sqrt{AC^2 + BC^2} = \sqrt{40^2 + 9^2} = \sqrt{1681}$.

$\therefore$ AB = 41 ft.

Prop. No 339.

4 A ship started from A southward and sailed 33 miles then reaching C she turned her course due west and sailed 56 miles. Her distance at B from A $= \sqrt{33^2 + 56^2} = \sqrt{4225} = 65$ miles.

Prop No 340.

5 A is the signal station from which two ships B and C are observed to bear respectively N. E. at a distance of 6 km, and N. W. at a distance of 1·1 km. now it is required to find out BC the distance between them.

The ∟ of bearing at A between both the ships is 90°.

Then $AC^2 + AB^2 = BC^2$ or $1.1^2 + 6^2 = BC^2$.

$\therefore BC = \sqrt{37.21} = 6.1$ km.

Prop No 341

6 AB a ladder 65 ft. long one end of which rests against a wall AC 63 ft high The distance BC of the other foot B of the ladder from the wall is to be known

Now in the rt ∟ed △ ACB, AC = 63 st and AB the hypotenuse = 65 ft.

∴ BC = $\sqrt{65^2 - 63^2} = \sqrt{2 \times 128} = 16$ ft.

Prop No. 342

7 $a = 55$ metres, $b = 73$ metres

$b^2 = a^2 + c^2$, or $b^2 - a^2 = c^2$

Then $c^2 = (b-a)(b+a)$

$= 18 \times 128 = 2304$.

$c = \sqrt{2304} = 48$ metres

Prop No. 343

8 A man travels from A 27 miles due South to B, and then 24 miles due West to C, finally 20 miles due North to D Join AD, and from D draw DE ∥ BC.

Now CD = BE = 20 miles

AE = 27 − 20 = 7 and DE = BC = 24 miles

Then in the rt angled △ ADE, AE = 7 miles, and DE = 24 miles.

∴ AD = $\sqrt{7^2 + 24^2} = \sqrt{625} = 25$ miles.

Prop No 344

9 From A draw AF ∥ BC meeting CD at F, and from E draw EG ∥ CD meeting AF at G

Now CB = AF = 60 metres, and GE = DF = 80 − 25 = 55 metres, and AG = 60 − 12 = 48 metres, for DE = FG

In the △ AGE, the ∟ G is a rt ∟, and GE and AG are known.

∴ AE = $\sqrt{55^2 + 48^2} = \sqrt{5329} = 73$ metres.

Prop No 345.

10. AC a ladder 50 ft long reaches the wall AB at A, a point 48 ft. from the ground BD, and by turning the ladder on its other end C over to the other side of the street it reaches a point E, 14 ft. high in the opposite wall DE.

There the two rt angled △s ABC and EDC, the ∟s at D and B are rt ∟s, and the ladder forms the hypotenuse in both the △s, and the walls as one side, then of the other sides or the two parts

DC and BC of the street BD $DC = \sqrt{EC^2 - DE^2} = \sqrt{50^2 - 14^2}$ and $BC = \sqrt{AC^2 - AB^2} = \sqrt{50^2 - 48^2}$

$\therefore BD = \sqrt{EC^2 - DE^2} + \sqrt{AC^2 - AB^2}$

$= \sqrt{50^2 - 14^2} + \sqrt{50^2 - 48^2} = 48 + 14 = 62$ ft.

PART II

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Prop. No 346

1 ABCD is a square, and AC a diagonal. Then $AC^2 = AB^2 + BC^2$ But $AB = BC$, and $AB^2 = BC^2$ $AC^2 = 2\ AB^2$ and the figure $ABCD = AB^2$.

$\therefore AC^2 =$ twice the figure ABCD, i e, the square on the diagonal is equal to double of the given square.

Prop No 347.

2. $AB = c$, $BC = a$, $AC = b$, and $AD = p$

$$p^2 = AB^2 - BD^2 = c^2 - BD^2$$

$$p^2 = AC^2 - DC^2 = b^2 - DC^2$$

$$c^2 - BD^2 = b^2 - DC^2$$

$$\text{or } c^2 - b^2 = BD^2 - DC^2.$$

Prop No 348.

3 Join OA, OB and OC

Then because $OA^2 = AZ^2 + OZ^2 = AY^2 + OY^2$ and

$$OB^2 = BX^2 + OX^2 = BZ^2 + OZ^2$$

$$OC^2 = CY^2 + OY^2 = CX^2 + OX^2$$

$\therefore$ by adding these together

$$OA^2 + OB^2 + OC^2 = AZ^2 + OZ^2 + BX^2 + OX^2 + CY^2 + OY^2$$

$$= AY^2 + OY_2 + BZ^2 + OZ_2 + CX^2 + OX^2$$

$$\therefore AZ^2 + BX^2 + CY^2 + OZ^2 + OX^2 + OY^2 = AY^2 + BZ^2 + CX^2 + OY^2 + OZ^2 + OX^2$$

Now taking away common $OX^2 + OY^2 + OZ^2$ from these equals there remains $AZ^2 + BX^2 + CY^2 = AY^2 + BZ^2 + CX^2$.

PART II

PAGE 123, THEOR. 29, 30.

Prop No. 349.

4. The $\angle$ A is a rt $\angle$

$\therefore BC^2 = AB^2 + AC^2$

$BC^2 + PQ^2 = AB^2 + AC^2 + AP^2 + AQ^2$ and $PQ^2 = AP^2 + AQ^2$

But $PC^2 = AC^2 + AP^2$, and $BQ^2 = AB^2 + AQ^2$

$\therefore BC^2 + PQ^2 = PC^2 + BQ^2$.

Prop No 350.

5 ABC is a rt ∟ed △, the ∟ A being the rt ∟, BD and CE are the two medians from the acute ∟s B and C

Now $BD^2 = AB^2 + AD^2$, $4BD^2 = 4AB^2 + 4AD^2$ and $EC^2 = AC^2 + AE^2$, $4EC^2 = 4AC^2 + 4AE^2$ By adding

$4\,BD^2 + 4\,CE^2 = 4\,AB^2 + 4\,AC^2 + 4\,AD^2 + 4\,AE^2$.

But $BE = AE$, and $AD = DC$.

$\therefore BE^2 = AE^2$, and $AD^2 = DC^2$

$\therefore 4AE^2 = AB^2$, and $4AD^2 = AC^2$

$\therefore$ by substituting AB^2 and AC^2 for $4AE^2$ and $4AD^2$

$4BD^2 + 4CE^2 = 4AB^2 + 4AC^2 + AB^2 + AC^2 = 5AB^2 + 5AC^2$

But $AB^2 + AC^2 = BC^2$

$\therefore 4BD^2 + 4CE^2 = 5BC^2$

Prop No 351

6. ABCD and EFGH are the two given squares. Draw a st. line OP = EF one side of the square EFGH. From O draw OQ at rt ∟s to OP, and make OQ = AB one side of the square ABCD. Join QP. Upon QP describe a square QPYX Now $QP^2 = OQ^2 + OP^2$ QPYX is the square on QP QPYX = ABCD + EFGH.

Prop No 352

7 Let ABCD and EFGH be the two squares as in the last preceding exercise. In the square EFGH describe a semi-circle FOG on one side FG With F as centre and radius = AB one side of the smaller square draw an arc cutting the semi-circle at D Join FO and GO. On OG describe a square OQ Then $OQ = FG^2 - FO^2$

Because the ∟ FOG is a rt ∟ $FG^2 = FO^2 + OG^2$.

But FO^2 = the square ABCD, and FG^2 = the square EFGH.

$\therefore$ the square OQ = square EG − sq AC.

Prop No. 353.

8. AB is the given st. line At A make an ∟BAC = quarter of a rt ∟. From B draw BC at rt. ∟s to AB meeting AC at C At the point C in AC make an ∟ ACD = the ∟ A, the side CD meeting AB at D. The st. line AB is divided at B, such that $AD^2 = 2BD^2$.

Because the ∠ BAC = the ∠ ACD, AD = CD, and the extr. ∠ CDB = two inter ∠s DAC and ACD. But each of the ∠s DAC and ACD is $\frac{1}{4}$ of a rt. ∠, ∴ the extr. ∠ CDB = $\frac{1}{2}$ a rt ∠

∴ BC = BD.

Now AD = CD, ∴ $AD^2 = CD^2$. But $CD^2 = BC^2 + BD^2$.

∴ $AD^2 = CD^2 = BC^2 + BD^2 = 2BD^2$

∴ AB is divided at D so that $AD^2 = 2BD^2$.

Prop No. 354. Prop. No. 355

9. AB is the given st. line, and CE the given square It is required to divide AB into such two parts that the squares on those two parts is equal to the given square

From the centre A with a radius = the side of the given square, describe an arc XO At B in AB make an ∠ ABO = $\frac{1}{2}$ the rt. ∠. The arm BO meeting the arc OX if possible at O and X. Join AO, and from O draw perpendiculars OP on AB. AB is divided at P so that $AP^2 + BP^2 =$ square CE

Then because ∠s at P are rt ∠s, and the ∠ B is half a rt ∠. ∴ the ∠ BOP = $\frac{1}{2}$ a rt. ∠ and BP = OP. But $AO^2 = AP^2 + OP^2$ for OPA is a rt ∠ed △, and AO^2 = figure CE and $OP^2 = BP^2$.

∴ the square CE = $AP^2 + BP^2$.

If from X the other point where BX cuts the arc, perpendicular XY be drawn on AB then as shown above the square CE = $AY^2 + BY^2$

In case any of the perpendiculars OP or XY falls on AB produced then AB can be said to be externally so divided.

There is another case where BO does not reach the arc, and then AB cannot be divided

Prop No 356

10. (i) $a^2 + b^2 = c^2$ $14^2 + 48^2 = 50^2$ $196^2 + 2304 = 2500$.

∴ This case forms a rt ∠ed △.

(ii) $40^2 + 10^2 = 41^2$. $1600 + 100 = 1681$.

∴ This case does not form a rt. ∠ed △.

(iii) $20^2 + 99^2 = 101^2$ $400 + 9801 = 10201$

∴ This is also a case of rt ∠ed △

Prop No ~~299~~ 356

11. In the △ ABC, the side AC = the side CB and ∠ C is a rt ∠.

∴ $AB^2 = AC^2 + BC^2$ or $= 2AC^2$.

ADEB is the square on AB, and ACFG is the square on AC Join AE, BD, and CG The whole figure BD is divided into four equal parts by the diagonals AE and BD The △ ABC = the △ CGA, and the △ ABC = the △ ABO the △ CGA = the △ ABO.

∵ But the square CG = twice the △ CGA and the square BD = four times the △ ABO

the square BD = twice the square CG When AC = BC = 2″, $AB = \sqrt{4+4} = 2\ 83''$, and by measurement AB is found nearly 2 83″.

Prop No 357.

12 AC = 6 cm is the diagonal, it is required to describe the square of which it is a diagonal Bisect AC at O, from O draw OD at rt ∟s to AC, and produce DO to B, making OD and OB = AO or OC Join AB, BC, CD, and AD Then ABCD is the required square

As the ∟ AOB is a rt ∟, ∵ $AB = \sqrt{AO^2 + OB^2} = \sqrt{2\ AO^2} = \sqrt{18} = 2\ 24$ cm

On measurement 2 3 cm nearly

Area of the figure ABCD $= 2 \times \frac{1}{2} \times 3 \times 6 = 18$ sq cm

13 In the Problem it is shown that if OP = OA = 1, then $PA^2 = OP^2 + OA^2 = 1 + 1 = 2$ ∴ $PA = \sqrt{2}$, i e, the diagonal PA of a square whose side OP = OA = 1, is $\sqrt{2}$ Then it is clear that if the given side of a square be multiplied by $\sqrt{2}$ it becomes equal to the diagonal of the square.

the diagonal of a square whose side is 50 metres $= 50 \times \sqrt{2}$ metres = 70 7 metres

Prop No. 358

14 ABC is an equil △, each side of which = $2m$ AD is a perpendicular from the vertex A on the base BC BD = DC = m AD = p Now $p^2 = (2m)^2 - m^2$ or $4m^2 - m^2 = 3m^2$

$p = \sqrt{3m^2} = \sqrt{3} \times m$

Suppose each side = 8 cm then $p = \frac{8 \times \sqrt{3}}{2} = 6\ 928$ cm

By drawing another equilateral △ ABC having each side equal to 8 cm. and then measuring AD the altitude, it is found nearly 6 9 cm.

Prop No 359.

15. $a = m^2 - n^2$, $b = 2mn$, and $c = m^2 + n^2$ Now $(m^2 - n^2)^2 = m^4 - 2m^2n^2 + n^4$.

$\therefore a^2 = m^4 - 2m^2n^2 + n^4$, and $b^2 = 4m^2n^2$ adding these two ·
$a^2 + b^2 = m^4 - 2m^2n^2 + n^4 + 4m^2n^2 = m^4 + 2m^2n^2 + n^4 = (m^2 + n^2)^2$

But $c^2 = (m^2 + n^2)^2$ $\therefore a^2 + b^2 = c^2$.

If $m =$	2	3	4	3	4	4	&c
and $n =$	1	1	1	2	2	3	&c
Then $a =$	3	8	15	5	12	7	&c
$b =$	4	6	8	12	16	24	&c
$c =$	5	10	17	13	20	25	&c

Prop. No 360

16 (i) $a = 25$ cm. $p = 12$ cm BD $= 9$ cm

$$c = \sqrt{p^2 + BD^2} = \sqrt{12 \times 12 + 9 \times 9} = \sqrt{225} = 15 \text{ cm.}$$

Similarly $b = \sqrt{12^2 + 16^2} = \sqrt{400} = 20$ cm

$\therefore b = 20$, and $c = 15$.

(ii) $b = 41''$ $c = 50''$, BD $= 30''$

$p = \sqrt{50^2 - 30^2} = \sqrt{1600} = 40''$

and $a = BD + DC = 30'' + \sqrt{b^2 - p^2} = 30'' + \sqrt{41^2 - 40^2}$

$= 30'' + 9'' = 39''$

$BD = \sqrt{c^2 - p^2}$, and $DC = \sqrt{b^2 - p^2}$

add $BD + DC = \sqrt{c^2 - p^2} + \sqrt{b^2 - p^2}$

But $BD + DC = a$

$\therefore a = \sqrt{c^2 - p^2} + \sqrt{b^2 - p^2}$

Prop No. 361

17 In the $\triangle$ ABC, AD is the altitude.

$p^2 = c^2 - BD^2$, and again $p^2 = b^2 - DC^2$

$\therefore c^2 - BD^2 = b^2 - DC^2$

If $a = 51$ cm, $b = 20$ cm, $c = 37$ cm.

Now $c^2 - BD^2 = b^2 - BC^2$ $\therefore c^2 - b^2 = BD^2 - DC^2$ or $c^2 - b^2 = (BD + DC)(BD - DC) = a(BD - CD)$

Now substituting the values.

$$BD - CD = \frac{c^2 - b^2}{a} = \frac{37^2 - 20^2}{51} = \frac{1369 - 400}{57} = \frac{969}{51} = 19 \text{ cm.}$$

Again $BD + CD = 51$

$$\begin{aligned} BD - CD &= 19 \\ \hline 2\,BD &= 70 \end{aligned}$$

$\therefore$ $BD = 35$ cm and $CD = 51 - 35 = 16$ cm

Now again $p = \sqrt{c^2 - BD^2} = \sqrt{(37-35)(37+35)} = \sqrt{2 \times 72} =$ 12 cm

$\therefore$ area of the $\triangle ABC = \frac{1}{2} \times a \times p$

$$= \frac{1}{2} \times 51 \times 12 = 306 \text{ sq. cm.}$$

Prop. No. 362

18. (i) $b^2 - c^2 = (DC + BD)(DC - BD)$

$$DC - BD = \frac{10^2 - 9^2}{17} = \frac{19}{17}$$

But $DC + BD = 17''$

add $2\,DC = \frac{19}{17} + 17 = \frac{308}{17}$

$\therefore$ $DC = \frac{308''}{34}$

And $BD = 17 - \frac{308}{34} = \frac{270}{34}$

Now $p = \sqrt{9^2 - \left(\frac{270}{34}\right)^2} = \frac{72}{17}$ cm.

$\therefore$ the area of $\triangle ABC = \frac{1}{2} \times \frac{72}{17} \times 17 = 36''$ sq in.

Prop. No. 363.

(ii) $b^2 - c^2 = (DC + BD)(DC - BD)$

$$\therefore DC - BD = \frac{17^2 - 12^2}{25} = \frac{289 - 144}{25} = \frac{145}{25} = \frac{29}{5}$$

But $DC + BD = 25$

Add $2DC = 25 + \frac{29}{5} = \frac{154}{5}$ ft.

$\therefore DC = \frac{154}{10}$ ft

Now $p = \sqrt{17^2 - \left(\frac{14}{10}\right)^2} = \frac{\sqrt{324 + 16}}{100} = \frac{72}{10}$ ft.

$\therefore$ the area of the $\triangle ABC = \frac{1}{2} \times \frac{72}{10} \times 25 = 90$ sq ft.

Prop No. 364.

(iii) $(DC+BD)(DC-BD)=b^2-c^2$.

$\therefore DC-BD=\dfrac{28^2-15^2}{41}=\dfrac{559}{41}$

$\therefore \dfrac{DC+BD=41}{2DC=41}-\dfrac{559}{41}=\dfrac{422}{41}$

$\therefore DC=\dfrac{1122}{82}$

Now $p=\sqrt{28^2-\left(\dfrac{1122}{82}\right)^2}=\dfrac{252}{41}$ cm.

$\therefore$ Area of $\triangle$ ABC$=\frac{1}{2}\times\dfrac{252}{41}\times 41=126$ sq cm.

Prop No 365

(iv) $(DC+BD)(DC-BD)=b^2-c^2$

$DC-BD=\dfrac{37^2-13^2}{40}=\dfrac{50\times 24}{40}=30$ yds

Sum $\dfrac{DC+BD=40 \text{ yds}}{2DC=70 \text{ yds}}$ $\therefore$ DC$=35$ yds

Now $p=\sqrt{37^2-35^2}=\sqrt{72\times 2}=12$ yds

$\therefore$ the area of the $\triangle$ ABC$=\frac{1}{2}\times 12\times 40=240$ sq yds

Prop No 366.

19. The angle POQ is a rt angle. $\therefore PQ^2=OP^2+OQ^2=5.6^2+3.3^2$ $\therefore PQ=\sqrt{5.6^2+3.3^2}=6.5$ cm Now PQ slides and takes the position as P′Q′ where OP′$=4$ cm.

$\therefore OQ'=\sqrt{6.5^2-4^2}=\sqrt{26.25}=5.1$ cm.

Prop No 367.

20. Area of the $\triangle=\frac{1}{2}\times a\times b$

and also $\triangle=\frac{1}{2}\times p\times c$

$\therefore \frac{1}{2}ab=\frac{1}{2}pc$

$\therefore ab=pc$

Now $pc=ab$ or $\dfrac{1}{p}=\dfrac{c}{ab}$

$\therefore \dfrac{1}{p^2}=\dfrac{c^2}{a^2b^2}$. But $c^2=a^2+b^2$

$\therefore \dfrac{1}{p^2}=\dfrac{a^2+b^2}{a^2b^2}$ or $\dfrac{1}{a^2}+\dfrac{1}{b^2}$

PART II

PAGE 127, PROB. 17.

Prop No 368

1 ABCD is a square described on BC = 5 cm. Join BD Then BD is a diagonal of the square, and the ∠ CBD = 45°. From C draw CE ∥ BD meeting AD produced at E DBCE is the parallelogram on the same base BC having the same altitude DC as the square

∴ the square ABCD = the parallelogram DBCE The diagonal BD which is also the oblique side of the parallelogram DBCE = $\sqrt{2 \times 5^2} = 5 \times \sqrt{2} = 7\ 1$ cm By measurement also BD = 7 1 cm nearly.

Prop No 369

2 On the base AB = 2 5″ describe a parallelogram whose opposite oblique sides AD = BC = 2″ From the points A and B as centres with a radius = 2 5″ draw two arcs cutting, DC, and DC produced at E and F respectively Join AE and BF Then EABF is the rhombus required on the same base AB and between the same parallels AB and DF.

Prop No 370

3 In the figure given on page ~~66~~ 127 to explain the definition of complements, AC is the diagonal of the parallelogram ABCD.

∴ the △ ABC = the △ ADC Again AK is the diagonal of the parallelogram EH, and KC of GF, ∴ the △ AHK = the △ AEK, and the △ KFC = the △ KGC From the △ ADC take away the △s AHK and KFC, and from the △ ABC take away the △s AEK, and KGC, then the remainder HF = the remainder EG

EG is a parallelogram, produce GK one of its sides to H, making KH equal to given st line HK From H draw HA ∥ EK or GB, meeting BE produced at A Join AK, as AH is ∥ EK, ∴ AK if produced will meet BG produced, and let them meet at C From C draw CD ∥ GH or AB meeting EK and AH produced at F and D respectively Then HF is the parallelogram equal and equiangular to the given parallelogram EG

Prop No 371

4 Let CDEF be the given rectangle, and AB the given st line Produce EF to G, making FG = AB. Proceed as in the

construction of the last preceding exercise, and complete the figure HDKM, in which FM is the required rectangle which is = the rectangle CDEF, because each of them are complements to the figures CG and EL parallelograms about the diagonal HK

The remaining side FL of the rectangle FM is by measurement equal to 4 cm.

Prop No 372.

5 ABCD is the given parallelogram in which AB = 2 4″, AD = 1 8″, and the ∟ A = 55° It is required to draw a parallelogram whose greatest side = 2 7″ Proceed as in last preceding exercise and complete the figure AFKG, in which CK is the required parallelogram equiangular to the given parallelogram ABCD.

The shorter side of the parallelogram CK measures 1 6″.

Prop No. 373

If the ∟ A is increased the area of the given parallelogram ABCD will also increase so that when the ∟ A becomes a rt ∟ the area of the figure will reach its maximum, for with the increase of the ∟ A, the altitude from D upon AB increases till AD itself becomes the altitude. With the increase of the area of ABCD, the area of the parallelogram CK also increases. In the similar manner with the decrease of the ∟ A the area also decreases, and it becomes zero when the ∟ A is = D, or the sides AB and AD coincide.

Prop. No. 374.

6. ABC is an equilateral △ on a side BC = 6 cm. From the vertex A draw AD perpendicular to BC From C draw CF ∥ AD, and from A draw AEK ∥ BC cutting CF at E, and make EF = 5 cm. From F draw FG ∥ AE or BC, meeting DA produced at G Join GE Produce GE to meet BC produced at H From H draw HKL ∥ CF or DAG, meeting AE and GF produced at K and L respectively The figure EL is the rectangle required, and it is described on EF = 5 cm.

The remaining side EK of the rectangle EL measures 3·1 cm. nearly.

The area of EL = 5 × 3·1 = 15 5 sq cm. approximately.

PART II

Page 130

Problems 18—19.

Prop No 375

1. ABCD is the quadrilateral. Join DB. From C draw CE ‖ DB, meeting AB produced at E. Join DE.

The triangle ADE is = in area to the figure ABCD

AE the base = 10 9 cm, and DF the altitude = 4·4 cm

∴ the area of the triangle ADE = $\frac{1}{2}$ × 4 4 × 10 9 = 23 98 sq cm.

Prop No 376

2. ABCD is the given quadrilateral and BD the diagonal. Proceed as in the above exercise, and measure out AE = 5 7" and the altitude DF = 2·9".

∴ the area of the triangle ADE = $\frac{1}{2}$ × 2 9 × 5 8 = 8 41" sq in.

Prop No. 377

3. ABCDE is a regular pentagon of which each side = 4 cm. Join DA and DB, and produce AB both ways to F and G From the points C and E draw CG and EF ‖s DB and DA respectively meeting AB produced at G, and BA produced at F. Join DF and DG. Then the △ FDG is equal to the pentagon ABCD. The altitude from D on FG = 6 1 cm and FG measures 9·2 cm

∴ area of the △ = $\frac{1}{2}$ × 6 1 × 9 2 = 28 06 nearly

Prop. No 378

4. From the point D draw DE ‖ AC meeting BA produced at E. From C draw CF perpendicular to AB produced. Join CE The ECB is the △ required AD = 365 m, and EB = 710 m.

∴ the area of the △ ECD = $\frac{1}{2}$ × 710 × 365
= 129575 sq metres.

Prop No 379. Prop. No. 380.

5 D the other extremity of the given base BD lies in BC or BC produced. Join AD From C draw CE ‖ AD, meeting BA produced or BA at E. Join DE Then EBD is the required △ The △ ADE = the △ DAC, for they are on the same base AD, and between the same parallels AD and EC. [Theor. 26] To each of these add the △ ABD in case (i) or the figure BEOC in case (ii) Then the whole △ ABC = the whole △ EBD

(i) Prop. No 381. (ii) Prop. No 382.

6. Let ABC be the given △, and P the altitude. If the given altitude = the altitude of the △ ABC, then proceed to describe a △ as given in Problem 8 But in case the given altitude be less or greater than the altitude of the △ ABC proceed thus. From the points B and C draw BE and CD at rt. ∟s to BC, and through A draw EAD ∥ BC, meeting BE and CD at E and D From CD or CD produced cut off CF = P. Join BF. Produce BF if necessary to meet EAD or EAD produced at G From G draw GK ∥ BE or CD, meeting BC or BC produced at K, and through F draw MF ∥ BC or ED, meeting BE, and GK or these produced at M and H Take any point O in MF, and join BO and KO. Then OBK is the required △. Then because (i) EK or (ii) MC is a rectangle and (i) BG or (ii) BF the diagonal, ∴ the complements EF and HC are equal. ∴ the rectangle EC = the rectangle MK.

But the △ ABC = ½ of EC, and the △ OBK = ½ of MK, because they are on the same base BC and BK, and between the same ∥s BC and ED or BK and MH. ∴ the △ ABC = the △ OBK.

Prop No 383. Prop No 384.

7. Let ABC be the given △ and X the given point. Through the points B and C draw BE and CD st. lines at rt ∟s to BC, and through A draw a st. line EAD ∥ BC, meeting BE and CD at E and D. From the given point X draw a st line XM ∥ BC, meeting BE and CD or these produced at M and F Join BF, meeting ED or ED produced at G. From G draw GK ∥ BE and CD, meeting BC at K, and produce if necessary to meet MX at H. Join BX and XK. Now EK (i), or MC (ii) the rectangle, and the complement EF = the complement FK, ∴ the figure BD = the figure MK, and also their halves are equal, ∴ the △ ABC half of BD = the △ XBK half of MK.

Prop No. 385.

8. ABCD is a quadrilateral, and X a given point in DC, it is required to construct a △ having its vertex at X, and its base being in the same st line with AB. First construct the △ ADE = the quadri figure ABCD [Prob 18] Then proceeding as in the last preceding exercise construct a △ AXN = the △ ADE.

Prop No 386

9 ABC is a given $\triangle$ Divide one of its sides BC into any n say 4 parts at D, E, and F points. Join these points with the $\angle$ opposite to them, *i e*, AD, AE, and AF, &c Now the $\triangle$ ABC is divided into n here four equal $\triangle$s ABD, ADE, AEF and AFC

Because AO is the altitude of the $\triangle$ ABC and AO is the common altitude for all these n here four triangles, and their bases are equal But the area $= \frac{1}{2} \times$ base $\times$ ht the areas of these triangles are equal Or, all these triangles are on equal bases and between the ||s BC and that drawn through A the common vertex, these triangles are *equal in area* [Theor. 26] O is the point of intersection of PQ and ZC.

10 Z is the middle point of AB, then ZC bisect the $\triangle$ ABC, *i e*, the $\triangle$ BZC = the $\triangle$ AZC But the $\triangle$ ZPQ = the $\triangle$ QCZ, take away the equal parts ZOQ, then $\triangle$ ZOP = $\triangle$ COQ Now by taking $\triangle$ COQ from the $\triangle$ BZC and adding $\triangle$ ZOP, and similarly by taking away the $\triangle$ ZOP from the $\triangle$ AZC and adding the $\triangle$ COQ, the $\triangle$ BPQ = the figure APQC

11 Let AP and HX intersect at O, and AQ and KX at O′, then with the same reasoning as given above the $\triangle$ BHX = the $\triangle$ APQ, and the $\triangle$ CKX the $\triangle$ APQ $\therefore$ the $\triangle$ BHX = the $\triangle$ CKX = the $\triangle$ APQ But as the $\triangle$ HOA = $\triangle$POX and the $\triangle$ KO′A = the $\triangle$ QO′X $\therefore$ the $\triangle$ BHX = the $\triangle$CKX = the figure AHXK

Prop No 387

12 ABC is a $\triangle$, and X is a point in the base BC It is required to cut off from the $\triangle$ ABC in an $\frac{1}{n}$th part by a st line drawn from the point X Make BD $= \frac{1}{n}$th part of BC, say here $\frac{1}{8}$th part Join AD and AX From D draw DE || AX, meeting AB at E and join EX Then BEX is the required part Because the $\triangle$ BAD is the $\frac{1}{n}$th part of the $\triangle$ ABC, and the $\triangle$ BEX can be proved on the analogy of the proof of the last preceding exercise = the $\triangle$ BAD as the $\triangle$ BAD is $\frac{1}{n}$th or near $\frac{1}{8}$th part

. the $\triangle$ BEX is the $\frac{1}{n}$th and here $\frac{1}{8}$th part of the $\triangle$ ABC

Prop. No 388.

13. ABCD is a quadrilateral. Join DB. From C draw CE ∥ DB meeting AB produced at E Join DE Then the △ ADE = the figure ABCD. [Prob 18]

Bisect the base AE of the △ ADE, at F, and join DF.

Then the triangle ADF = the triangle FDE.

∵ the triangle ADF is half of the triangle ADE and ∴ the triangle ADF is also half of the figure ABCD.

Prop No 389.

14. ABCD is a quadrilateral. Construct a triangle ADE = the figure ABCD [Prob 18.] Bisect the base AE of the triangle ADE into n parts, and make AF = $\frac{1}{n}$th of AE. Join DF Because the △ ADF = $\frac{1}{n}$th part of the △ ADE

∴ the △ ADF is also $\frac{1}{n}$th part of the quadrilateral ABCD.

PART II.

Page 134.

Prop No 390.

1. XOX′ and YOY′ are the axis of reference and O the point of origin.

(i) Along OX mark off OM, 4 units in length, and at M draw MA perp. to OX, making MA = 6 units of length. Then A is the point whose co-ordinates are (6, 4).

Similarly mark off A in the following three cases whose co-ordinates are (– 6, 4), (– 6, – 4), (6, – 4)

(ii) Prop. No. 391.

(iii) Prop No 392.

2 (i) Prop No 393. (ii) Prop. No. 394.

Prop No 395.

3 (i) Co-ordinates of the middle point are (8, 5).

(ii) „ „ (10, 10).

Prop. No. (i) 396 (ii) 397. (iii) 398 (iv) 399

4 Co-ordinates of mid-points.

(i) (4, 5), (ii) (4, 5), (iii) (– 4, – 5), (iv) (– 4, – 5).

Prop. No 400.

5. The co ordinates of the points of trisection of the line joining (O, O) to (18, 15) are (6, 5) and (12, 10)

Prop No 401

6. The abscissa of the points P in (i) is the same while ordinate changes, ∴ the position of the P points lies on the line parallel to YOY' While in case (ii) the abscissa changes but ordinate is the same throughout, ∴ the line of position of points P remains parallel to XOX' If the line of position of P points be produced it intersects that of P' points, the co-ordinate of which are (5, 8.)

Prop No. 402

7 (i) The distance $OP = \sqrt{8^2 + 15^2} = 17$.

From the centre O with a radius = OP, describe an arc PQ cutting the abscissa at Q which is 17 parts distant from the origin O ∴ DP = 17.

Prop. No 403

(ii) Here the distance $OP = \sqrt{(-8)^2 + (-15)^2} = 17$.

From the centre O with radius OP describe an arc PQ meeting the ordinate of X at Q which is 17 parts from the origin. ∴ OP = 17.

Prop No. 404

(iii) The distance $OP = \sqrt{2\cdot 4^2 + \cdot 7^2} = 2\cdot 5''$

From the centre O with the radius OP, describe an arc PQ meeting the ordinate of X at Q which is 2·4" from the origin O. ∴ OP = 2·5"

Prop. No. 405

7. (iv) $OP = \sqrt{\cdot 7^2 + 2\cdot 4^2} = 2\cdot 5''$

From the centre O and with radius OP draw an arc PQ meeting line of X at Q which reads 2·5". ∴ OP = 2·5".

Prop No 406.

(i) $PP' = \sqrt{3^2 + 4^2} = 5$ From the centre O with a radius = PP' draw an arc cutting OX' at Q which is 5 parts distant from O. ∴ PP' = 5.

Prop No 407

(ii) From P′ draw P′M ∥ OX′ meeting PS at M

$P'M=9-5=4$, and $PM=8-5=3$. ∴ $PP'=\sqrt{3^2+4^2}=5$. From the centre O with a radius = PP′ draw an arc cutting OX′ at Q, i e., at 5th division from O.

∴ $PP'=5$.

Prop No 408

(iii) $OP'=8$, $OP=15$ ∴ $PP'=\sqrt{8^2+15^2}=17$.

From the centre O and with a radius PP′ draw an arc cutting OX′ at Q which point is at the 17th division from O. ∴ $PP=17$

Prop No. 409.

(iv) From the point P draw PM ∥ XOX′ meeting P′ 5 at M. Now $PM=10+5=15$, and $P'M=12-4=8$ ∴ $PP'=\sqrt{15^2+8^2}=17$

From the centre O with radius PP′ draw an arc cutting XO at Q, a point 17 divisions apart from O.

∴ $PP'=17$.

Prop No 410.

(v) $PP'=\sqrt{8^2+35^2}=36$ approximately.

From the centre O with radius = PP′ draw an arc cutting OX′ at Q just near the 36th division from O.

∴ $PP'=36$ nearly.

Prop No 411

8. (vi) From P′ draw $P'M=XOX'$ meeting P20 produced at M Now $P'M=20+15=35$, and $PM=15\times 3=18$. ∴ $PP'=\sqrt{35^2+18^2}=39\cdot 4$. By measuring PP′ in the compasses and then applying the legs of the compasses along XOX′ it covers something above 39 divisions. ∴ $PP'=39$ nearly

Prop No 412

9. Join PP′, P′P″, and PP″ As P and P″ are 2 divisions on the Y ordinate, and hence PP″ ∥ XOX′, and $=7+3=10$. From the centre O with radius PP′ draw an arc cutting OX′ at X′, a point 10 divisions from O.

∵ $PP'=10$, and $PP''=10$ also ∴ PP′ + PP″ are the equal sides of the isosceles △ PP′P″.

Prop No 413

10 The co ordinates of A = (0, 5) $\therefore$ OA = 5

" B = (3, 4) $\therefore$ OB = $\sqrt{3^2+4^2} = 5$

" C = (5, 0) $\therefore$ OC = 5

" D = (4, − 3) $\therefore$ OD = $\sqrt{4^2+(-3)^2} = 5$

" E = (− 5, 0) $\therefore$ OE = 5

" F = (0, − 5) $\therefore$ OF = 5.

" G = (− 4, 3) $\therefore$ OG = $\sqrt{(-4)^2+3^2} = 5$

" H = (− 4, 3) OH = $\sqrt{(-4)^2+(-3)^2} = 5$

Hence it appears that the distance of all these 8 points from O is 5, and $\therefore$ if a circle be drawn from the centre O with a radius = the distance of one of these points from D, it will pass through all other points

Prop No 414.

11 (i) Suppose $oa = 4$, $ob = 8$

Then $ab = \sqrt{4^2+8^2} = \sqrt{a^2+b^2}$

(ii) $ob = b$, $oa = a$ join ab

$\therefore ab = \sqrt{a^2 \times b^2}$

(iii) join bo, then $ob = \sqrt{a^2+b^2}$

$\therefore$ the distances between these points are equal.

Prop No 415

12. These points when plotted become the angular points of a square, and the st lines joining them become diagonals of that square, and hence they bisect each other.

Prop No 416

13. When these points are plotted they occupy the places indicated in the figure by A, B, &c, respectively The distance between B and C = 9 + 4 = 13 From the centre A with a radius AB, draw an arc cutting the parallel through A at Q, i e., 13 divisions from A $\therefore$ AB = 13 AB = BC

The base AC is cut by the axis of X at 6th division which divides AC into two equal parts

Prop. No 417

14. The co-ordinates of the fourth vertice is (0, 0) and the co-ordinates of the intersection of the diagonals are (7, 5).

Prop No 418

15. By joining the four points ABCD, as the co-ordinates of D (5, 12), $\therefore$ AD $= \sqrt{5^2+12^2} = 13$, which is = AB.

$\therefore$ the four sides of the figure ABCD are equal, but the $\angle$s are not right $\angle$s, $\therefore$ the figure is a rhombus Join AC and B, and they intersect each other at 2 the co-ordinates of which are (9, 6)

16 The locus of the point is the st line bisecting OP at rt $\angle$s, and the locus cuts the axis at the points (4, 0) and (0, –4)

17. (i) ABCD is a rectangle, side AB $= 17 - 4 = 13$, and AD $= 12 - 3 = 9$. $\therefore$ the area $= 9 \times 13 = 117$.

(ii) AB $= 15 - 2 = 13$ and AD $= 6 + 3$ $\therefore$ area $= 9 \times 13 = 117$.

(iii) AB $= 5 + 8 = 13$, and AD $= 8 + 1 = 9$.

$\therefore$ area $= 9 \times 13 = 117$.

18 The quadrilateral formed is a square AC and BD are the diagonals $\therefore$ area $= \frac{AC^2}{2} = \frac{2^2}{2} = 2$ sq in Now joining the middle points of the sides of the above square, we get another smaller square PQRS each side of which $= 1''$.

$\therefore$ the area of PQRS $= 1^2$ or 1 sq inch

19 ABC is a $\triangle$, BC $= 18 - 4 = 14$, and altitude AD $= 10$.

$\therefore$ the area of $\triangle$ ABC $= \frac{1}{2} \times 14 \times 10 = 70$ units of area. The above rules apply to all the four $\triangle$s which have the equal bases and altitudes

Prop No 415

20. (i) ABC is the $\triangle$, AC is the base $= 6$, while B5 the altitude $= 3$. area of $\triangle = \frac{1}{2} \times 3 \times 6 = 9$ units of area

Prop No. 116

(ii) In this $\triangle$ base AB $= 3$, and altitude AC $= 6$.

$\therefore$ the area of the $\triangle = \frac{1}{2} \times 3 \times 6 = 9$ units of area. The $\angle$s in the $\triangle$ in (i) are 31°, 71° and 78°

Prop No 417

21 (i) The side BC joining two points B and C the co-ordinates of which are (12,10) and (12, –6) lie on the line 12 units distant from O, and $\parallel$ the axis Y. The area of the $\triangle = \frac{1}{2} \times (10 + 6) \times 12 = 96$ units of area.

Prop No 418.

(ii) In this $\triangle$ the side BC is $\parallel$ the axis of X

The area $= \frac{1}{2} \times (5 + 15) \times 8 = 80$ units of area.

Prop. No. 419

(iii) In the $\triangle$ ABC, BC is $\parallel$ the axis of Y

The area $= \frac{1}{2} \times (12 + 8) \times 12 = 120$ units of area.

Prop. No 420.

(iv) In this $\triangle$ base BC is $\parallel$ the axis of X

The area $= \frac{1}{2} \times (6 + 20) \times 8 = 104$ units of area.

Prop No 421

22. (i) The area of the $\triangle$ ABC $= \frac{1}{2} \times (15-5) \times (15-5) = 50$ units of area.

Prop No 422.

(ii) „ „ $= \frac{1}{2} \times 8 \times (18 - 3) = 60$ „ „

Prop No 423

(iii) „ „ $= \frac{1}{2} \times (8 + 4) \times (16 + 4) = 120$ „ „

Prop No 424

(iv) „ „ $= \frac{1}{2} (15 + 7) \times (11 + 1) = 132$ „ „

Prop No 425

23. Plot the points A, B, C, and D, and join AB, BC, CD, and AD. Then ABCD is a parallelogram From the centre D with the radius = DA draw an arc AP cutting the st line DF drawn $\parallel$ the axis of X at P, then DP $= 7 - 2 = 5$ units of length, i e, AD $= 5$. In the same manner from the centre D with radius = DC, draw an arc CQ cutting the st. line PD produced at Q, then DQ = DC $= 11 + 2 = 13$.

$\therefore$ the adjacent sides of the parallelogram are 5 and 13 respectively.

Area of the parallelogram $= (15 \times 9) - 2\{(\frac{1}{2} \times 12 \times 5) + (\frac{1}{2} \times 4 \times 3)\}$

$= 135 - 2\{30 + 6\} = 135 - 72$

$= 63$ units of area.

Prop No. 426.

24. (i) ABDC is a trapezium of which AB $\parallel$ CD. AC $= 9 - 3 = 6$,

$\therefore$ area $= \frac{1}{2} \times 6 \times (3 + 6) = 27$ units of area.

Prop. No 427.

(ii) ABCD is a trapezium. AD $= 3 + 3 = 6$.

$\therefore$ area $= \frac{1}{2} \times (5 + 2) \times 6 = 30$ units of area.

Prop. No. 428

(iii) ABDC is a trapezium. DC $- 11 - 3 = 8$. From B draw BE $\parallel$ AC, and B 4 if produced is the altitude, $\therefore$ BE $= 5$.

$\therefore$ The area of the parallelogram AE $= 5 \times 4 = 20$ units of area,

and the area of the $\triangle$ BDE $= \frac{1}{2} \times 4 \times 5 = 10$ units of area.
$\therefore$ the area of the trapezium $= 20 + 10 = 30$ units of area.

Prop. No. 429

(iv) From C draw CE ∥ AB, and BF $= 5$ is the altitude. $\therefore$ the area of the figure BE $= 5 + 3 = 8$ and the area of the $\triangle$ CDE $= \frac{1}{2} \times (8 - 3) \times 5 = 12.5$ $\therefore$ the area of the trapezium $= 8 + 12{\cdot}5 = 20{\cdot}5$ units of area.

Prop No 430.

25. (i) From A and B draw AP and BQ ∥ YY′, and through C draw PCQ ∥ XX′, meeting AP and BQ at P and Q. Now area of the trapezium APQB $= \frac{1}{2}(9 + 4) \times 15 = 97.5$. From this subtract the area of two $\triangle$s APC and BQC $= \frac{9 \times 7}{2} + \frac{4 \times 8}{2} = 31{\cdot}5 + 16 = 47.5$.

$\therefore$ the area of the $\triangle$ ABC $= 97.5 - 47.5 = 50$ units of area.

Prop No 431

(ii) Draw AP and BQ ∥ YY′ and QCP ∥ XX′ similar to the case above

Now the area of the trapezium BQPA $= \frac{1}{2}(7 + 9) \times 17 = 136$. From this subtract the area of $\triangle$s APC and BQC $= \frac{9 \times 11}{2} + \frac{7 \times 6}{2} = \frac{99}{2} + \frac{42}{2} = \frac{141}{2} = 70.5$

$\therefore$ the area of the $\triangle$ ABC $= 136 - 70.5 = 65{\cdot}5$ units of area

Prop. No 432.

(iii) From C draw CP ∥ XX′ meeting YY′ at P. The area of the $\triangle$ APC $= \frac{1}{2} \times 11 \times 14 = 77$ From which subtract the area of $\triangle$ BPC $= \frac{1}{2} \times 14 \times 8 = 56$

the area of the $\triangle$ABC $= 77 - 56 = 21$ units of area.

Prop. No 433.

(iv) Complete the trapezium as in cases (i) and (ii). Then the area of the trapezium $= \frac{1}{2}(9 + 19) \times 13 = 182$ subtract the area of two triangles APC and BQC $=$ $\frac{19 \times 8}{2} + \frac{9 \times 5}{2} = \frac{152 + 45}{2} = 98.5$.

$\therefore$ the area of the $\triangle$ ABC $= 182 - 98{\cdot}5 = 83{\cdot}5$ units of area.

Prop No 434.

26 Join BD Then AC and BD are the diagonals, but AC lies along the axis XX', BD at rt ∟s to AC is ∥ YY'

∴ area of the rhombus $ABCD = \frac{10 \times 24}{2} = 120$ units of area.

From the centre D with radius = DC, draw an arc CQ cutting the st line DQ which is parallel to the axis of X, the co-ordinates of the point Q are (20, − 5)

∴ the length of $DQ = 20 - 7 = 13$

∴ each side of the rhombus is = 13 units

Prop No 435

27 $CB = \sqrt{CE^2 + EB^2} = \sqrt{6^2 + 8^2} = \sqrt{100} = 10$

$AB = \sqrt{5^2 + 12^2} = \sqrt{169} = 13$

$CD = CG + GD = \sqrt{6^2 + 8^2} + \sqrt{4^2 + 3^2} = 10 + 5 = 15$ By measuring AD is found = 8 3 The area of BCFO = area of △ BCG − △FOG $= (\frac{1}{2} \times 16 \times 6) - (\frac{1}{2} \times 3 \times 4) = 48 - 6 = 42$ units of area, and the area of △ AOB $= \frac{1}{2} \times 12 \times 5 = 30$ units of area

Prop No 436

28 In the figure ABCD produce DA and CB to meet at F, the co-ordinates of F are (− 10, − 10) $AB = \sqrt{3^2 + 4^2} + \sqrt{3^2 + 4^2} = 5 + 5 = 10$

$BC = 13 - 4 = 9$, and $CD = \sqrt{15^2 + 8^2} = \sqrt{289} = 17$

From the centre A with radius = AD draw an arc DQ cutting AQ at Q the co-ordinates of which are (9 − 4) $AD = 4 + 9 = 13$ nearly or by measuring AD with the help of a decimal diagonal scale AD = 12 7

From D draw DE ∥ XX' meeting CF at E

The area of the △ $FCD = \frac{1}{2} \times DE \times CF = \frac{1}{2} \times 15 \times 23 = 172\ 5$ sq units, and

The area of the △ $ABF = \frac{1}{2} \times AG \times BF = \frac{1}{2} \times 6 \times 14 = 42$ sq units

∴ the area of the figure $ABCD = 172\ 5 - 42 = 130\ 5$ sq units

Prop No. 437

29 The points B and D are on the same ∥s, join BD, = 8 + 4 = 12, and CD = 8 − 3 = 5.

$\therefore$ $AB = \sqrt{8^2+6^2} = 10$, $BC = \sqrt{12^2+5^2} = 13$, $CD = 5$ and $DE = \sqrt{4^2+3^2} = 5$, and $AE = 3$.

The area of the figure ABCDE = area of $\triangle$ ABF + area of $\triangle$ BCD + area of $\triangle$DEF $= \frac{1}{2} \times 8 \times 6 + \frac{1}{2} \times 12 \times 5 + \frac{1}{2} \times 4 \times 3 = 24 + 30 + 6 = 60$ sq units.

Prop No 438.

30 For want of space the scale has been reduced to $\frac{1}{2}'' = 100$ yds or $1'' = 200$ yds From B and C draw CE and BF $\parallel$ YY′, and from A draw EAF $\parallel$ XX′

The area of the trapezium CEFB $= \frac{1}{2}$ (CE + BF) $\times$ EF.

But CE = 100 and BF = 700, and EF = 800 yds.

$\therefore$ the area of the trapezium $= \frac{1}{2}(100 + 700) \times 800 = 320000$ sq. yds.

and the area of the $\triangle$ ABF $= \frac{1}{2} \times 400 \times 700 = 140000$ sq. yds

and that of the $\triangle$ ACE $= \frac{1}{2} \times 100 \times 400 = 20000$

Sum of the area of both the $\triangle$s = 160000.

$\therefore$ the area of the $\triangle$ ABC = 320000 − 160000 = 160000 sq yds.

From the centre C with radius = CB draw an arc cutting the st. line through C $\parallel$ XX′ at Q the co-ordinates of which are $(5'', -2'')$.

$\therefore$ $CQ = 5'' + 5'' = 10''$ or 1000 yds. (in the plan $\frac{1}{2}''$ represents $1''$ of the question)

From A draw AP perpendicular on BC, and measure it. $\therefore$ AP = 320 yds.

Prop. No. 439

31. On measuring the lines that join the points it is found that they are all = one another, and the $\angle$s they contain are rt. $\angle$s. $\therefore$ the figure is a square

From the centre A with AB as radius draw an arc AQ cutting AX at Q, then AQ = 15 approximately, and $\therefore$ the area = 225 sq. units approximately

(i) From C draw ECF $\parallel$ XX′, meeting YY′ at E, and from B draw GBF $\parallel$ YY′ meeting XX′ at G and ECF at F. Each side of this square EOGF = 20, $\therefore$ area $= 20^2 = 400$ sq units. and the area of each of the $\triangle$s ADO, ABG, BFC and CED $= \frac{1}{2} \times 6 \times 14 = 42$ sq. units. $\therefore$ area of the 4 $\triangle$s $= 4 \times 42 = 168$ sq. units. Subtract this area of $\triangle$s from that of the square, *i. e*, 400 − 168 the area of the square ABCD = 232 sq. units.

(*ii*) Divide the square ABCD, as given in example 1, page 120, into four equal $\triangle$s and one middle square

The area of the middle square $= 8^2 = 64$ sq units

The area of the 4 rt $\angle$ed $\triangle$s $= 4 \{\frac{1}{2} \times 14 \times 6\} = 4 \times 42 = 168$ sq units

the area of the given square ABCD $= 64 + 168 = 232$ sq units

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PART II.

PAGE 138.

Miscellaneous.

Prop No 440

1. The side AB > the side AC From C draw CE $\parallel$ AP, meeting BA produced at E Because AP $\parallel$ CE, the $\angle$ BAP = the $\angle$ AEC, and the $\angle$ PAC = ACE But the $\angle$ BAP = $\angle$ PAC $\therefore$ the $\angle$ AEC = the $\angle$ ACE. Hence AE = AC Now in the $\triangle$ BCE the st line AP is $\parallel$ CE $\therefore$ AP divides BC and BE proportionally, *i e*, BP : PC :: BA : AC. But AE = AC $\therefore \frac{BP}{PC} = \frac{BA}{AC}$ But BA > AC $\therefore$ BP > PC

But BX = XC (Hyp) $\therefore$ BP > BX, again AB > AC, then the $\angle$ ACB > the $\angle$ ABC, add to each one of the = $\angle$s BAP, and CAP Then the $\angle$s ACB and CAP are > the $\angle$s ABC and BAP. But these four $\angle$s = 4 rt $\angle$s $\therefore$ the $\angle$s ABC and BAP = the exterior $\angle$ APC are less than a rt $\angle$ $\therefore$ the $\angle$ APD is < the $\angle$ ADP, $\therefore$ AP is > AD, or AP lies towards B from AD the perpendicular, $\therefore$ AP lies between AX and AD, and it is also intermediate in magnitude

Prop. No 441

2 ABC is a $\triangle$, AP bisects the $\angle$ BAC. From C draw CQ perpendicular to AP or AP produced. Produce CQ to meet AB or AB produced at E Then because AD the bisector of the $\angle$ BAC is perpendicular on CE, $\therefore$ AE = AC, and the $\angle$ AEC = the $\angle$ ACE.

(*i*) The exter $\angle$ AEC = inter $\angle$s CBE and BCE To these add the $\angle$ ACE $\therefore$ the $\angle$s AEC and ACE = the $\angle$s ABC and ACB $\therefore$ each of the $\angle$s AEC or ACE = $\frac{1}{2}$ of the $\angle$s ABC and ACB.

(ii) The $\angle$ AEC = the $\angle$ ACE Add the $\angle$ BCE $\therefore$ the $\angle$s AEC + BCE = the $\angle$s ACE + BCE = the $\angle$ ACB. But the $\angle$s AEC and ACE are equal. and the $\angle$ AEC = the $\angle$s ABC and BCE

$\because$ the ACB = 2 $\angle$ BCE + the $\angle$ ABC

Hence twice the $\angle$ BCE = the $\angle$ ACB − the $\angle$ ABC.

$\therefore$ the $\angle$ BCE = $\frac{1}{2}$ ($\angle$ ACB − $\angle$ ABC).

Prop No 442

3 In the figure of the last preceding exer. 2, draw AD perpendicular to CB The $\angle$s APD and PAD are = the $\angle$ ADP , for the $\angle$ ADP is a rt $\angle$. Hence the $\angle$ PAD is complementary to the $\angle$ APD.

But also in $\triangle$ PQC, the $\angle$ PQC is a rt $\angle$, $\because$ the $\angle$ PCQ is complementary to the $\angle$ CPQ or DPA . the $\angle$ PAD = the $\angle$ PCQ

But the $\angle$ PCQ = $\frac{1}{2}$ (the $\angle$ ACB − the $\angle$ ABC) by the last preceding exercise . the $\angle$ PAD = $\frac{1}{2}$ ($\angle$ACB − $\angle$ABC)

Prop No 443

4 Let C be the hypotenuse and AB the difference of the other two sides of a rt $\angle$ed $\triangle$ At the point A make an $\angle$ BAO = 45° or $\frac{1}{2}$ rt $\angle$ From B as centre and with radius = C the hypotenuse Draw an arc cutting AO at O From O drop OP perpendicular on AB produced. Then BOP is the $\triangle$ required. In the $\triangle$ APO, the $\angle$ P is rt $\angle$, and the $\angle$ A = 45°, · the remaining $\angle$ AOP = 45°, $\therefore$ PO = AP. For AB = AP − BP = PO − BP, $\because$ PO and PB are the two sides of the rt $\angle$ed $\triangle$ BOP, BO = C is the hypotenuse.

Prop No 444

5. Let the $\angle$ A be the difference of the base $\angle$s, and B the difference of the two sides, and CD the given base. It is required to describe the $\triangle$. Bisect the $\angle$ A At the point C make an $\angle$ DCE = $\frac{1}{2}$ A. From the centre D with radius = B draw an arc cutting CE at E. Join DE Bisect CE at O, and draw OP at rt. $\angle$s to CE, meeting DE produced at P Join CP Then CPD is the required $\triangle$. Since OP is drawn from the middle point of CE at rt. $\angle$s to CE, $\therefore$ PC = PE. ED = PD − PE = PD − PC.

Now the exter $\angle$ CEP = the $\angle$s CDE + DCE or the $\angle$ PCE = the $\angle$s CDE + DCE. Add the $\angle$ DCE.

$\therefore$ the whole $\angle$ PCD = 2 $\angle$ DCE + $\angle$ CDE

$\because$ Twice the $\angle$ BCE = the $\angle$ PCD − the $\angle$ CDE, or the $\angle$ A = the $\angle$ PCD − the $\angle$ CDE

Prop No 445.

(ii) Let B be the sum of the two sides and others as given above At C make an $\angle$ DCE = $\frac{1}{2}$ the $\angle$ A Draw CO at rt. $\angle$s to CE From the point D as centre and with a radius = B draw an arc cutting CO at O Join OD cutting CE at E Bisect OE at O, and join CO. Then because the $\angle$ OCE is a rt $\angle$ and CO is drawn from the rt $\angle$ to the middle point of OE the hypotenuse, $\therefore$ CO = PO = OE [Exer 10 to cor 2 Theor 16, page 47] Then PCD is the $\triangle$ required CD is the base PD and PC are the two sides, the sum of which = DO = B, and the $\angle$A = the difference of the $\angle$s PCD and PDC

Prop No 446

6 Let BC be the base and A the sum of one side and the altitude Bisect the base BC at D, and draw DE at rt $\angle$s to BC, making DE = A, join BE. Bisect BE at F, and draw FG at rt $\angle$s to BE, meeting DE at G Join BG and CG Then GBC is the $\triangle$ required In the $\triangle$s BGF and EGF, the $\angle$s at F are rt $\angle$s, the side BF = EF, and FG is common, $\therefore$ the $\triangle$s BGF and EGF are congruent, and BG = EG $\therefore$ ED = BG + GD But ED = A, $\therefore$ BG + GD = A Now BD = DC, and GD is common, and the $\angle$s at D are rt. $\angle$s $\therefore$ the $\triangle$s BGD and CGD are congruent, and BG = CG. $\therefore$ GBC is the required isosceles $\triangle$

Prop. No 447.

7. Let AB be the given st line At B in AB draw BC at rt $\angle$s to AB At the point A make an $\angle$ BAD = 22½° or ¼ rt $\angle$ AD meeting BC at D At D in AD make the $\angle$ ADP = the $\angle$ BAD DP meeting AB at P. The P is the point where AB is divided so that $AP^2 = 2BP^2$, now because the $\angle$ BAD = the ADP (const) AP = DP The exter $\angle$ DPB = the $\angle$s PAD and ADP = 45° or half a rt $\angle$ $\therefore$ the $\angle$ BPD = the $\angle$ BDP, and hence BP = BD. The $\angle$ at B is a rt $\angle$ $\therefore$ $PD^2 = BD^2 + BP^2$, but BD = BP $\therefore$ $PD^2 = 2BP^2$, and DP = AP. $\therefore$ $PA^2 = 2BP^2$.

Prop No 448

8 *(i)* The point O is outside the $\angle$ BAD or its vertical opposite $\angle$. Join OA, OD, OC, AC and OB

From O draw EOF $\parallel$ AD, meeting BA and CD produced at E and F respectively. Join EC and ED. The $\triangle$ AOD = the $\triangle$ EAD, and the $\triangle$ AEC = the $\triangle$ AED

$\therefore$ the $\triangle$ AEC = the $\triangle$ AOD

In the same manner the $\triangle$ OBE = the $\triangle$ OCE

$\therefore$ The sum of the $\triangle$s AEC + OCE = $\triangle$s OAD + OBE.

From these equals take away the part AEO.

$\therefore$ the $\triangle$ AOC = the $\triangle$s AOD + OBA

Prop. No 449.

(ii) Let the point O be within the $\angle$ BAD The same construction being made The $\triangle$ AOD = the $\triangle$ ACE or the $\triangle$ AOD = the $\triangle$s ACO + OCE + AOE But the $\triangle$ OCE = the $\triangle$ OBE $\therefore$ the $\triangle$ AOD = the $\triangle$s ACO + OBE + AOE $\therefore$ the $\triangle$ ACO = the $\triangle$ AOD − the $\triangle$ OBA.

Prop No 450.

9 Let ABCD be the given quadrilateral, of which AC and BD are the diagonals, intersecting each other at E. Produce CA to F and make AF = CE, so that EF = CA Join DF and BF. Produce DB to G, and make BG = DE so that EG = BD. Join FG Then EFG will be the $\triangle$ required.

Then because the base AC = EF, the $\triangle$ ADC = the $\triangle$ DEF, and the $\triangle$ ABC = the $\triangle$ BEF [Theor. 26]

$\therefore$ the triangle BDF = the triangles ABC + ADC = the figure ABCD.

But the triangle EFG = the triangle BDF, because they are on equal bases EG and BD, and between the same $\parallel$s [Theor 26].

$\therefore$ the triangle EFG = the figure ABCD, and the side EF = the diag AC, and the side EG = the diag BD, and the angle AEB is common.

Prop No 451.

10. Let the $\triangle$s ABC and DBC be on the same base BC and of given area, i e, between the same parallels BC and AD. Bisect

BC at E and join AE and DE Then AE and DE are the medians on the base BC in the triangles ABC and DBC According to the Cor III, page 97, the medians of a triangle are concurrent about $\frac{1}{3}$ of the median from the base. In the △ ABC the medians are concurrent at the point O, OE being $\frac{1}{3}$ of AE, and in the △ DBC the medians are concurrent at P, a point about $\frac{1}{3}$ of DE from BC

Now in the △ AED, the point O is $\frac{1}{3}$ of AE from E, and P is $\frac{1}{3}$ of DE from E, the line joining OP is parallel to AD or BC, and it is therefore the locus of the intersection of the medians of △s described on BC and having the same area

Prop No 452 also Hall

11 Let ABC be the given △, and D the given st line It is required to draw a △ on the base BC equal in area to the △ ABC and having its vertex at the given line D From A draw AE ∥ BC, meeting the st line D, or D produced at E Join EB and EC Then EBC is the △ required Since they are = for they are on the same base BC and between the same parallels BC and AE, and the vertex E rests on the st line D

In case the parallel AE does not meet D or D produced, then D must be ∥ BC and either above or below AE, and then the construction fails.

Prop No 453

12 Let ABCD be a parallelogram of rods turnable at all the corner points, but the side AB is fixed, and E is the middle point of DC Bisect AB at F, and join EF

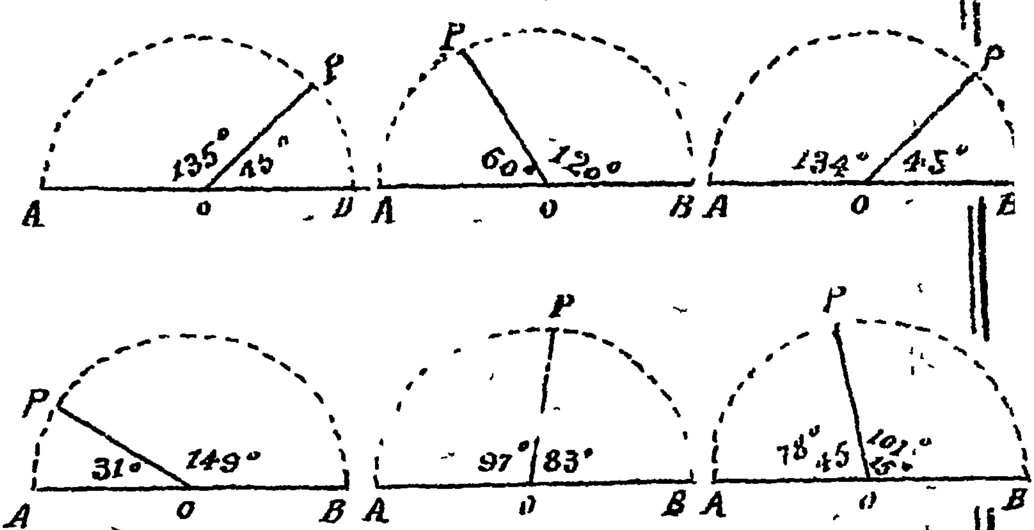
As the rods AD and BC remain constant, and when turn round the points A and B, they move in a circle round A and B Similarly the st line joining the middle points of AB and CD moves round the point F, and E the middle point of DC describes a circle round F, and hence the locus of E is the circle described round F with radius = FE

PART I.

PAGE 13.

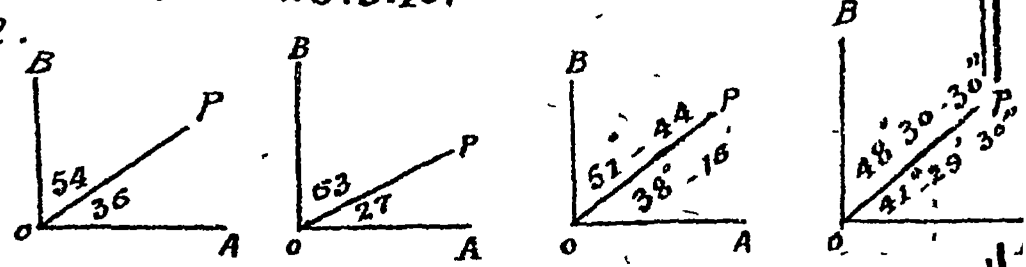
THEOR. 1 & 2.

Exer 1. *Prop. No. 1 to 6.*

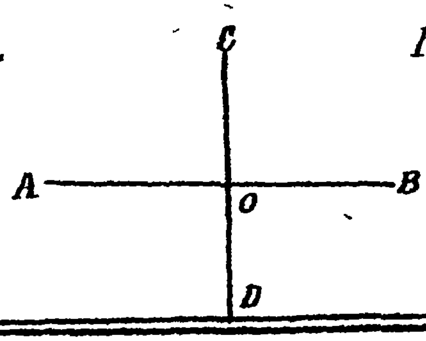


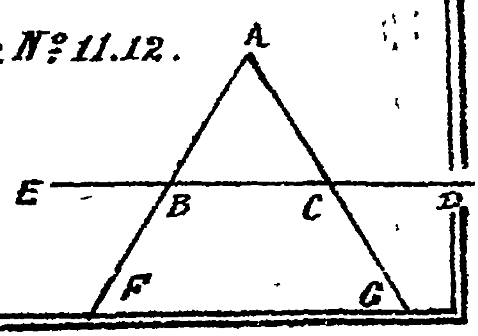
Prop. Nos. 7.8.9.10.

2.



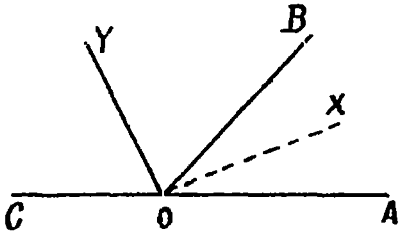
3. *Prop. No. 11.12.*





Prop. No. 13.

6.



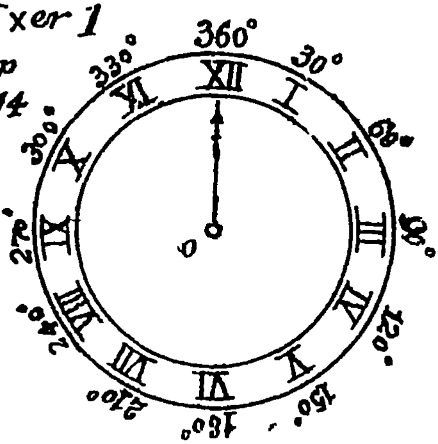
PART. I

PAGE 15.

Theor. 3.

Exer 1

Prop No. 14 15.



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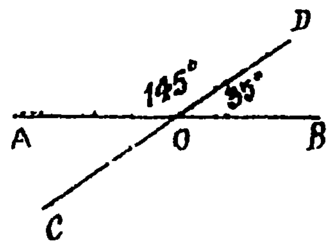

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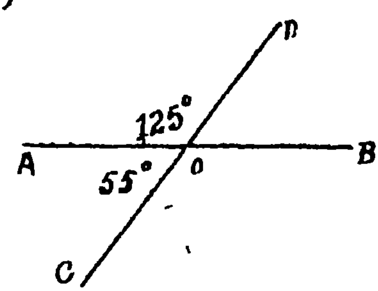

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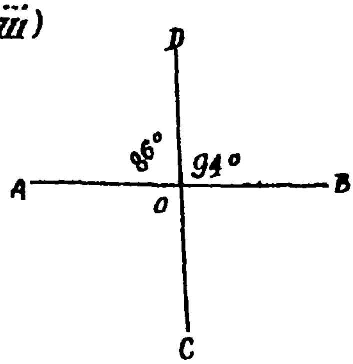
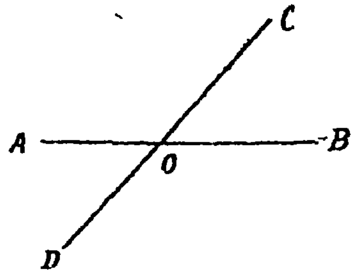


Prop.
№ 16
17.

5



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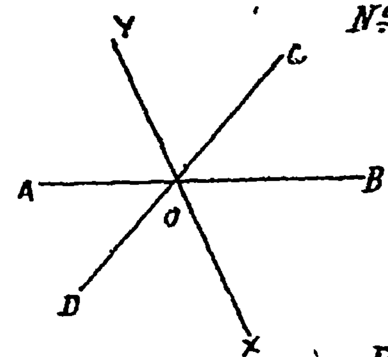
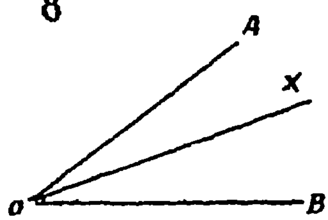


Prop № 19.

8



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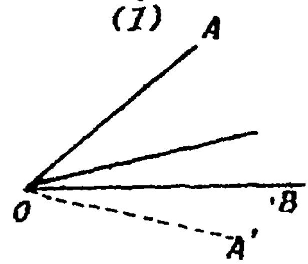

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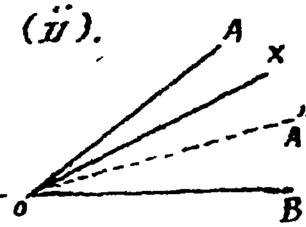

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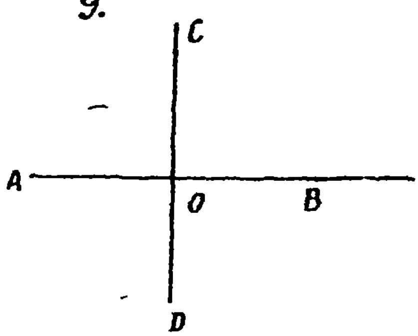
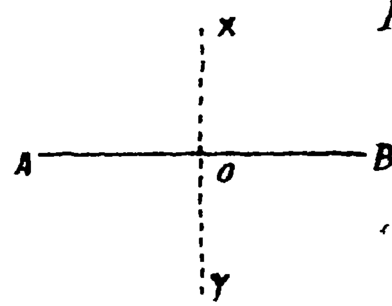


Prop.
№ 20

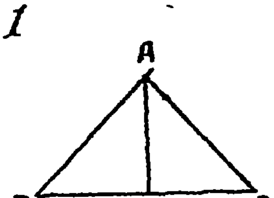


PART I.

Page 19.

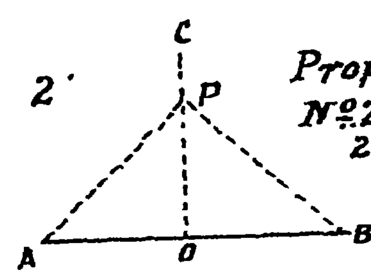
Theor 4.

Exer. 1



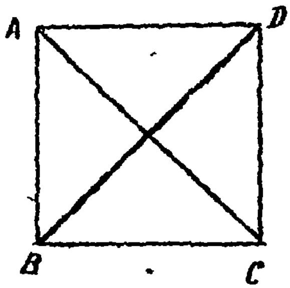
2

Prop.
№ 22
23



Prop` Nº 24. 25.

3



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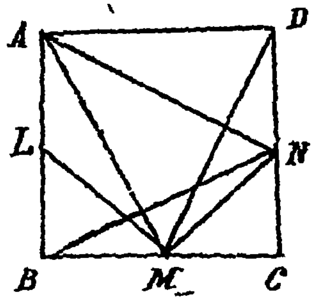
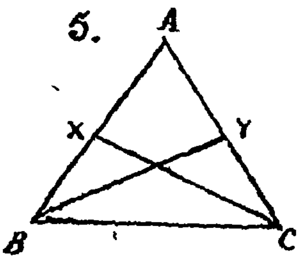


Prop.
Nº 26.

5.



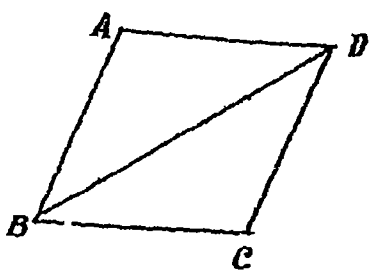
PART I.

PAGE 21.

THEOR 5.

Exer. 1.

Prop.
Nº 27.
28.



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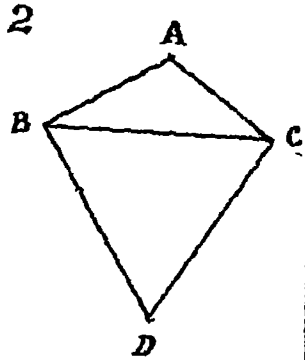
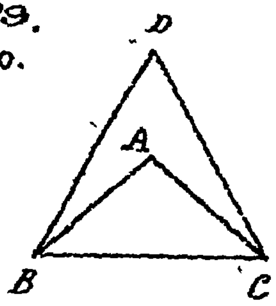


Prop.
No. 29.
30.

3.



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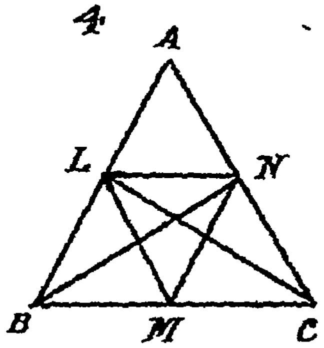


PART. I.

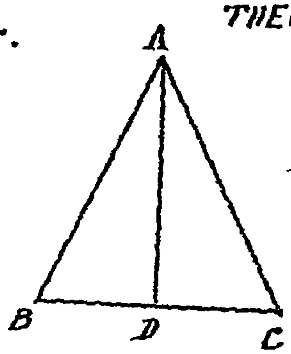
PAGE 26.

THEOR. 4 & 7.

Exer.

1.

PROP.
No.
31.
32.



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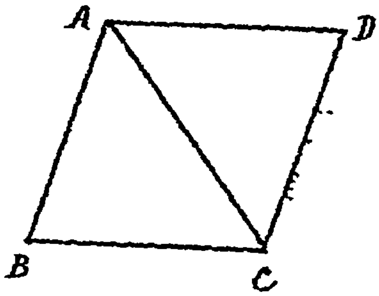
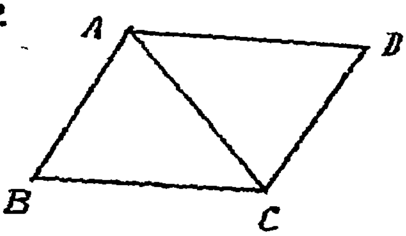
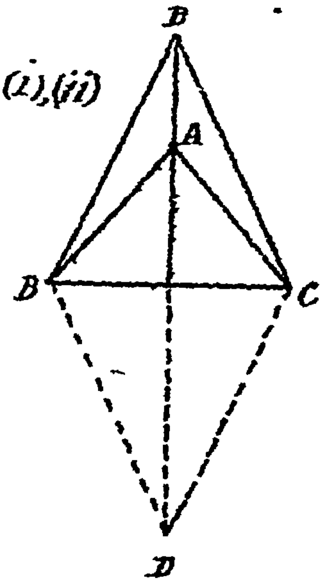


3

PROP.
No.
33.
34.
35.

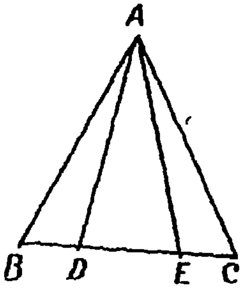


4. (i), (ii)



Prop. №
36-37

7



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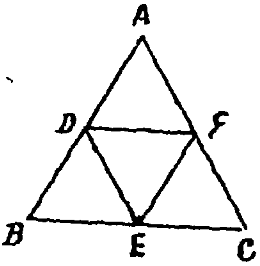

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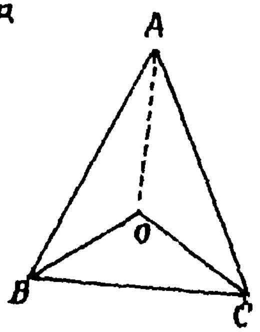

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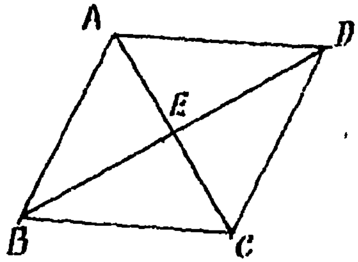

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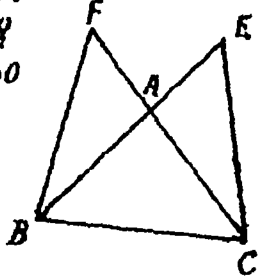

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Exer on triangles

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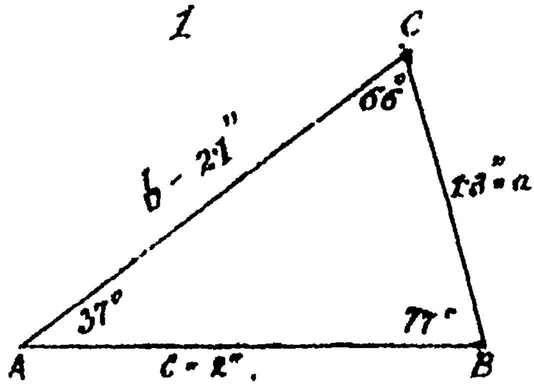

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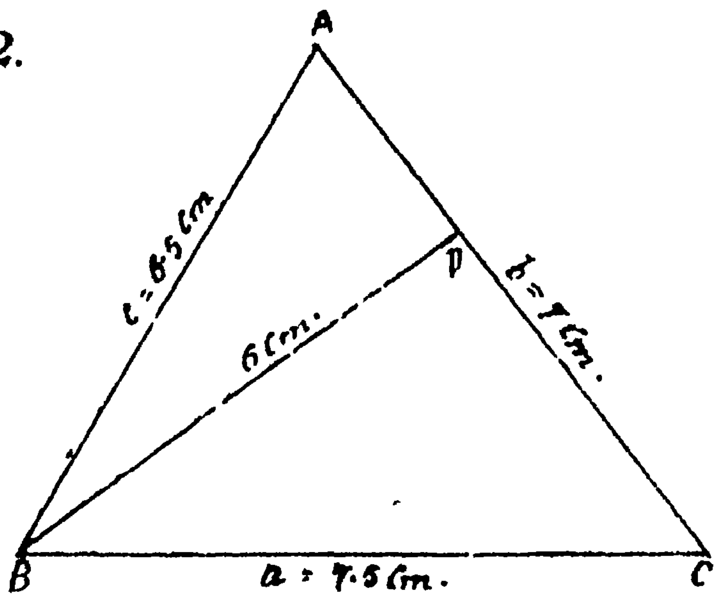

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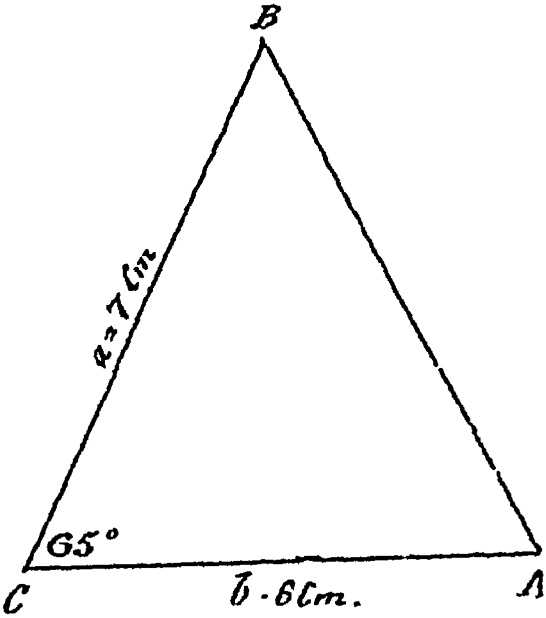

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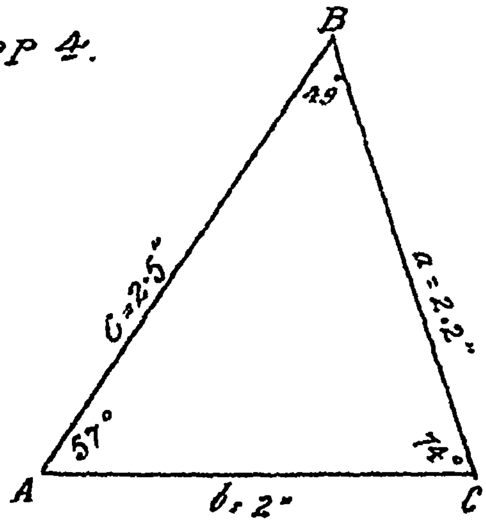

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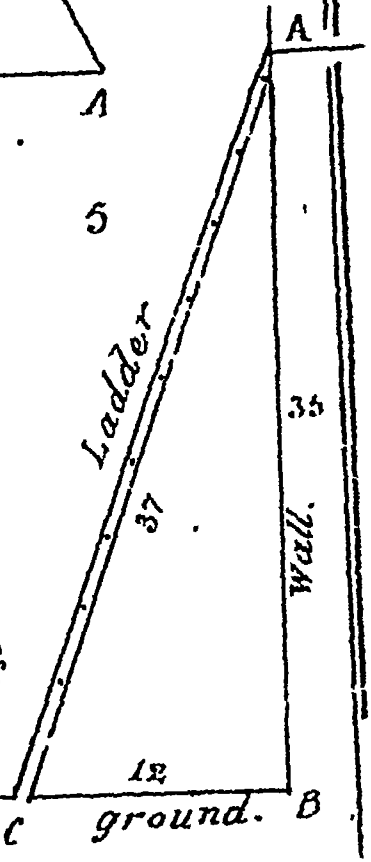

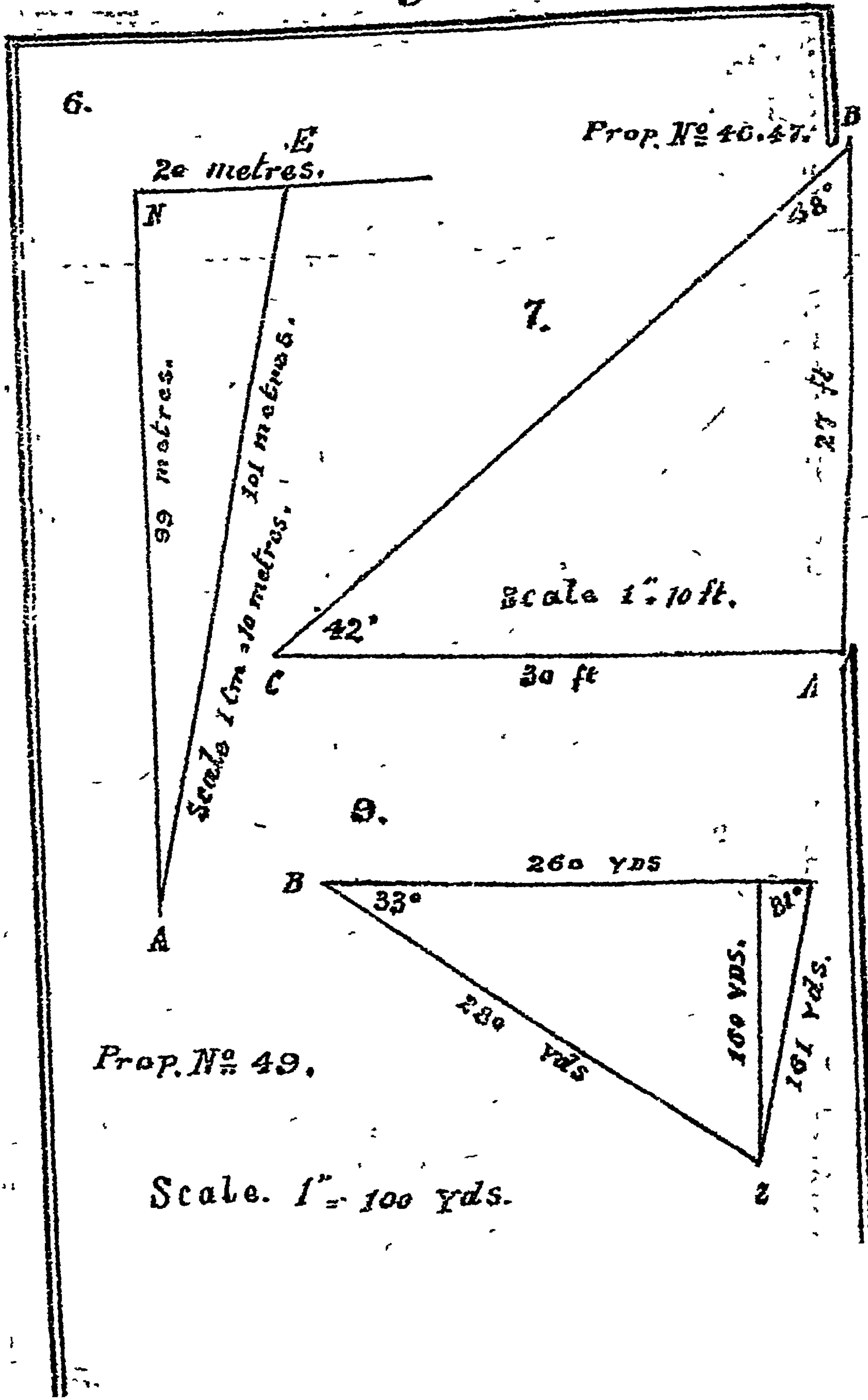
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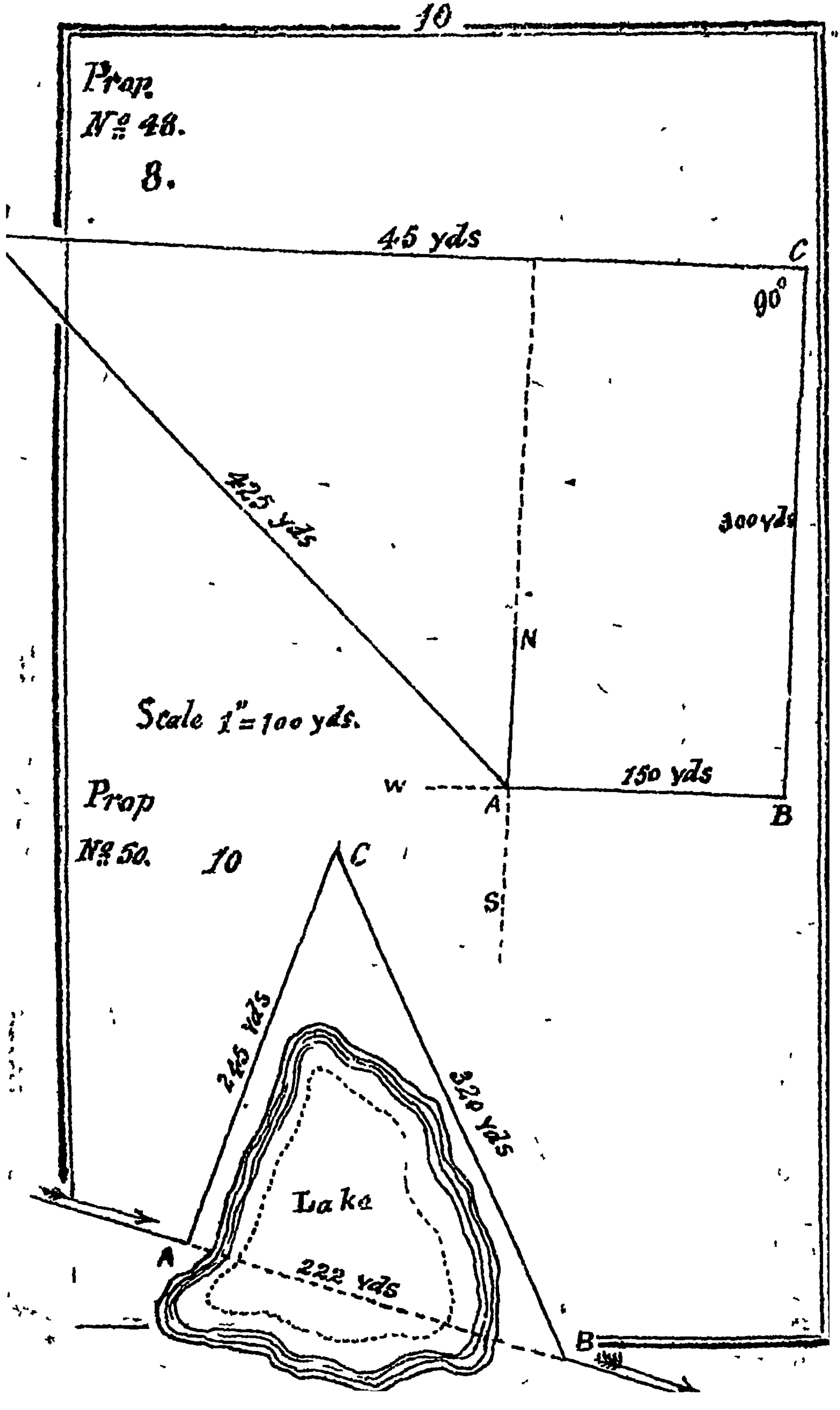
E
20 metres.
N
99 metres.
101 metres.
Scale 1 Cm = 10 metres.
A
Prop. No 46.47.
B
38°
7.
27 ft
Scale 1" = 10 ft.
42°
C
30 ft
A
8.
B
260 YDS
33°
81°
160 YDS.
161 Yds.
280 Yds
Prop. No 49.
Scale. 1" = 100 Yds.
Z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Prop.
No. 48.
8.
45 yds
C
90°
425 yds
300 yds
N
Scale 1" = 100 yds.
W
A
150 yds
B
Prop
No. 50.
10
C
S
245 yds
320 yds
Lake
A
222 yds
B

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Theor 8.

Exer.

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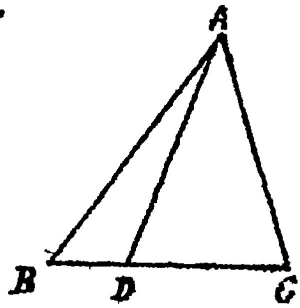

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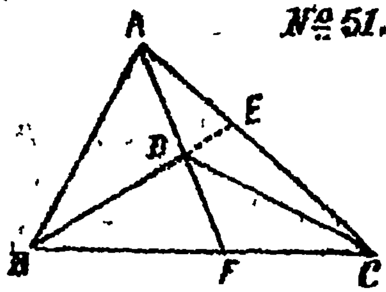

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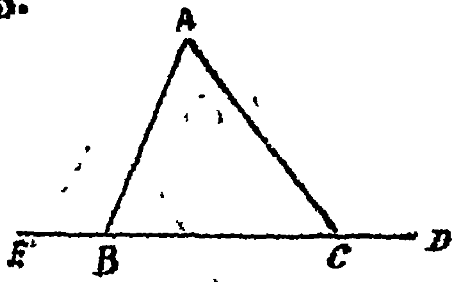

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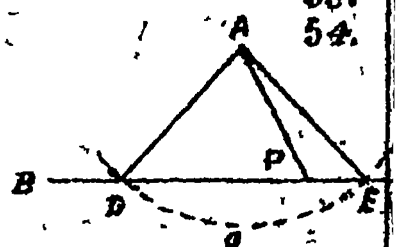

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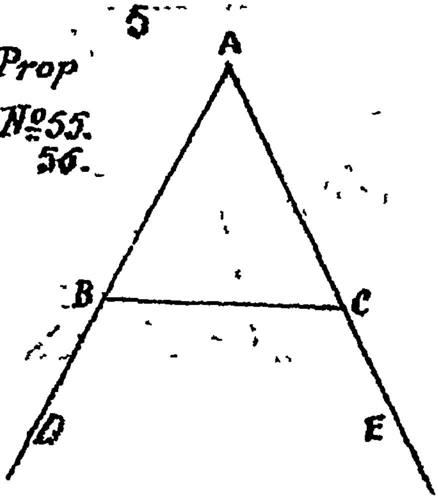

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Theor. 9-12.

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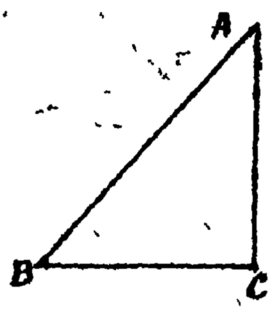

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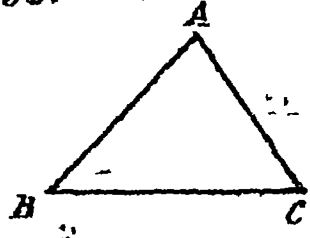

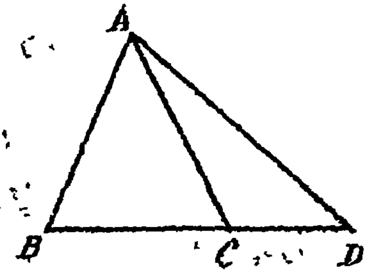

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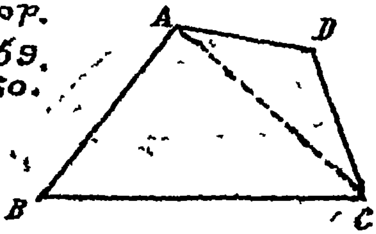

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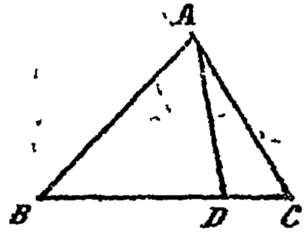

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64

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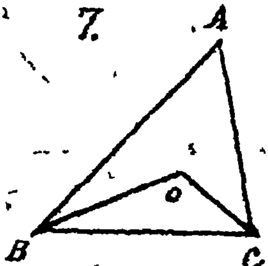

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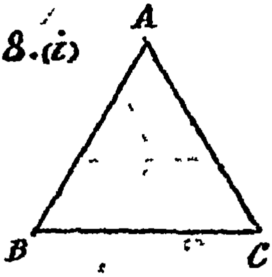

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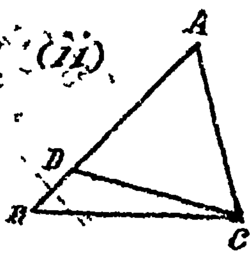

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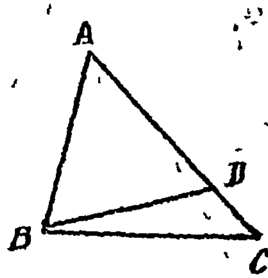

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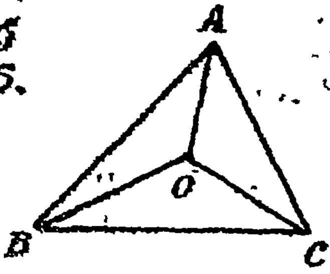

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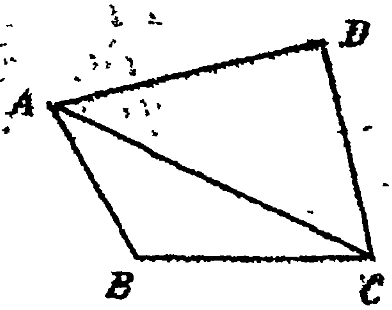

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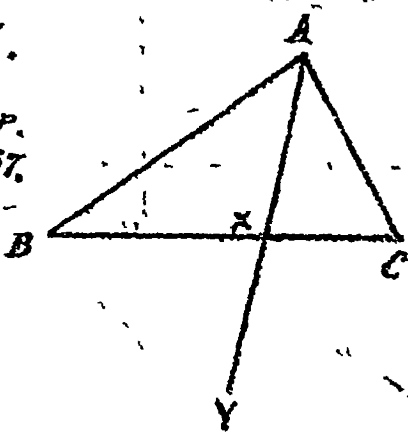

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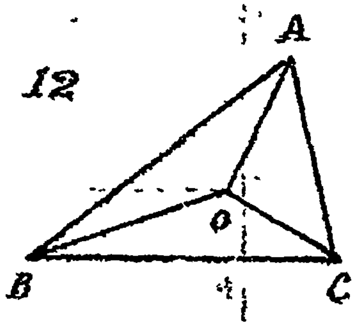

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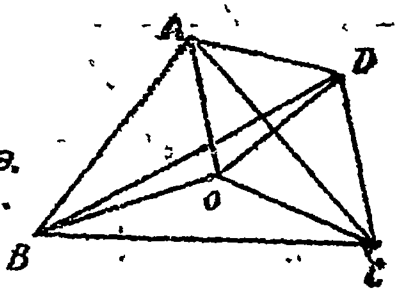

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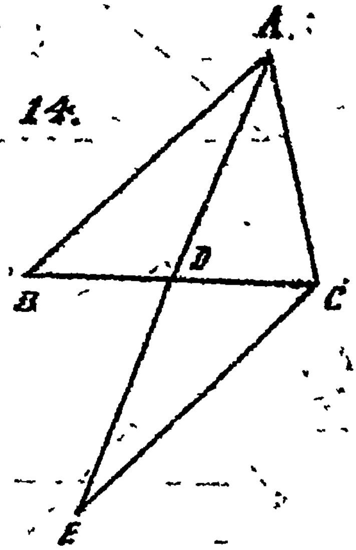

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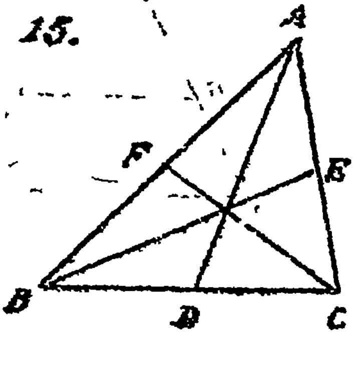

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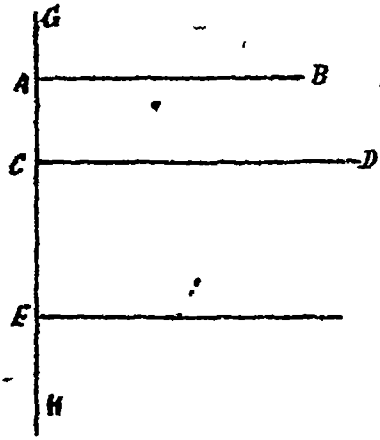

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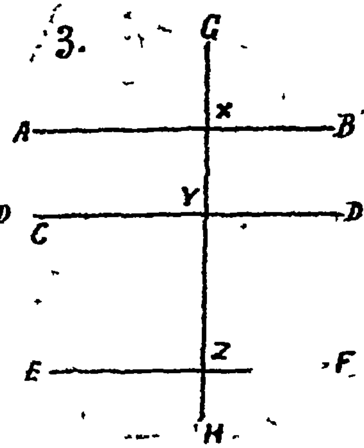

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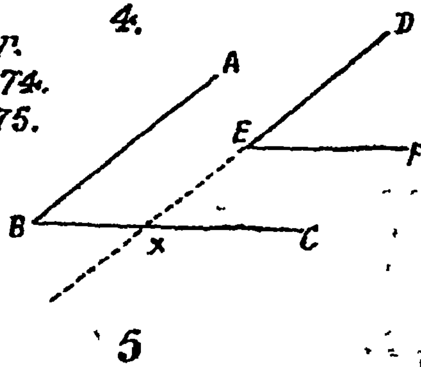

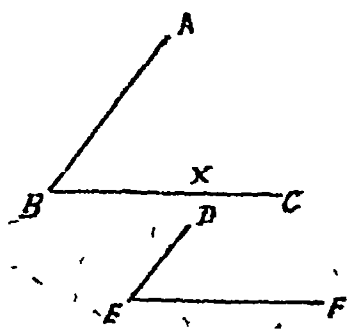

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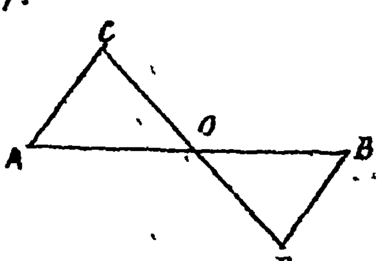

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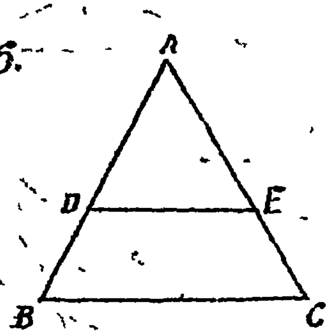

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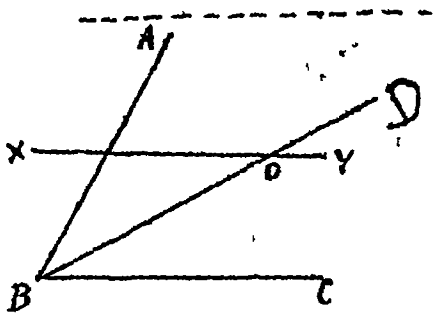

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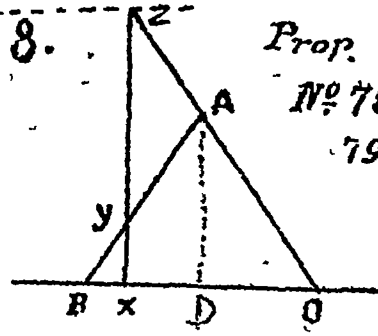

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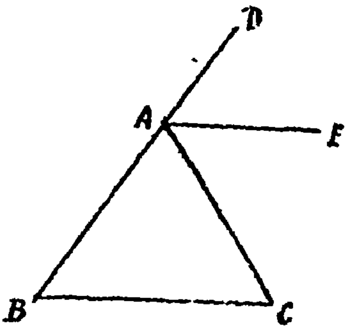

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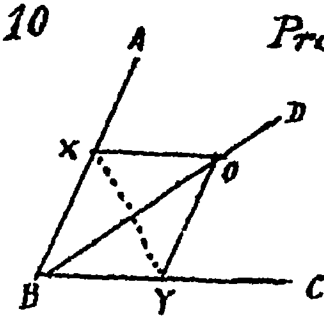

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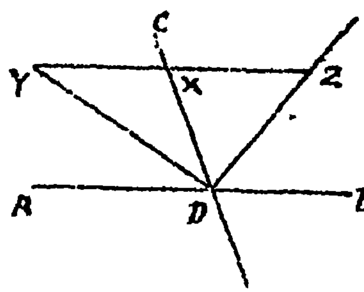

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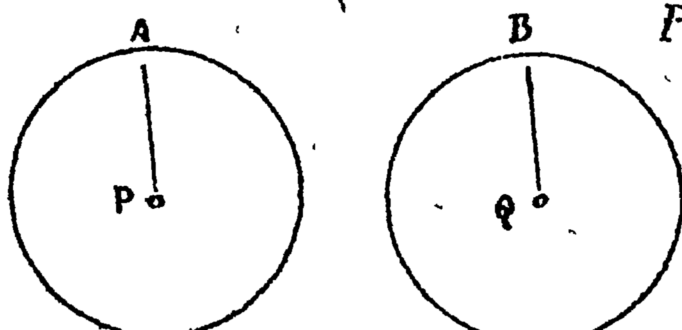

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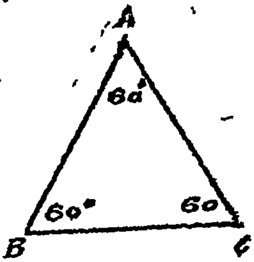
87.



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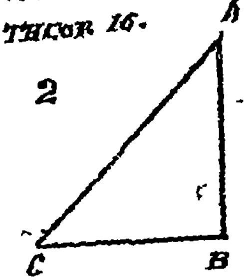

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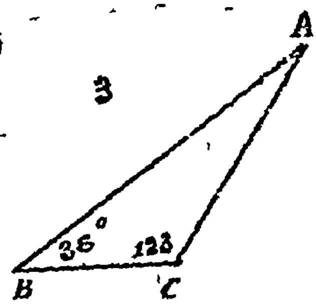

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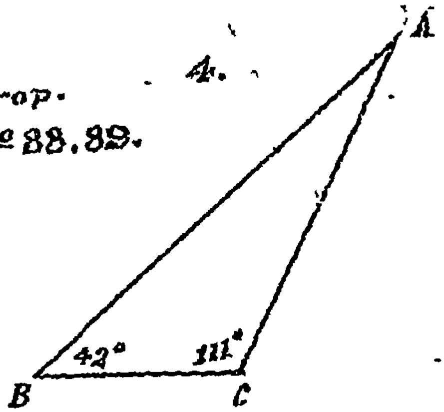

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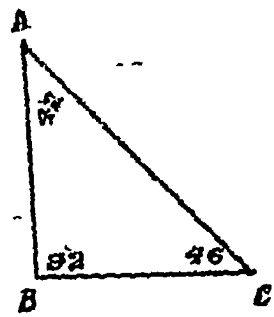

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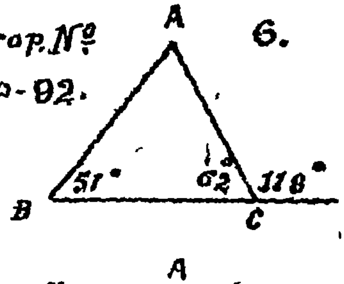

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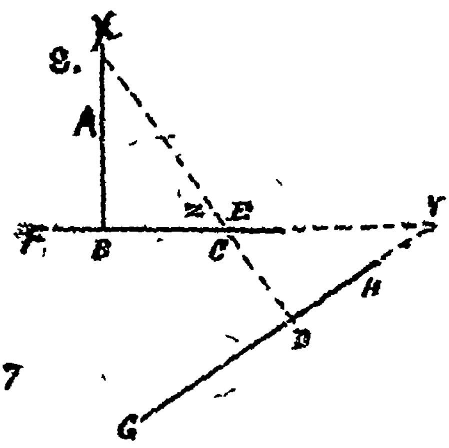

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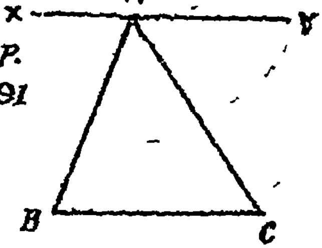

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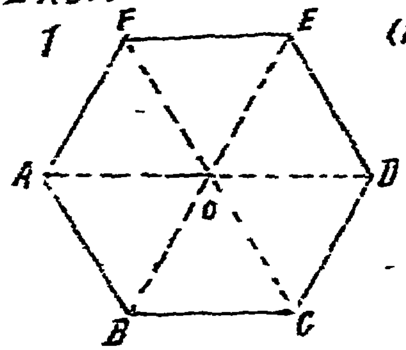

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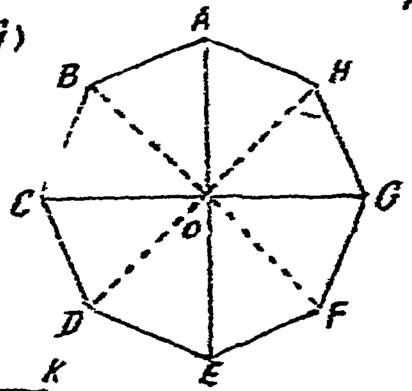

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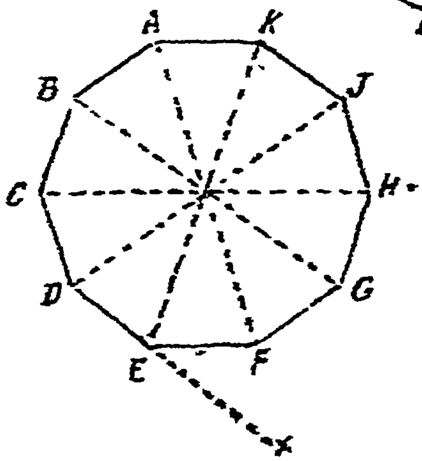

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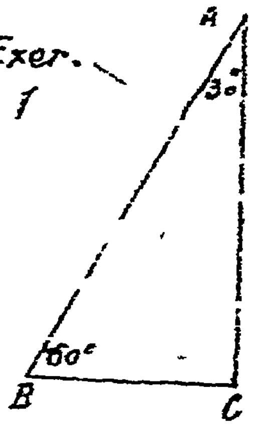

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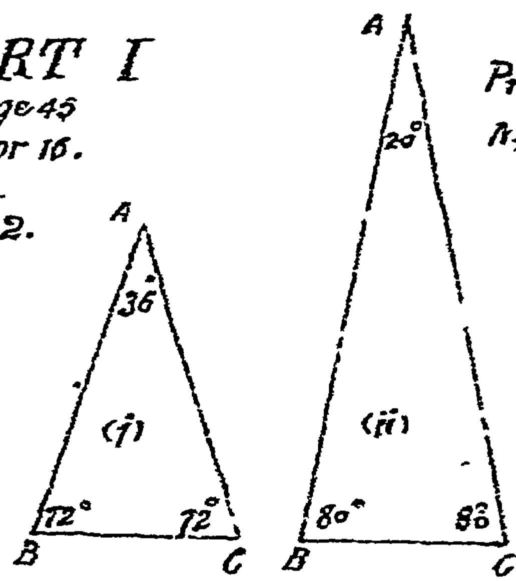

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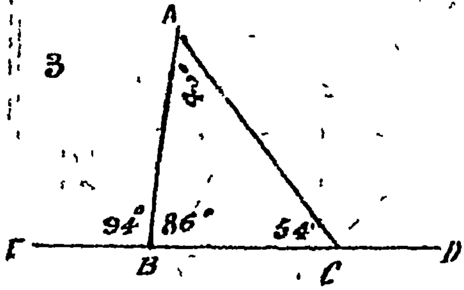

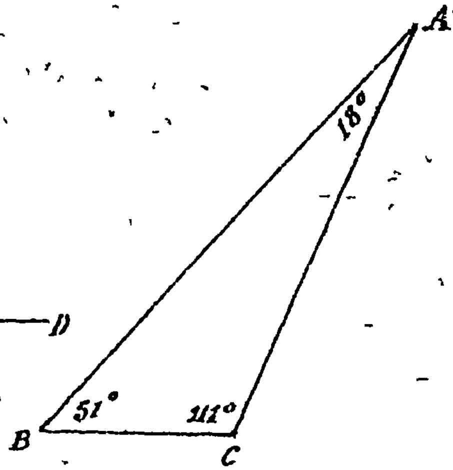

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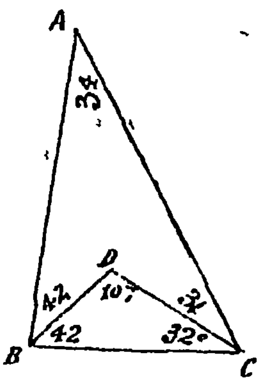

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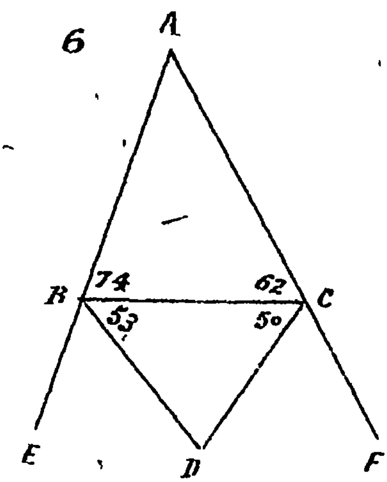

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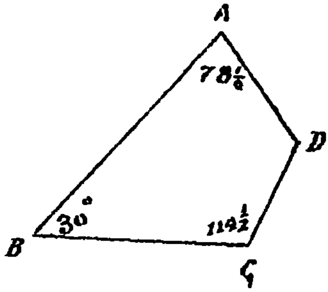

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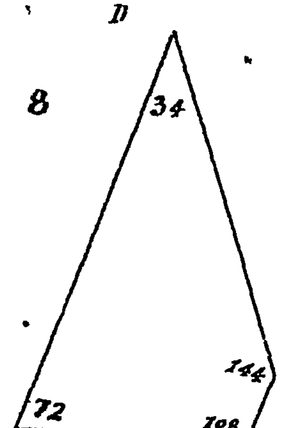

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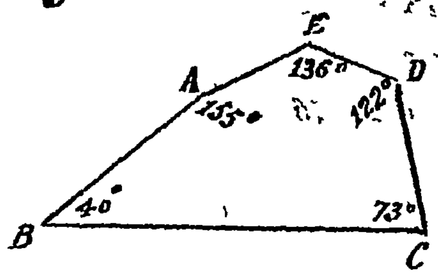

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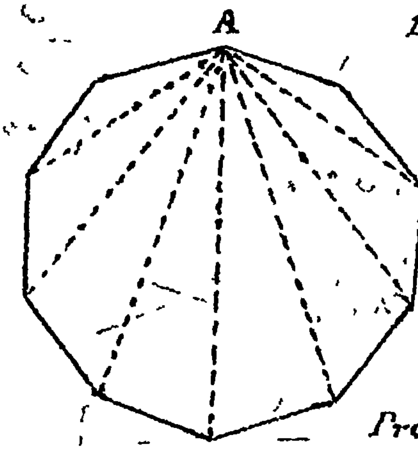

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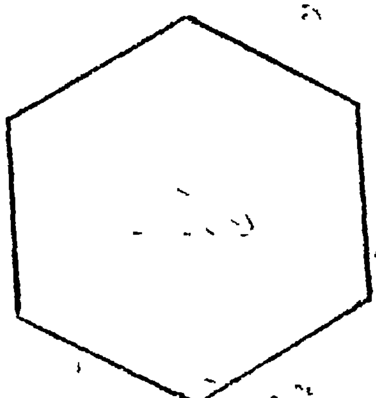

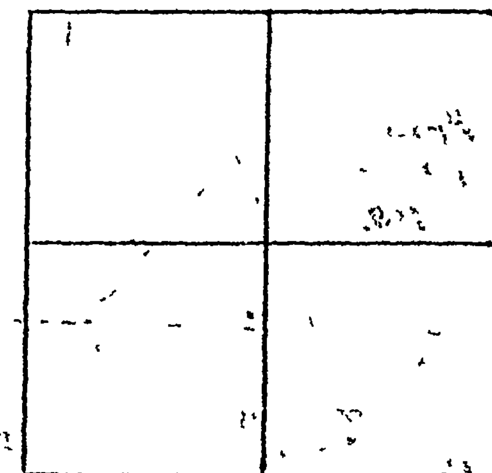

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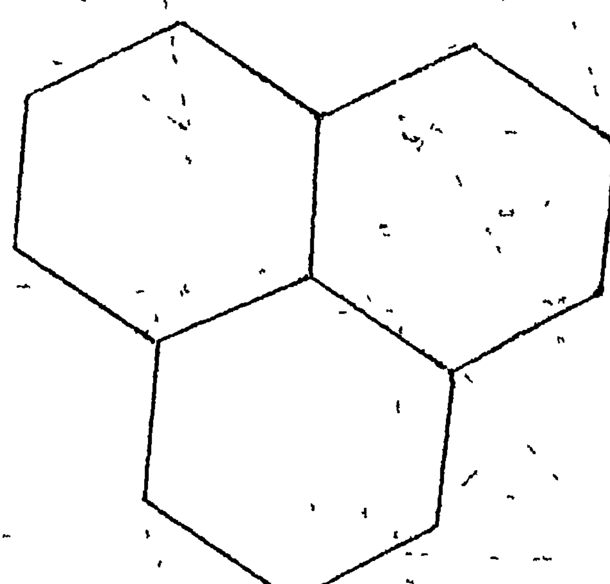

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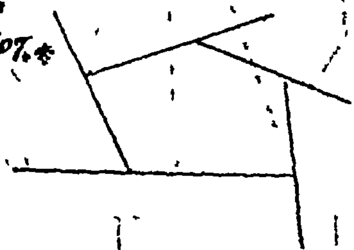
Cor 2 Theor 16.

Exer: 1

Prop No 107.*



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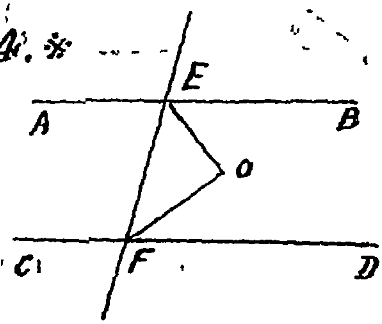

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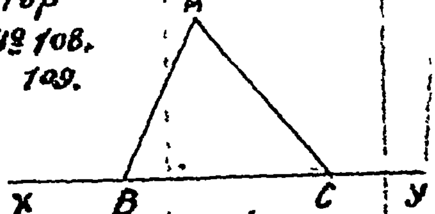
Prop No 108. 109.



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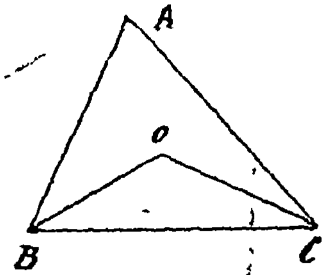

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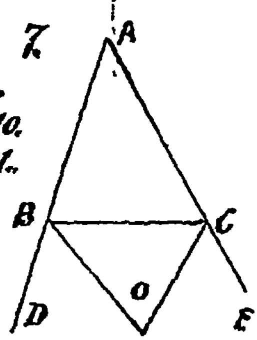
Prop. No 110. 111.



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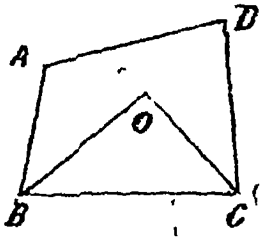

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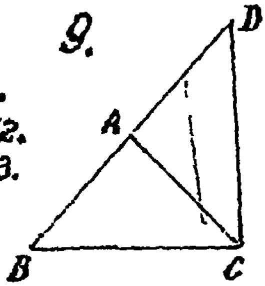

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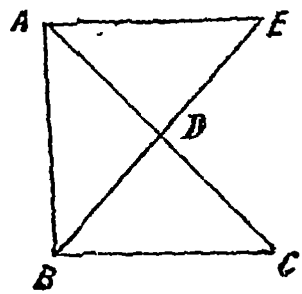

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Theor. 17.

Prop
№ 114
115.

Exer.

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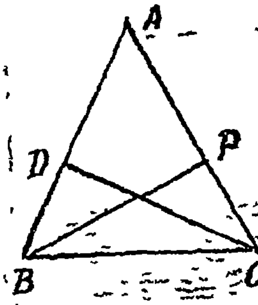

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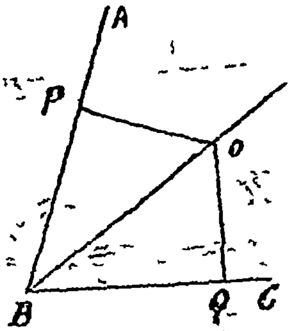

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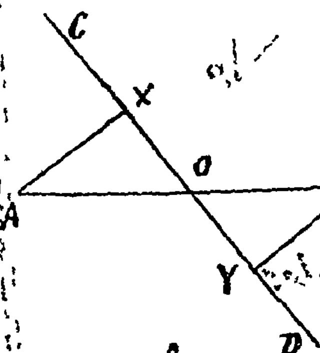

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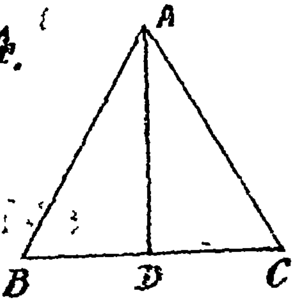
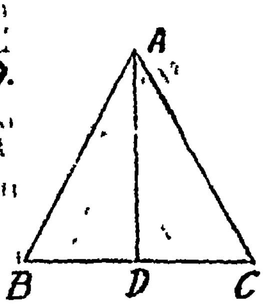


Prop.
№
116.
117.

5.



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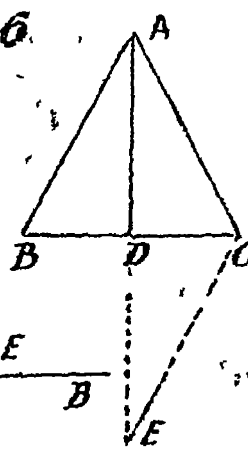
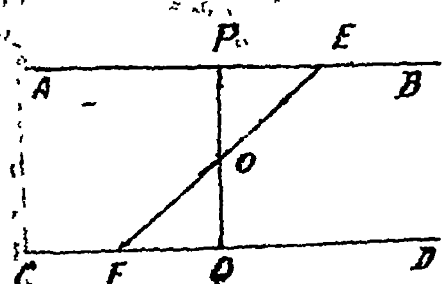


Prop.
№
118.
119.

7.



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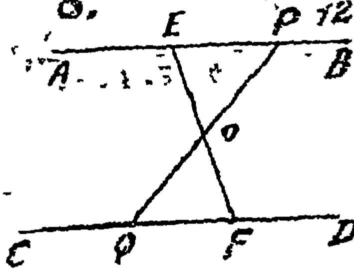
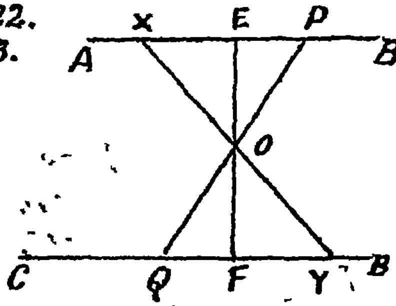


Prop.
№ 120.
121.

Prop. № 122. 123.

9



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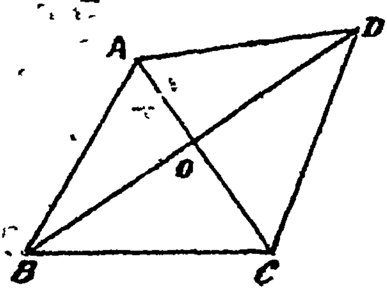

Prop. № 124.

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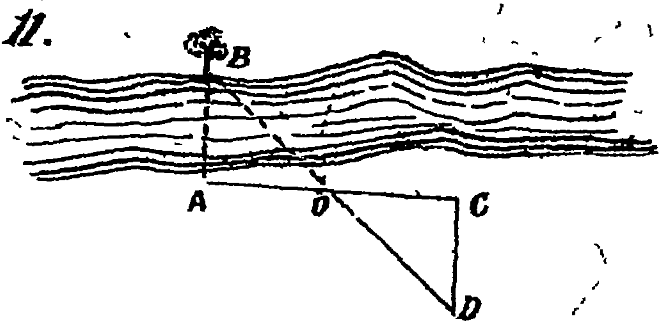

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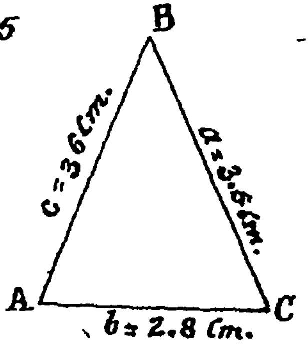
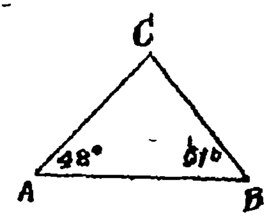
On Triangles

Prop. № 125 126 127.

Exer. 3



4 (i)



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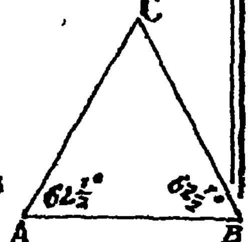

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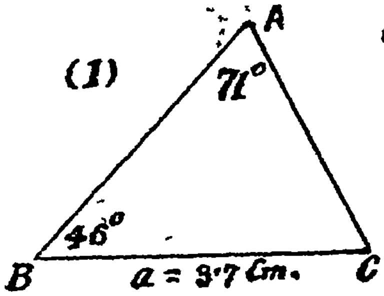
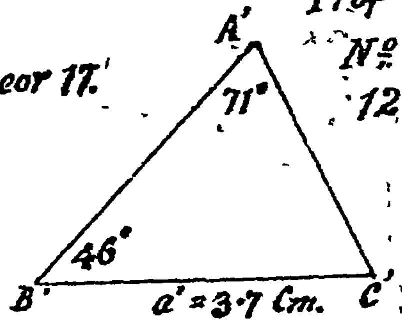
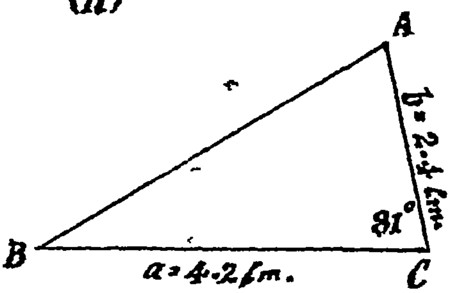
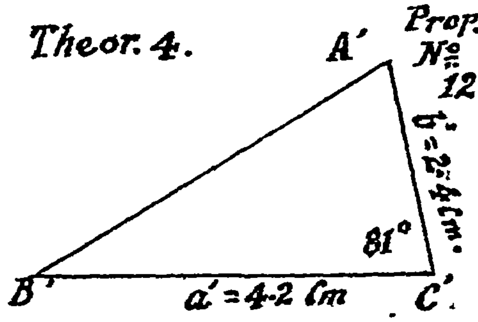
(I) Theor 17. Prop No. 128.



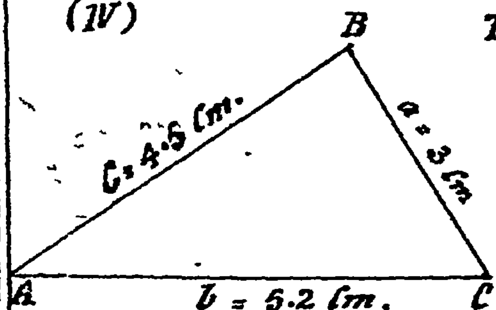


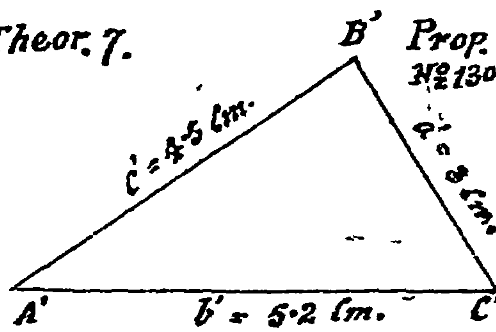
(ii) Theor. 4. Prop. No. 129.



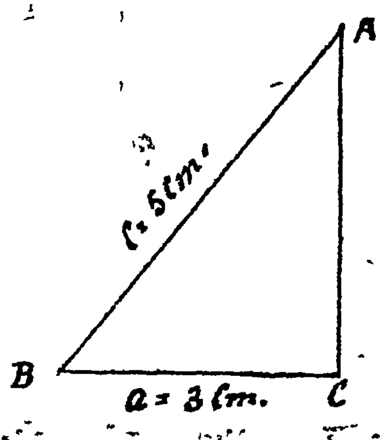


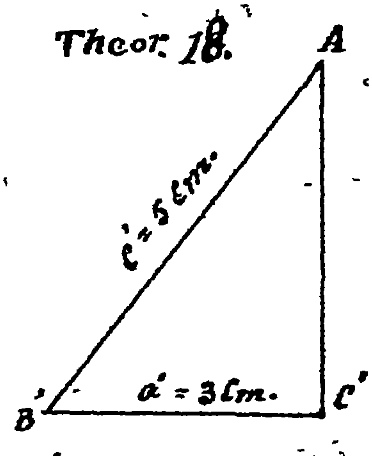
(IV) Theor. 7. Prop. No 130





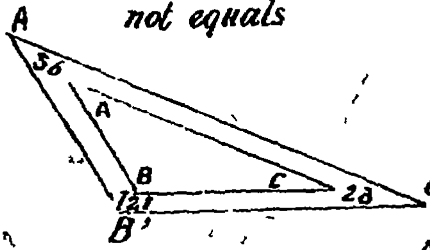
(VI) Theor. 18. Prop. No 131

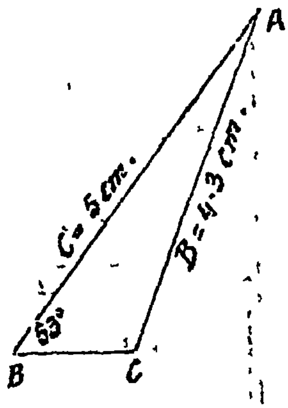


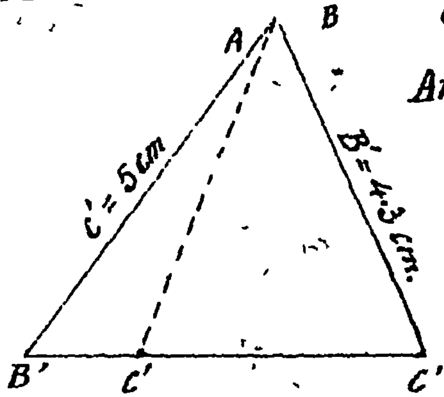


Prop. (iii)
No. 132.

Triangles not equals

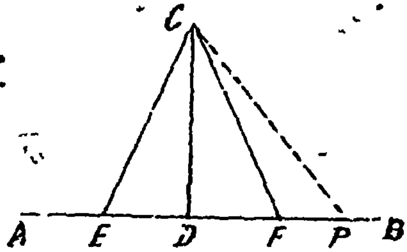






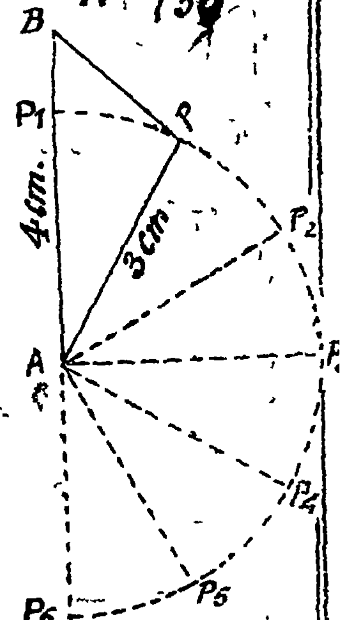
Ambiguious case.

Prop. 8.
No. 134.
~~136.~~



No 130

11.



Prop. 12
No 137.

Scale
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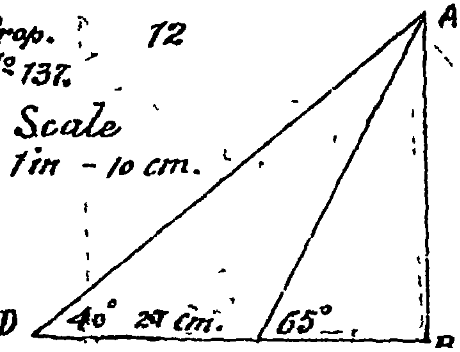
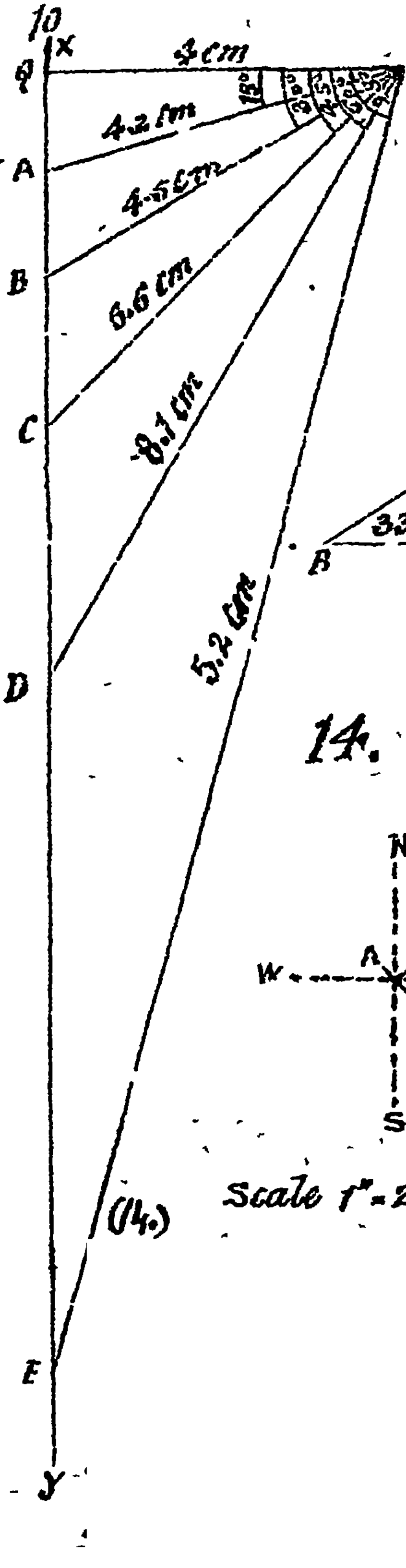
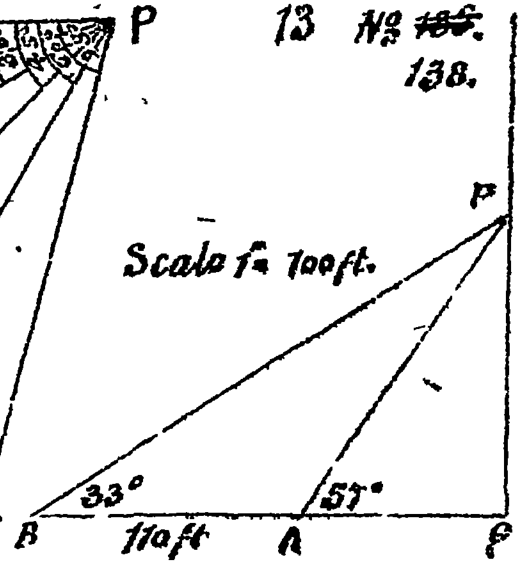


10

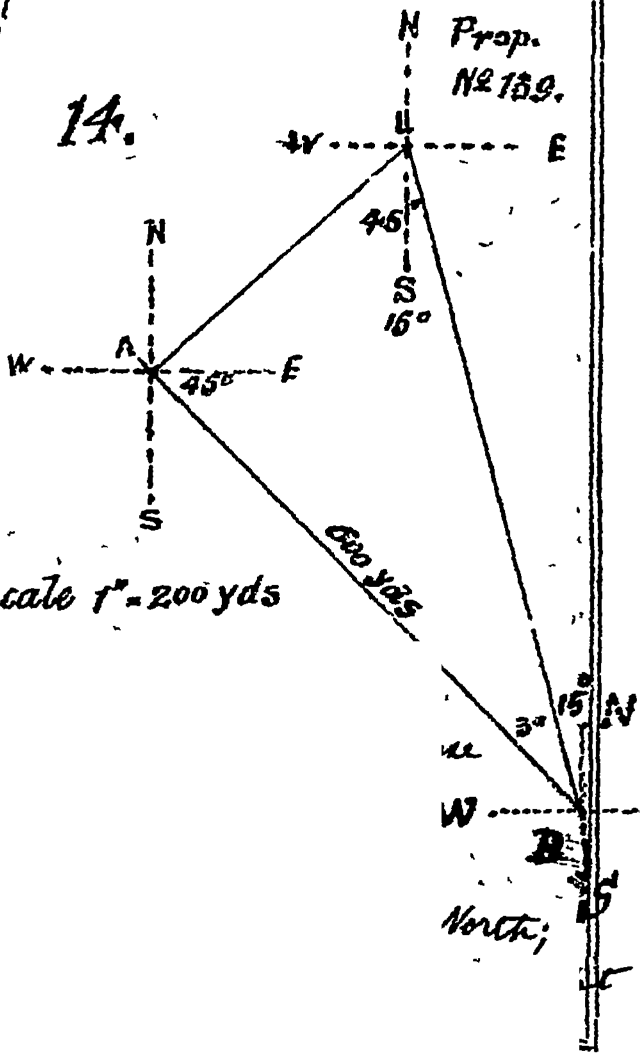
Prof. Nº 135



13 Prop. Nº ~~135.~~ 138.



14. Prop. Nº 139.



Scale 1" = 200 yds

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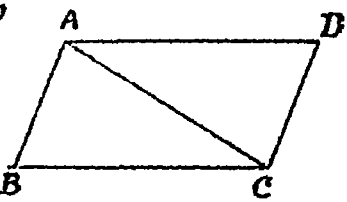
Prop. No. 140 141.



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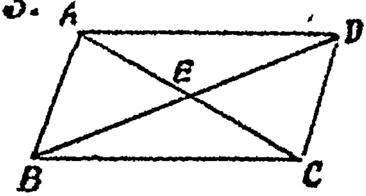

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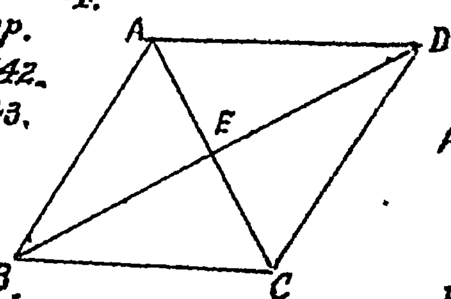
Prop. No. 142. 143.



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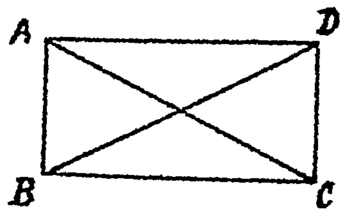

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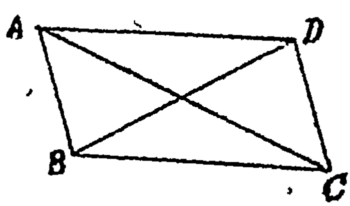
Prop. No. 144.



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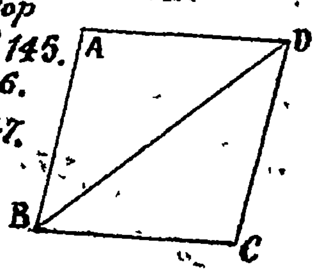
Prop No. 145. 146. 147.



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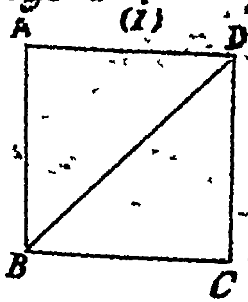
(i)



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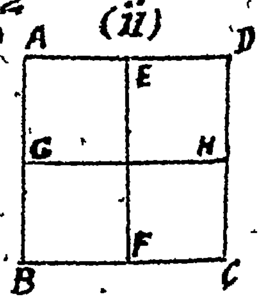

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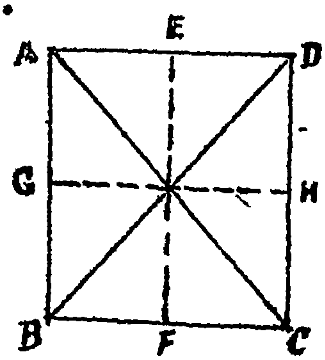

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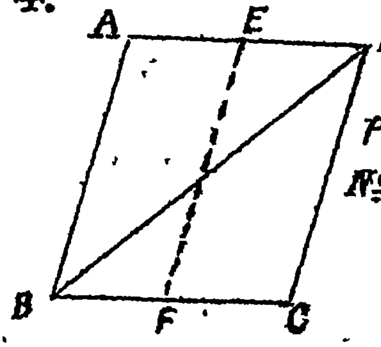
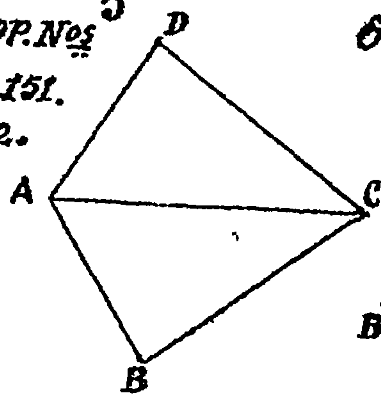


Prop
№ 148
149.

Prop. №s
150.151.
152.

5



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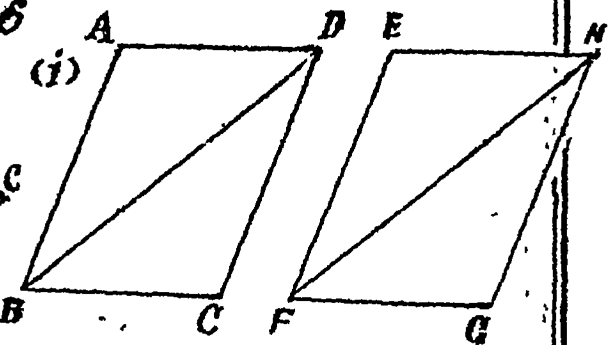

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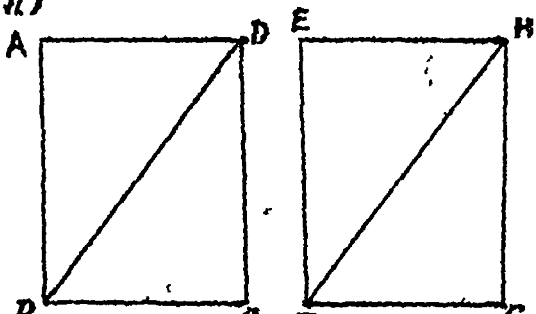

Prop. №
153. 154.

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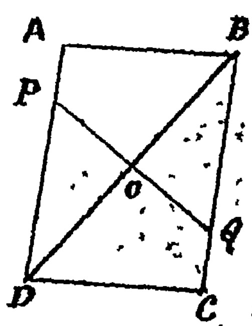

Prop.
№ 159.

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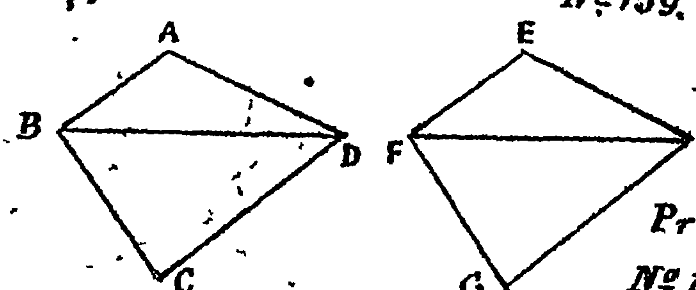
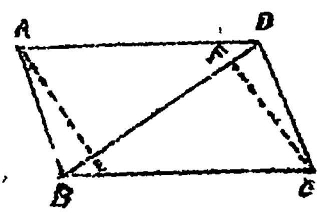


Prop
№ 155. 156.

Prop
№ 160, 161

9



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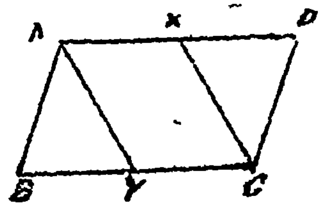

Prop
№ 162, 163

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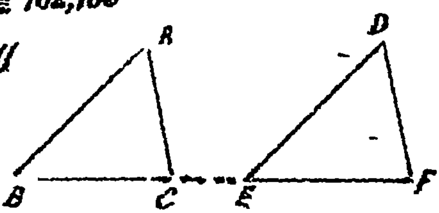

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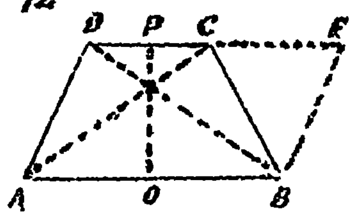
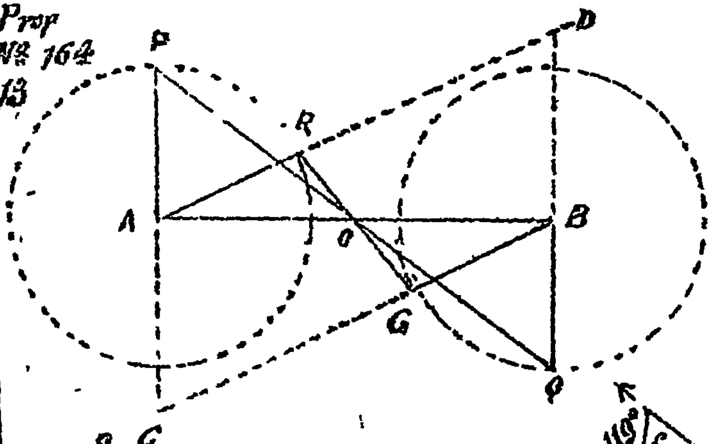


Prop
№ 164

13



Prop
№ 165
166

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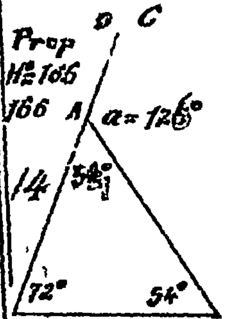

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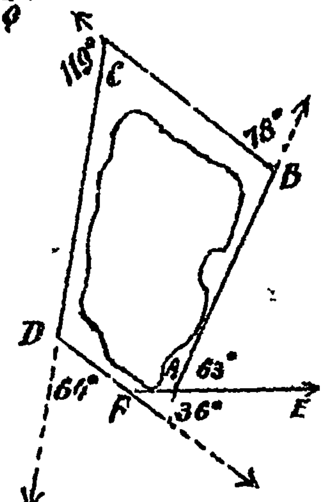

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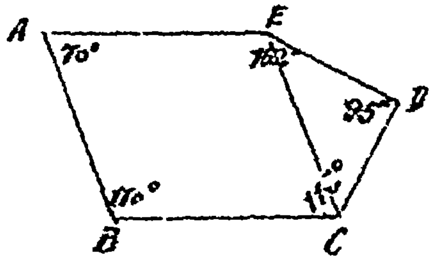
Prop.
№ 167



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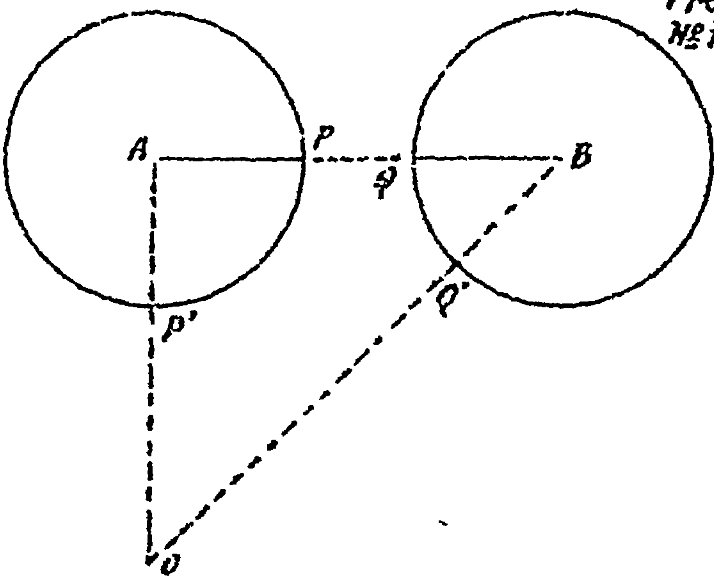
Prop
№ 168.



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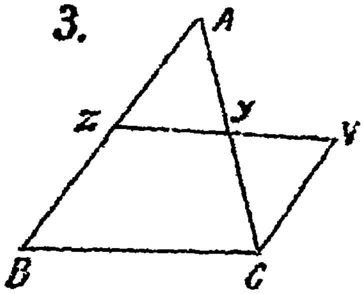
Theor 22.

Exer

1 & 2. one solved in the Book.

Prop
№ 169.
170.

3.



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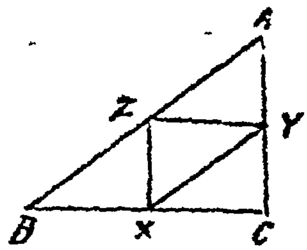

Prop. No 171. 172.

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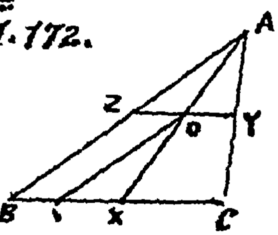

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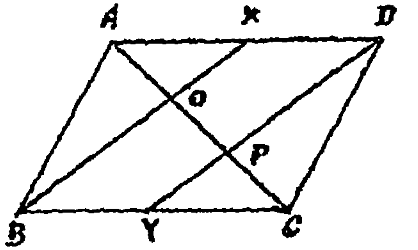

Prop. No 173-174.

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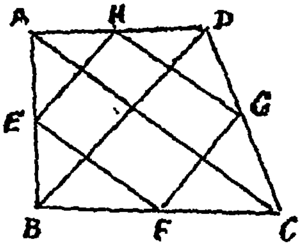

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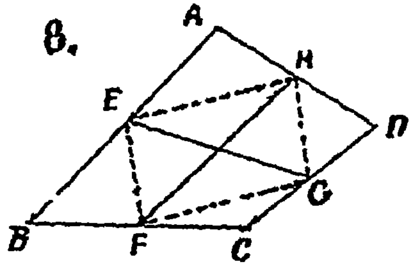
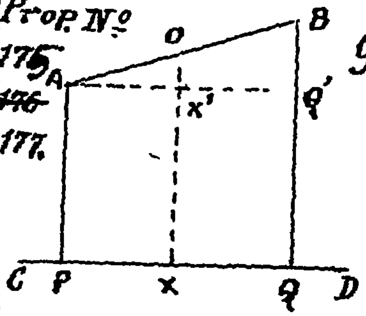
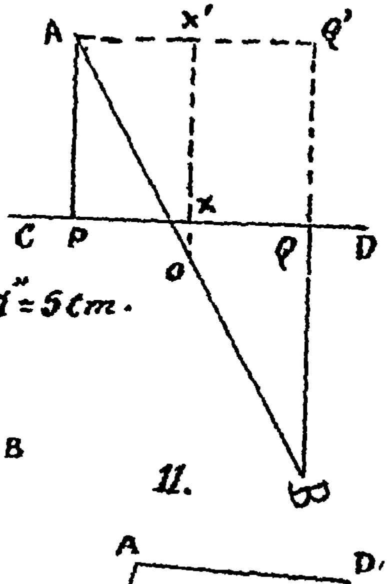


Prop. No 175
~~176~~
177.

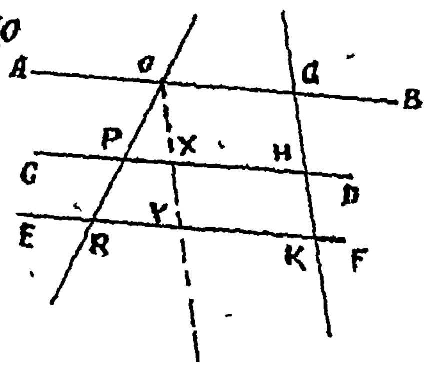


9.

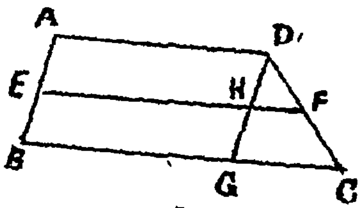


Prop. No 178. 179 Scale 1" = 5 cm.

10

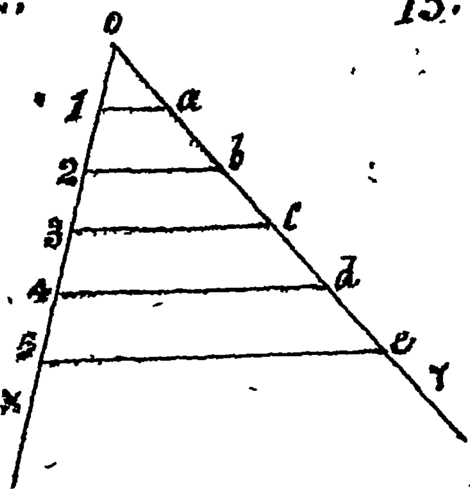


11.

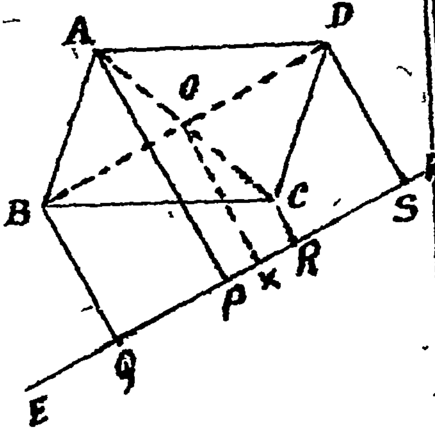


Prop. No. 180.
181

12.

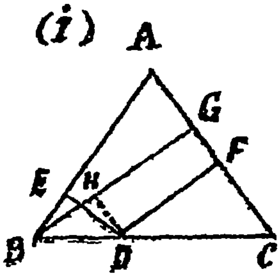


13.

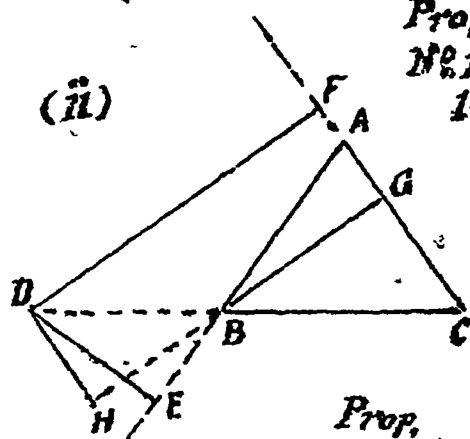


Prop.
No. 182.
183.

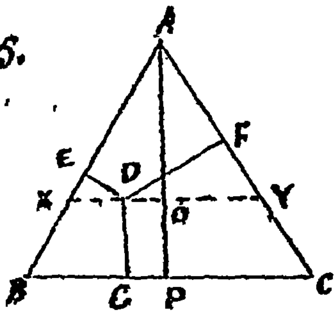
14. (i)



(ii)

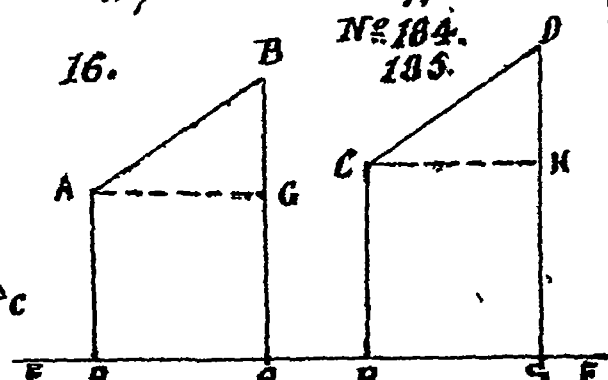


15.



Prop.
No. 184.
185.

16.



PART I

Page 68.

On linear measurement

Exer.

1

1·25″

2·72″

3·08″

2

2·68″

6·8 cm. or $\frac{2\cdot68}{0\cdot3937} = 6\cdot75$ cm.

3.

5·7 cm. $5\cdot7 \times \cdot3937$

2·25″ nearly $= 2\cdot44''$

4

A B

The line represents 3·15″

By measure A.B. is found 7·93 cm.

∴ 1 cm = 0·39″ in.

5 A 2·9 cm B

C 6·2 cm. D

(i) By measure AB = 1·15″ in.

(ii) " " CD = 2·47″ in.

∴ from (i) 1″ = 2·52 cm.

& from (ii) 1″ = 2·57 cm.

$2\,\underline{)\,5\cdot09}$

∴ average = 2·54 cm.

6.

3.36" = 336 miles

4.08" = 408 miles

7.

0.85" = 850 Km.

2.98" = 2980 metres.

1.01" = 1010 metres.

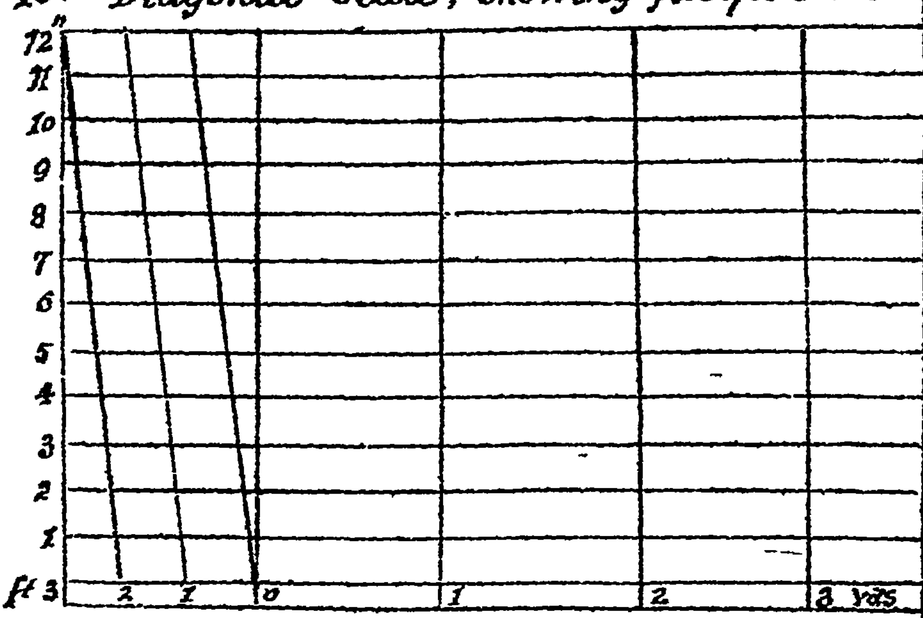
8.

4.17" = 417 Links = 10.6 cm.

9.

42.5 Km = 8.5 cm. = 3.35"

13. Diagonal Scale, showing yds. fts & ins



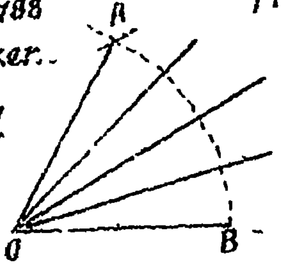
PART. I.

Page 79.

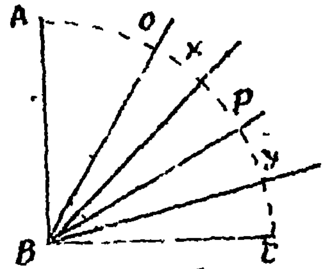
Prob 1.7.

Prop
No 187
188
Exer.

1

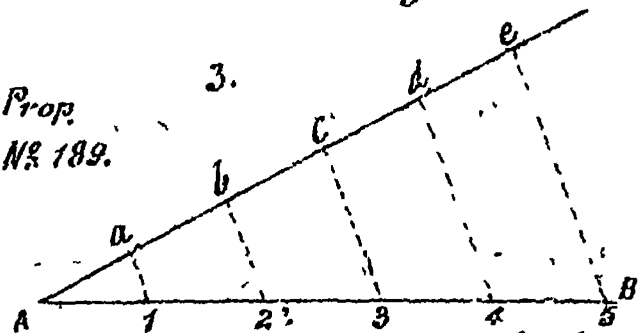


2



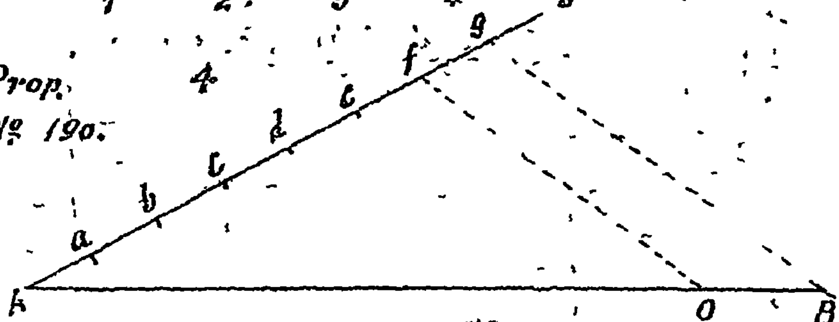
Prop.
No 189.

3.

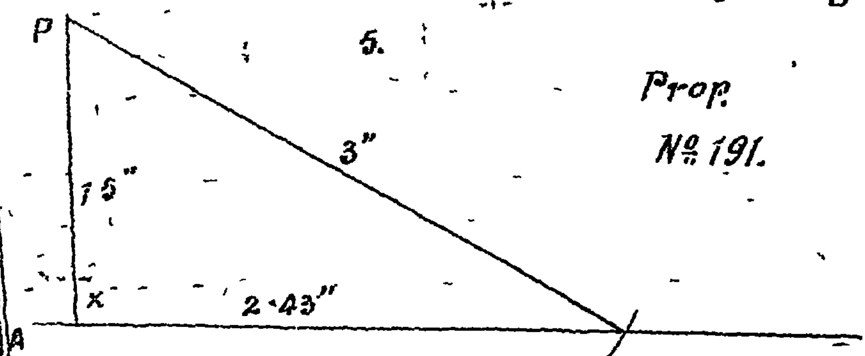


Prop.
No 190.

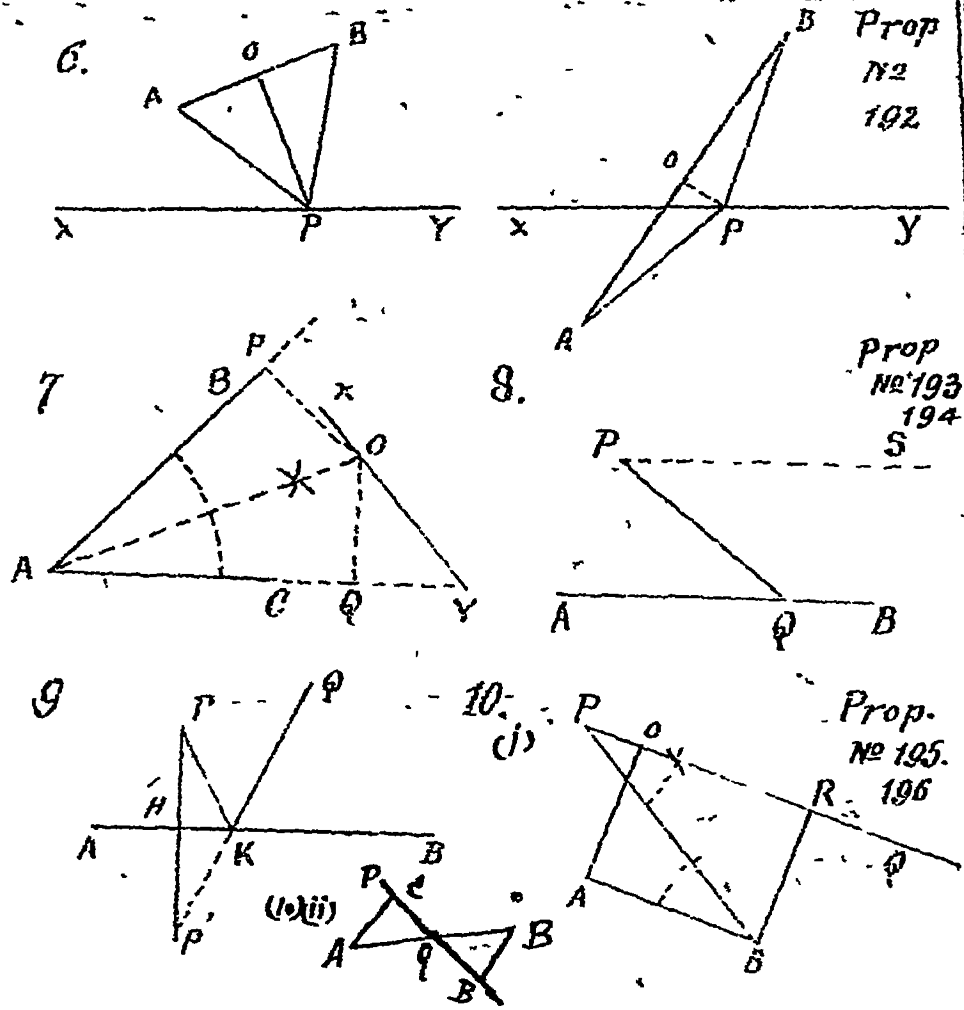
4



5.

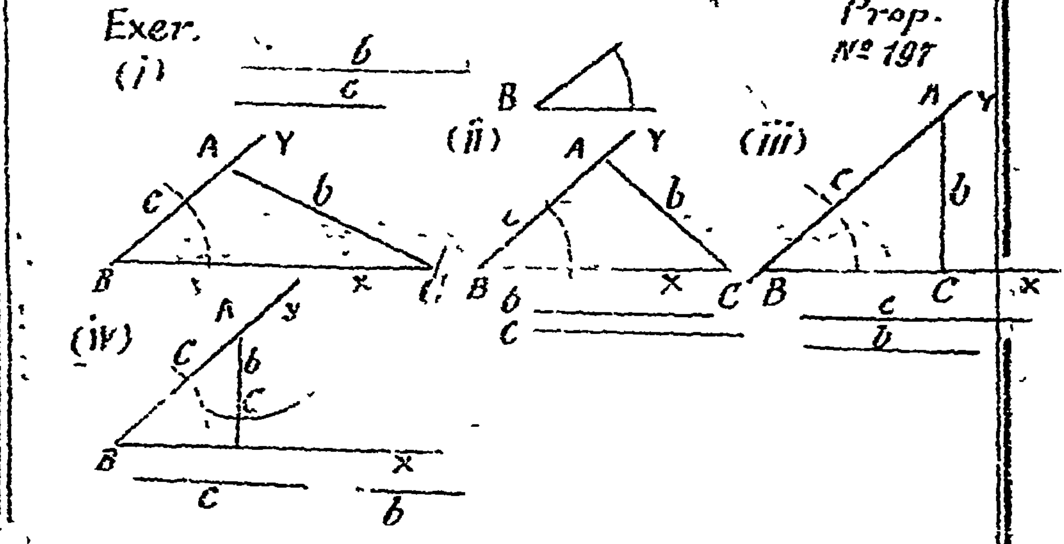


Prop.
No 191.



PART I.

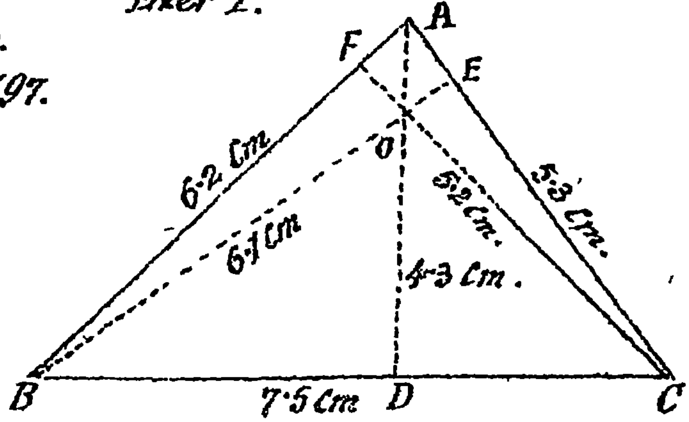
Page 82



PART I
Page 84
Prob. 8-10.

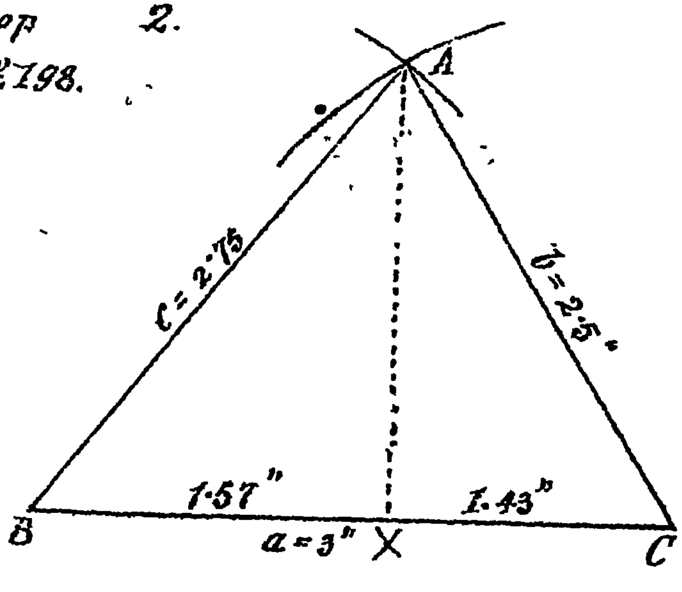
Exer 1.

Prop.
No. 197.



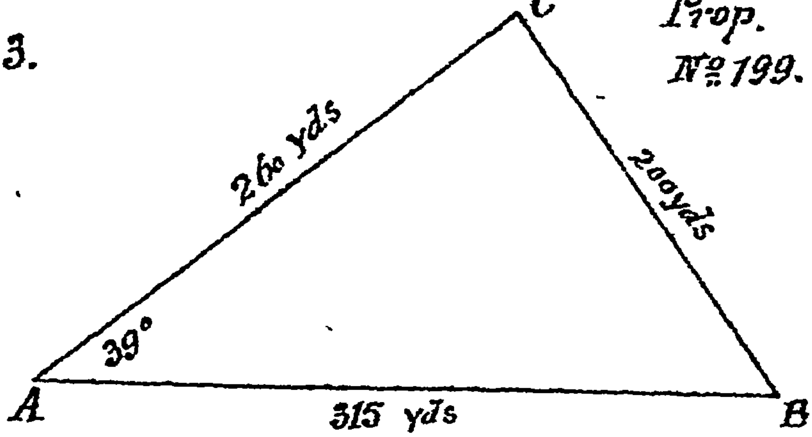
Prop
No. 198.

2.



3.

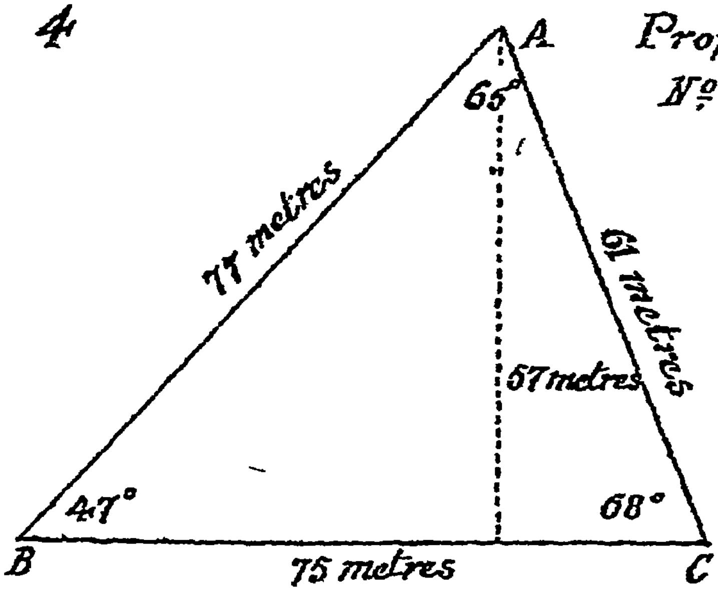
Prop.
No. 199.



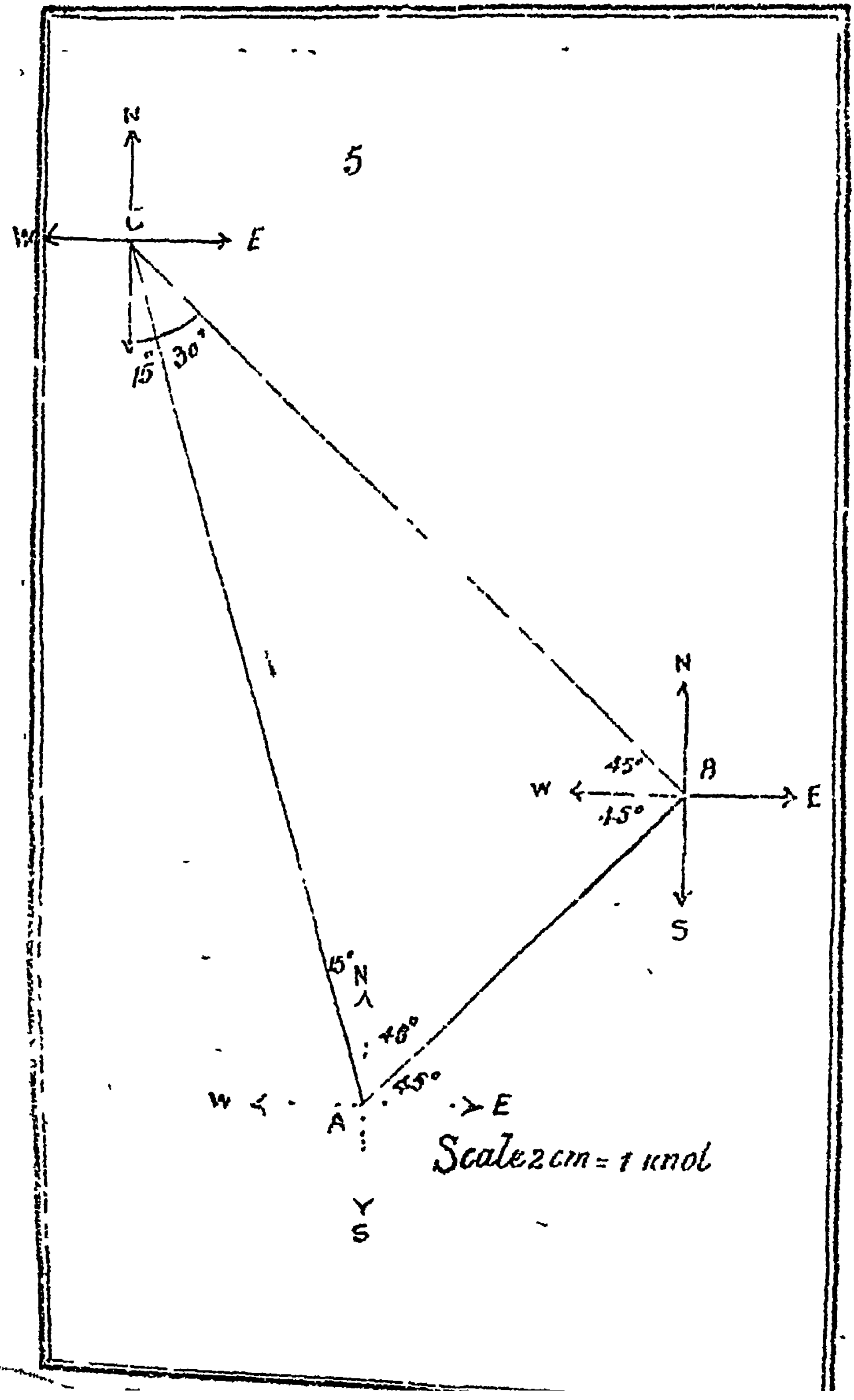
Scale 1" = 100 yds.

4

Prop.
No. 200.

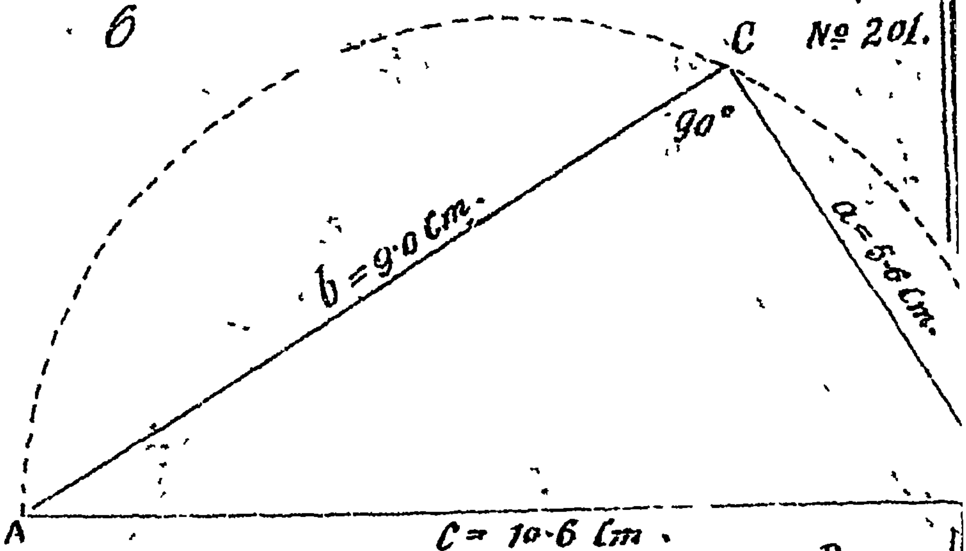


Scale 1 cm = 10 metres.


5
N
W
E
C
15°
30°
N
45°
B
W
E
15°
S
15°
N
48°
45°
W
E
A
S
Scale 2cm = 1 knot

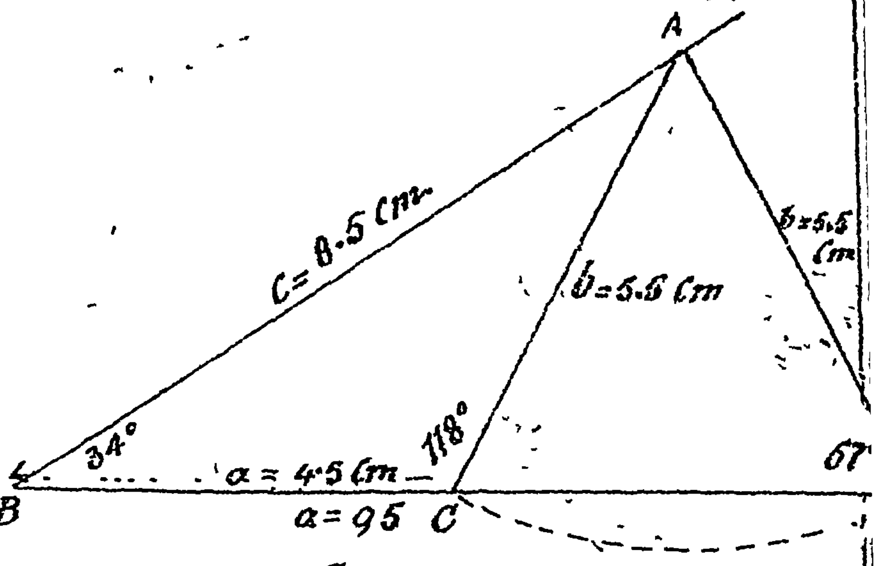
6

Prop. № 201.



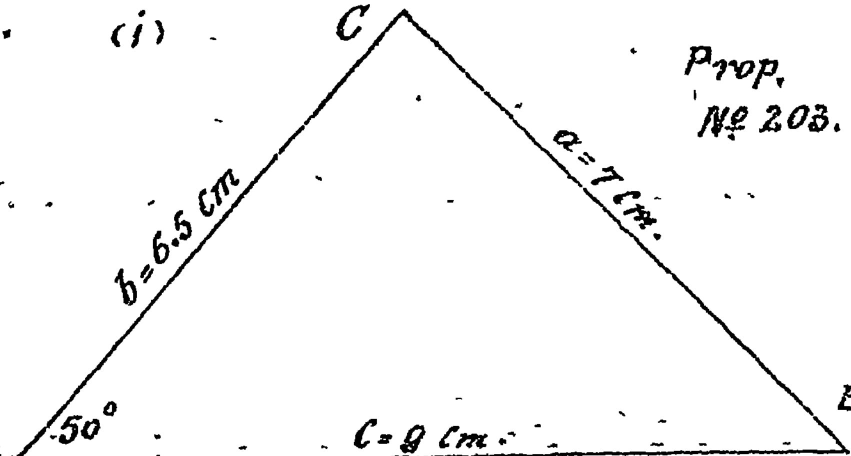
7.

Prop. № 202.



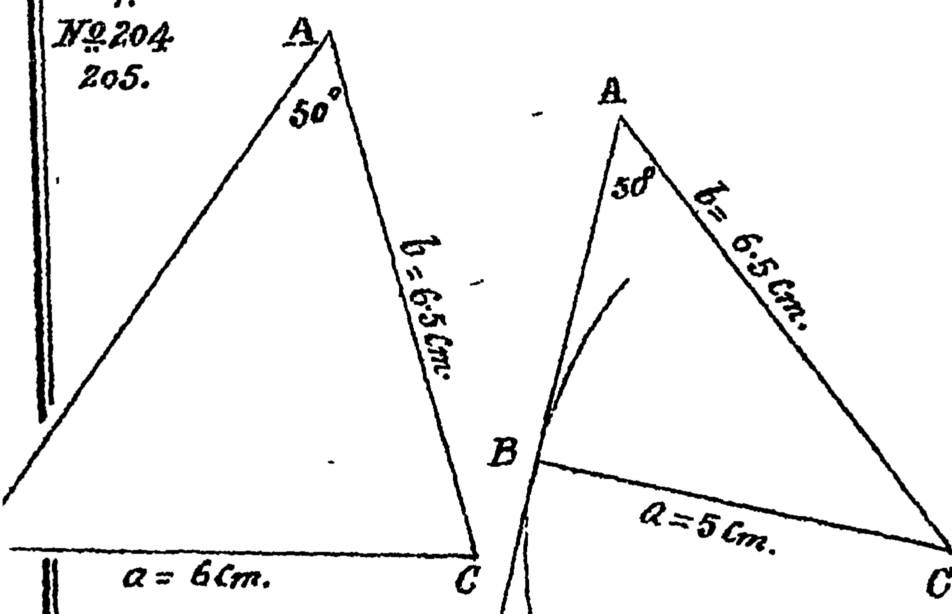
8. (i)

Prop. № 203.



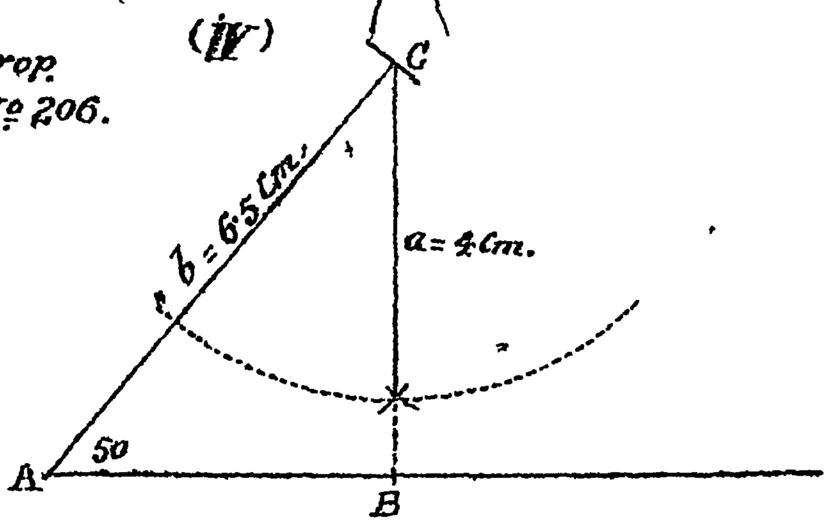
Prop.
No. 204
205.

(ii) (iii)



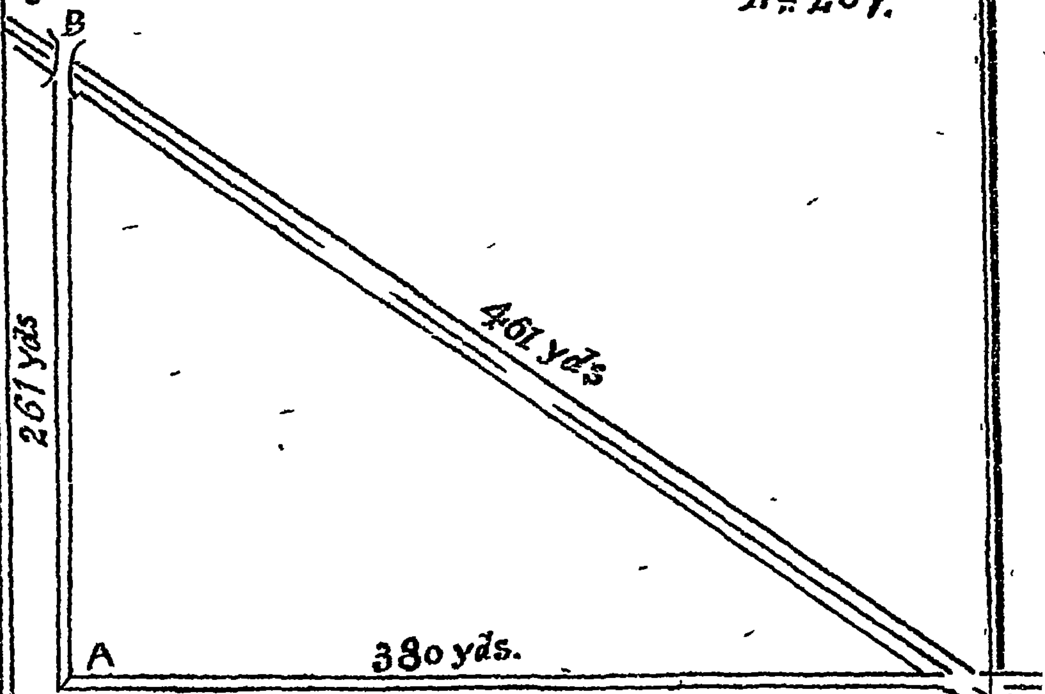
Prop.
No. 206.

(iv)



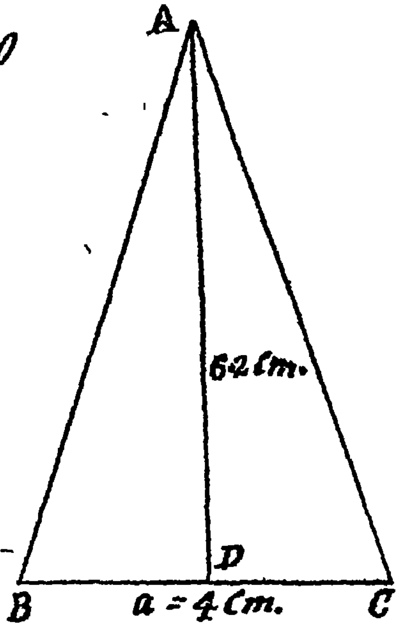
Prop.
No. 207.

9



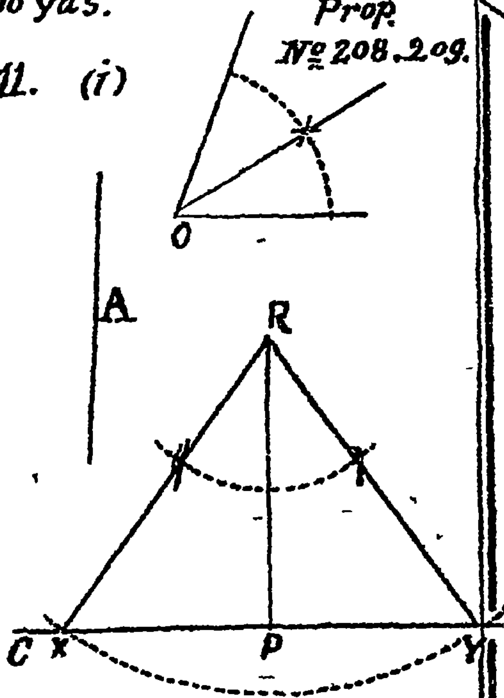
Scale 1" = 100 yds.

10



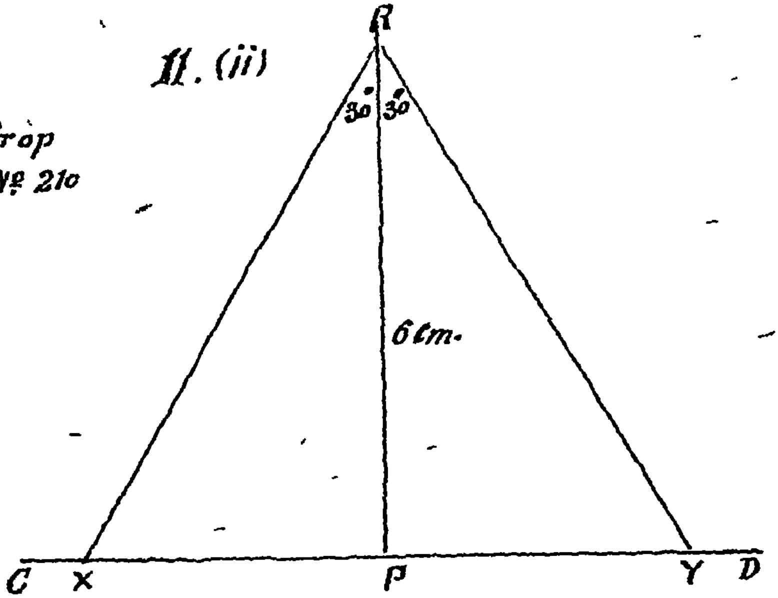
11. (i)

Prop.
No. 208. 209.



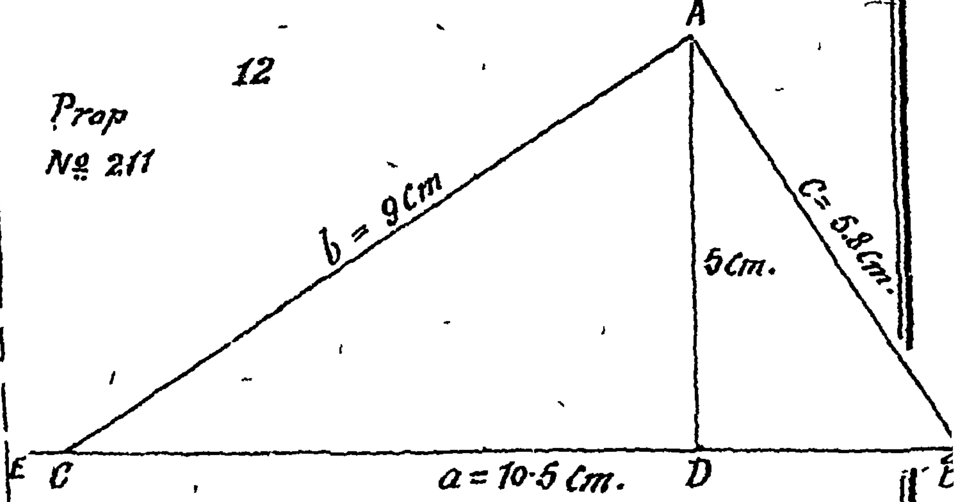
11. (ii)

Prop
No. 210



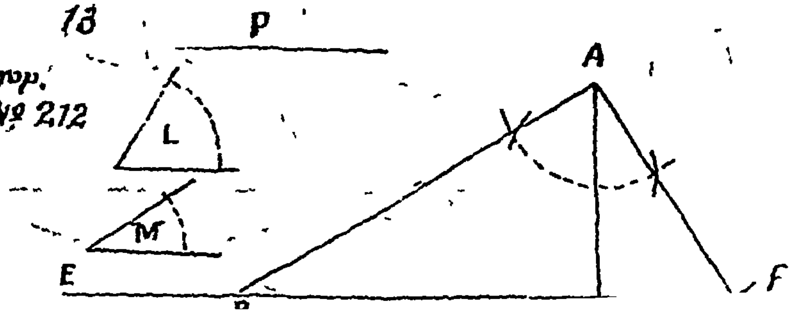
12

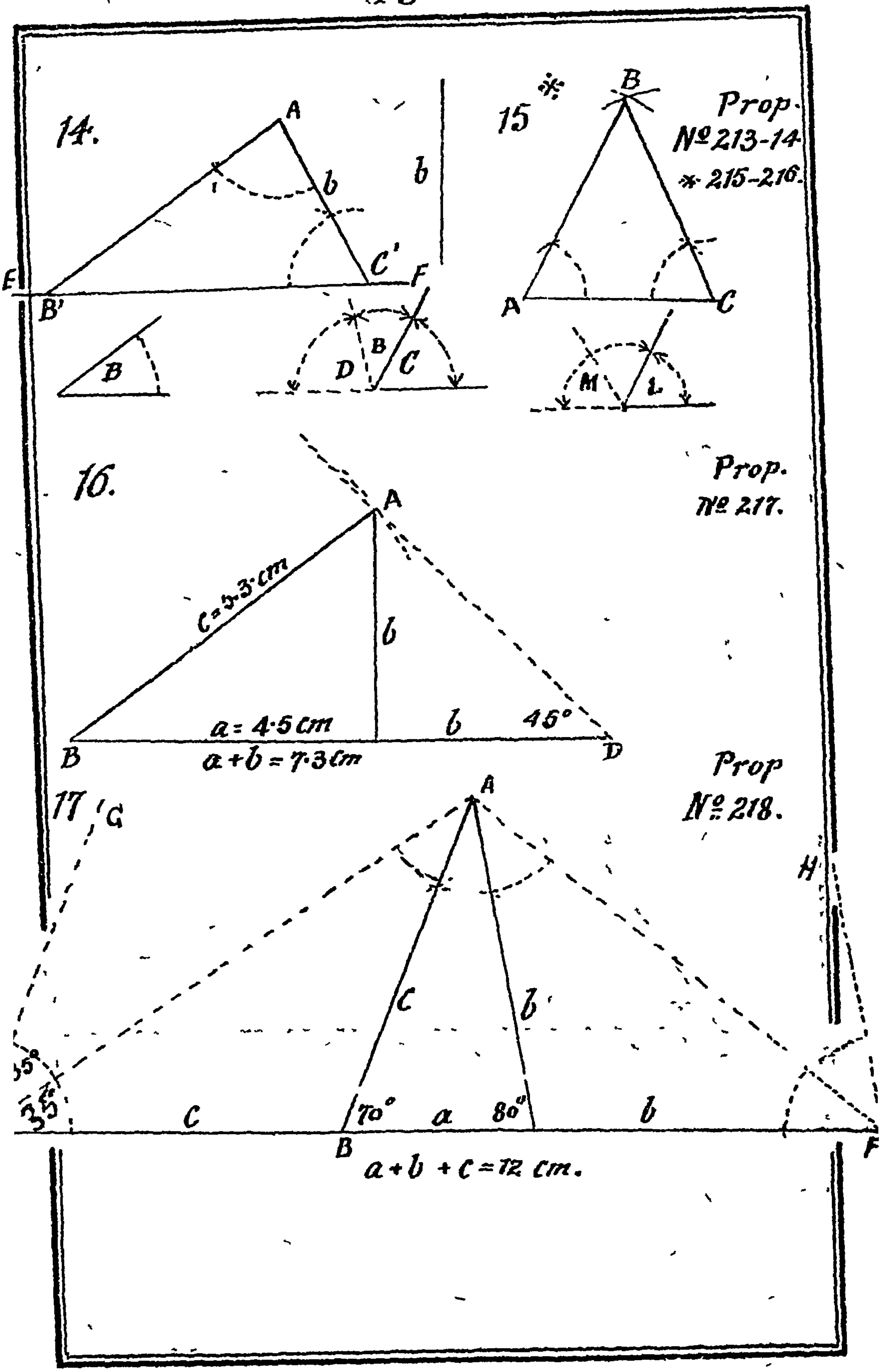
Prop
No. 211



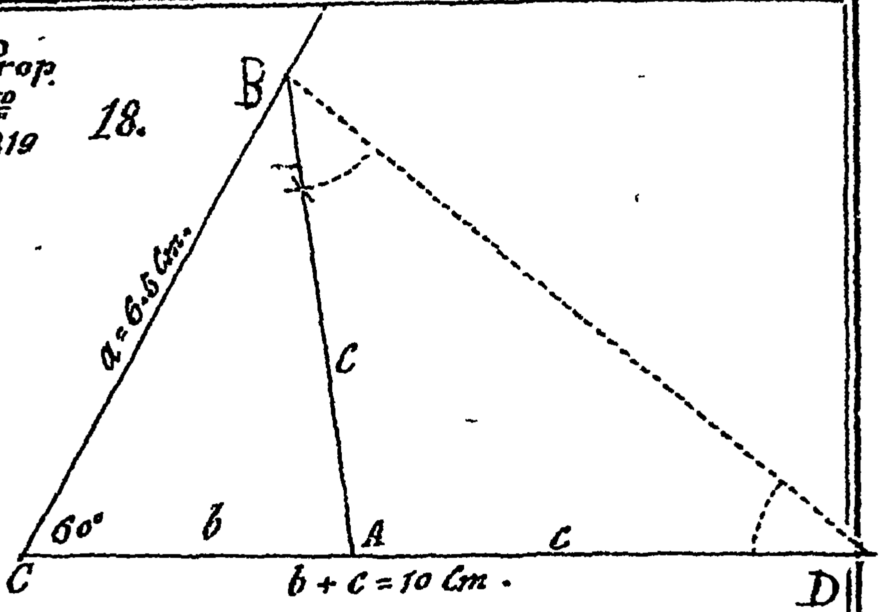
13

Prop.
No. 212

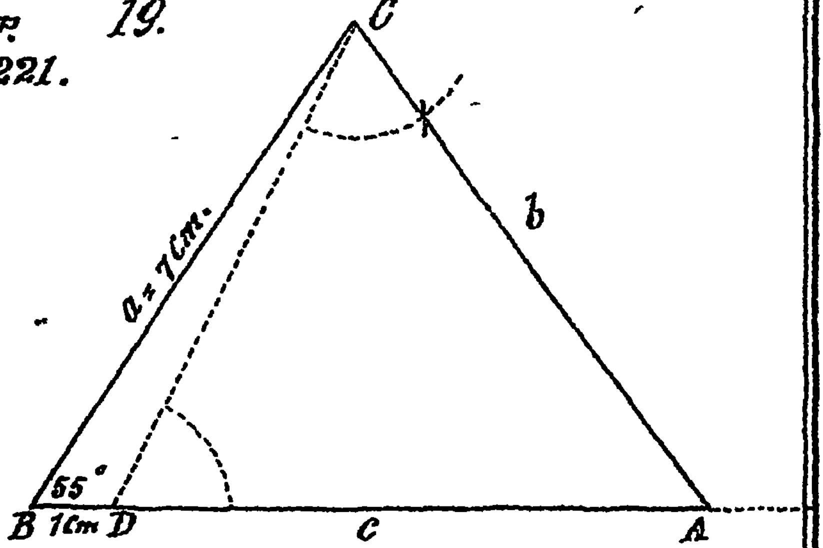



14.
A
b
b
C'
E
F
B'
B
D
B
C
15 *
B
Prop.
№ 213-14
* 215-216.
A
C
M
L
16.
A
Prop.
№ 217.
c = 5.3 cm
b
a = 4.5 cm
b
45°
B
D
a + b = 7.3 cm
17
G
A
Prop
№ 218.
H
c
b
35°
35°
c
70°
a
80°
b
B
F
a + b + c = 12 cm.

Prop.
Nº 219 18.



Prop. 19.
Nº 221.



PART I.

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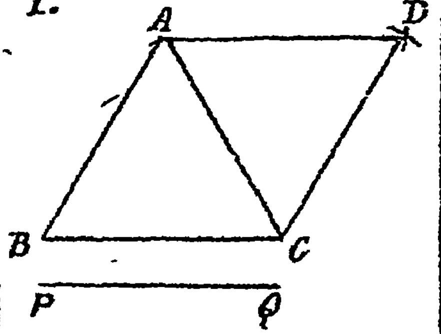
Construction of Quadrilaterals.

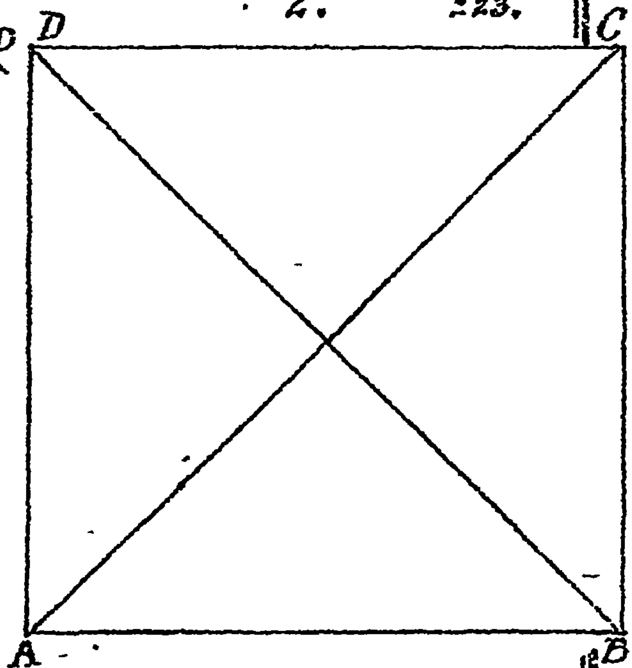
Prop. № 222
223.

Exer.

1.

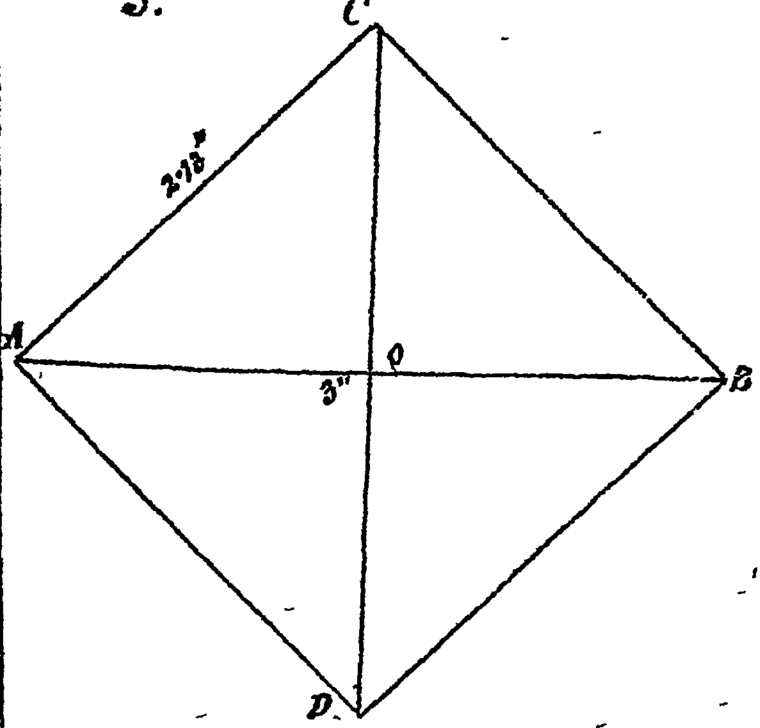


2.



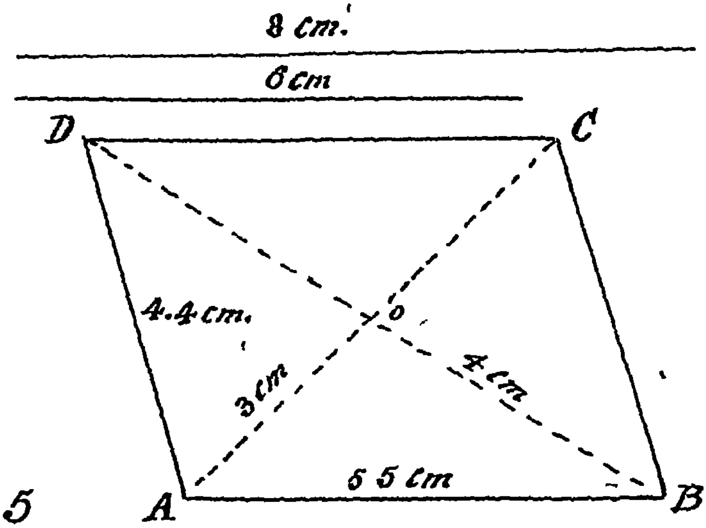
3.

Prop. № 224.



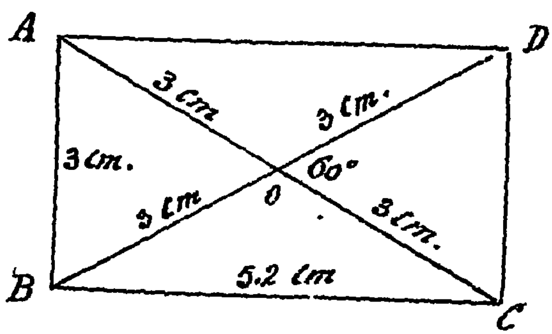
4.

Prop
№
225



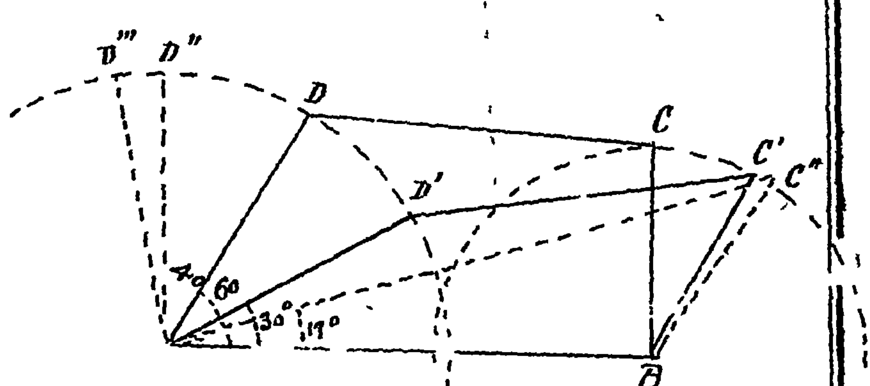
5

Prop.
№
226



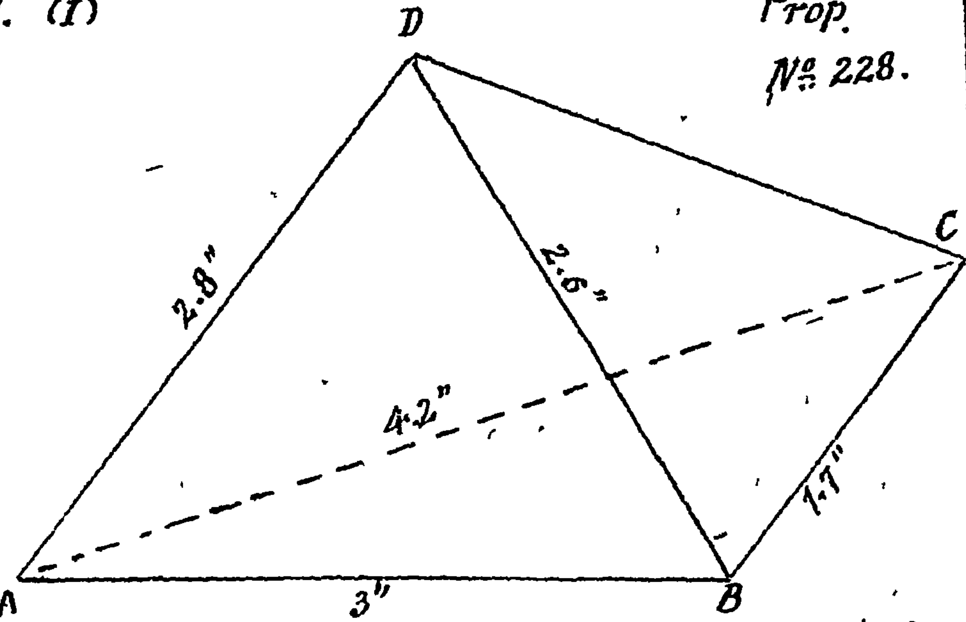
6

Prop.
№
227



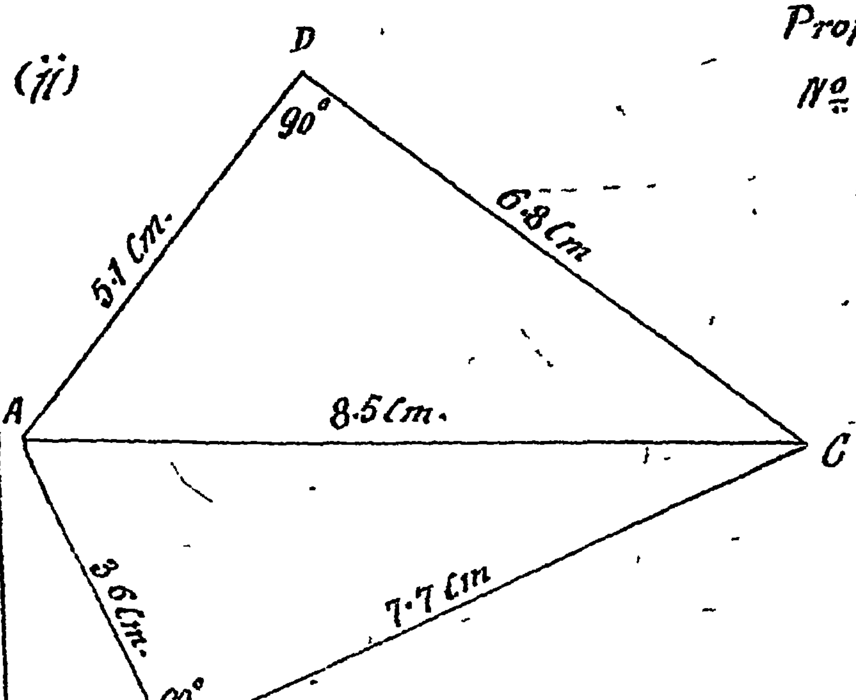
7. (i)

Prop. No. 228.



(ii)

Prop. No. 229.

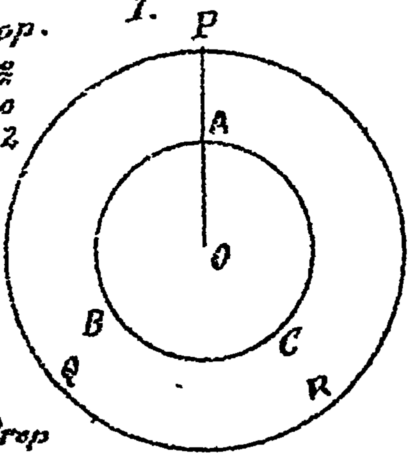


PART I.

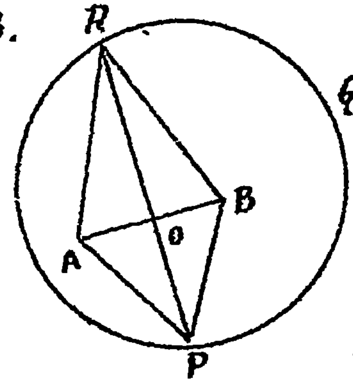
Page 94
On Loci

Exer. 1.

Prop. No 230 232

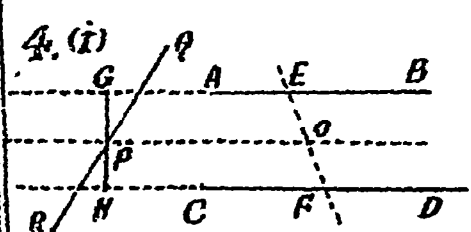


3.

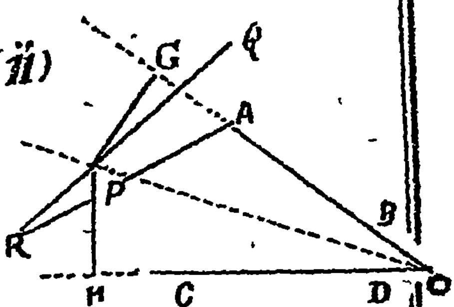


Prop No 233.234.

4. (i)

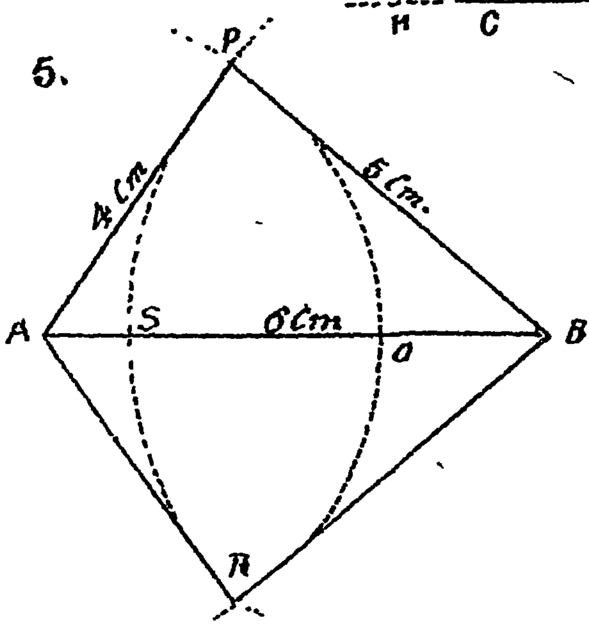


(ii)



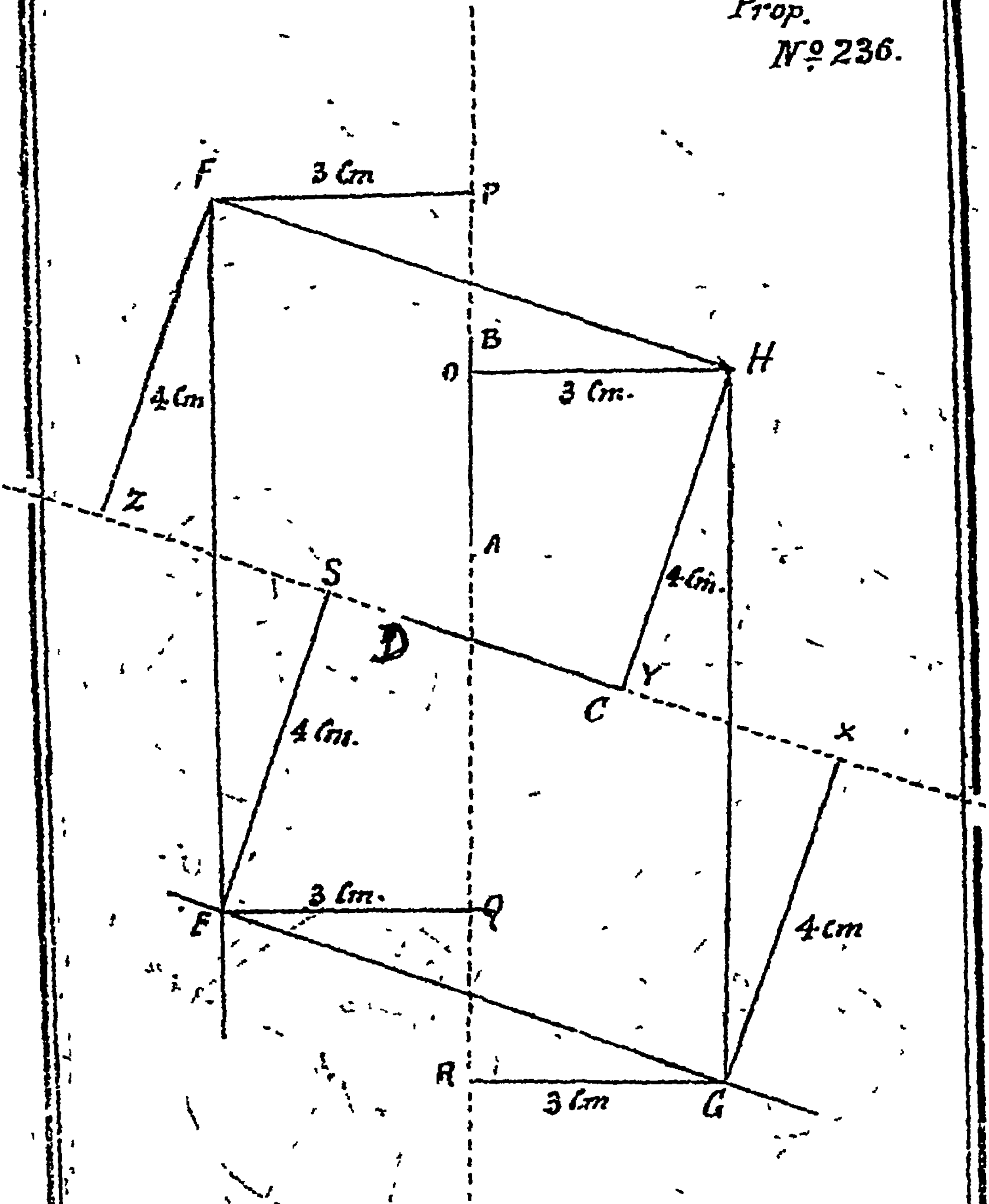
Prop. No 235

5.



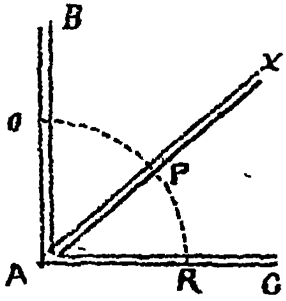
6.

Prop.
N° 236.

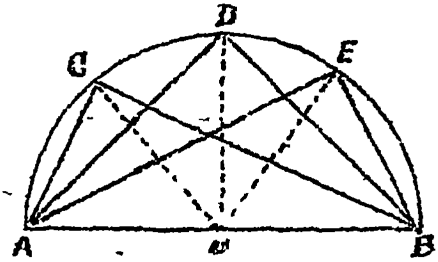


Prop.
No. 237 238

7

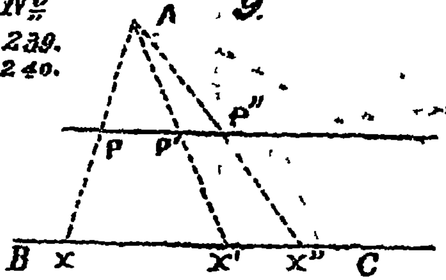


8.

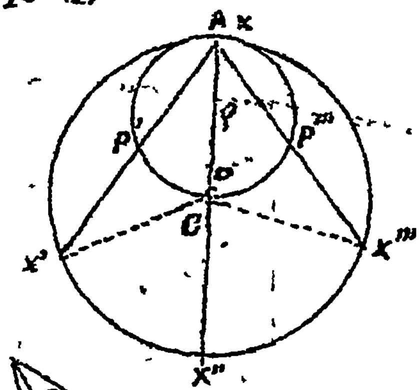


Prop.
No.
239.
240.

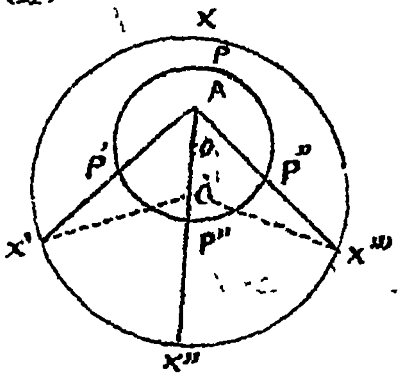
9.



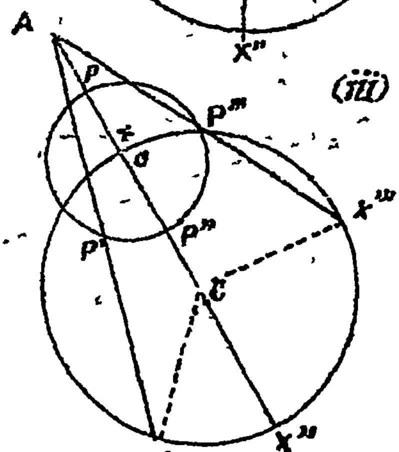
10 (i)



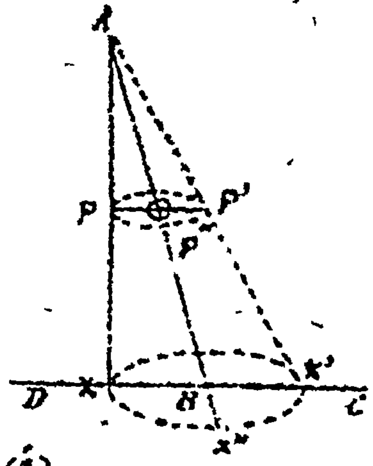
(ii)



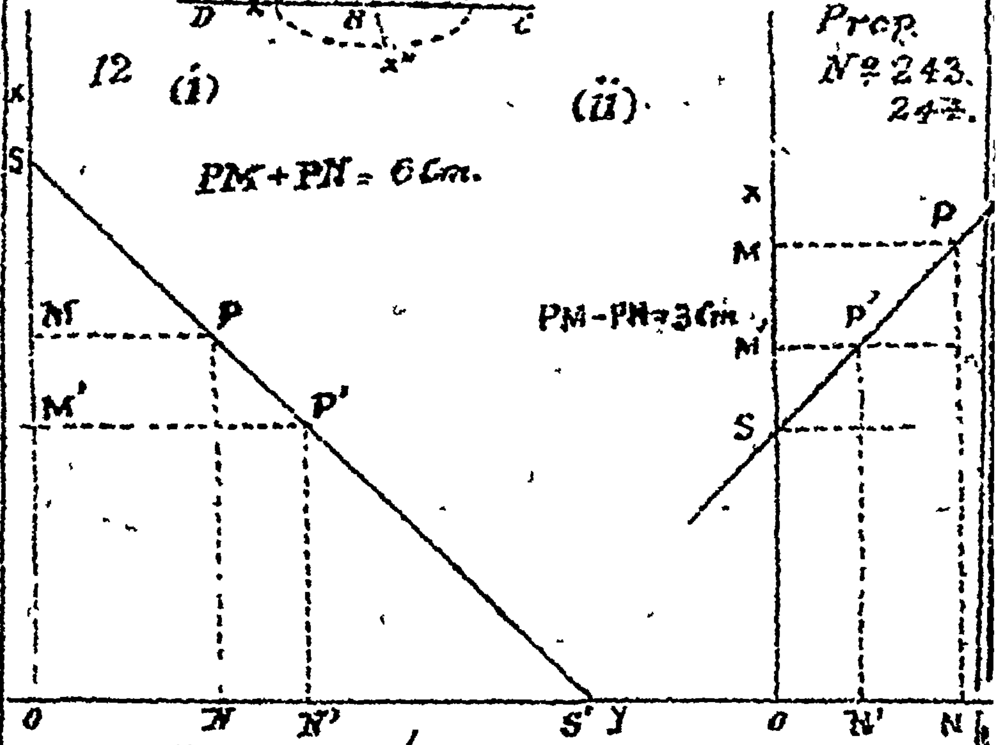
(iii)



11.

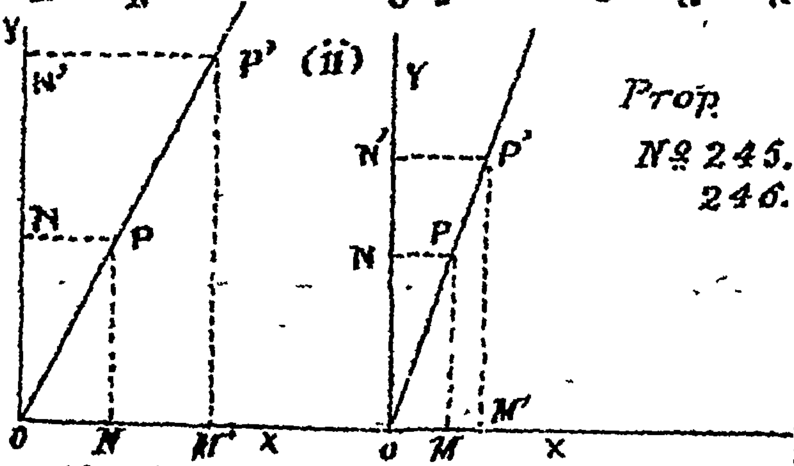


12 (i) (ii)



Prop. No. 243. 247.

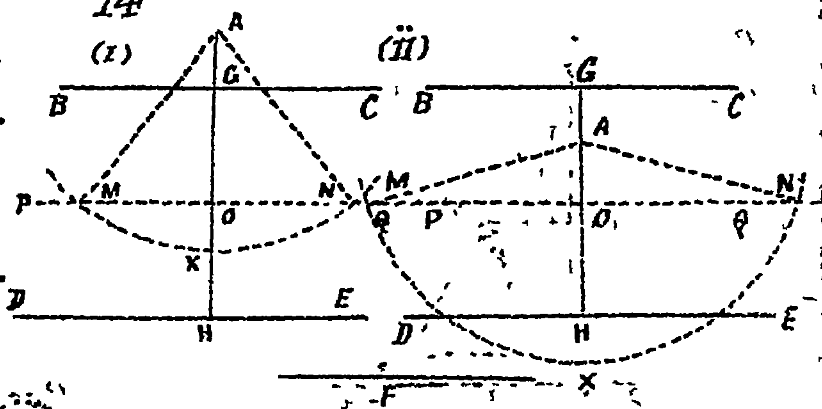
13. (i) (ii)

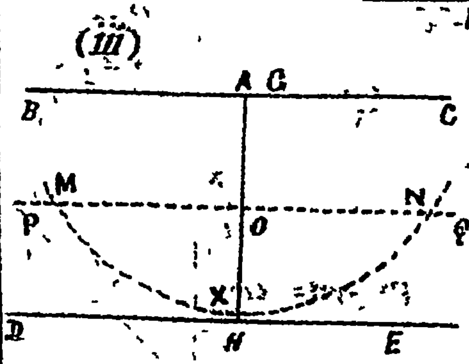


Prop. No. 245. 246.

Prop. No. 247 248.

14

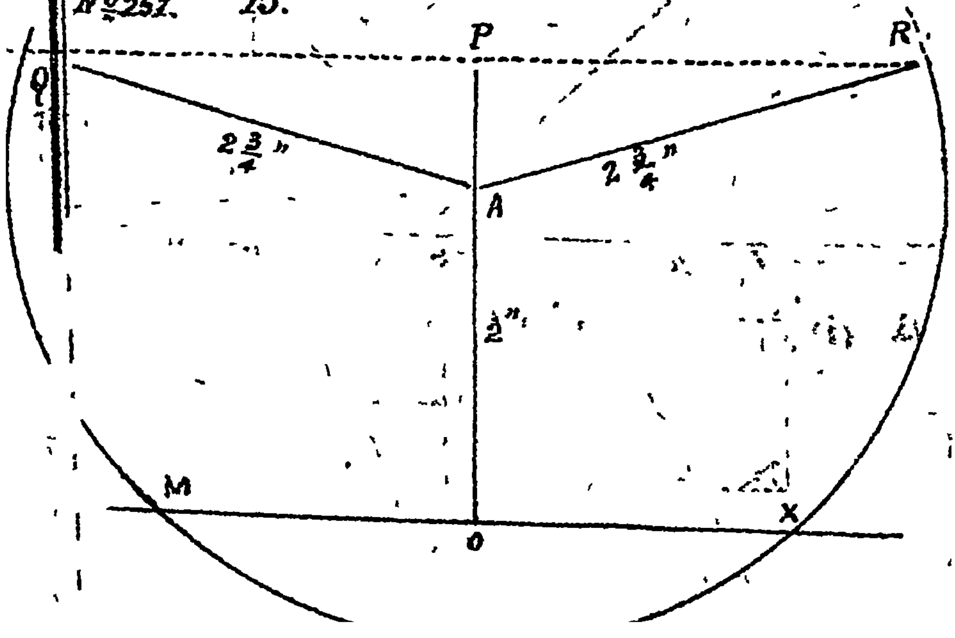




Prop. No. 249.

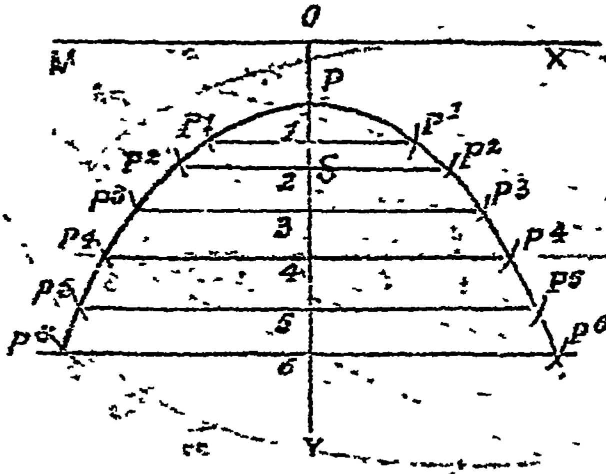
Prop. No. 251.

15.

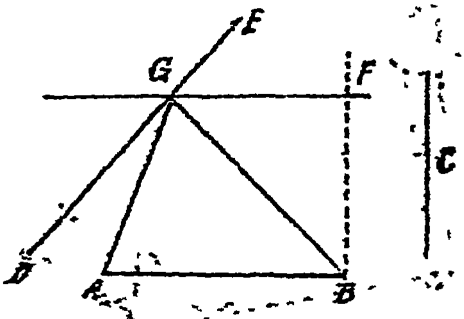


16.

Prop.
№ 252.



17.



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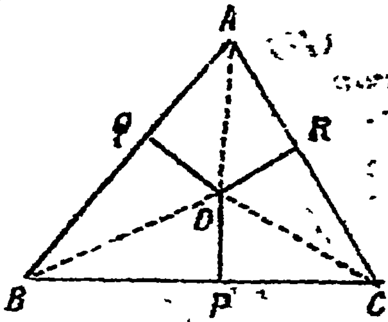

19 (i) (ii)

Prop.
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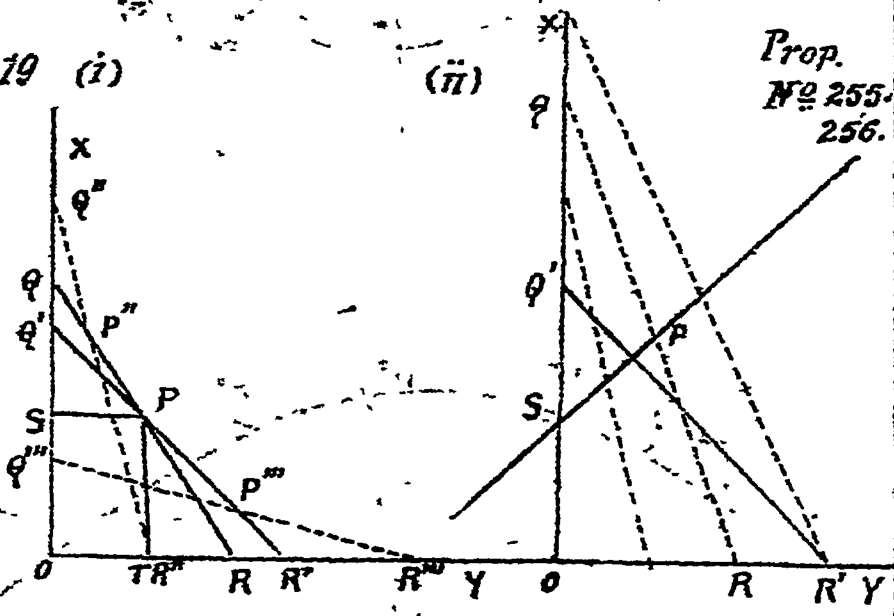
256.



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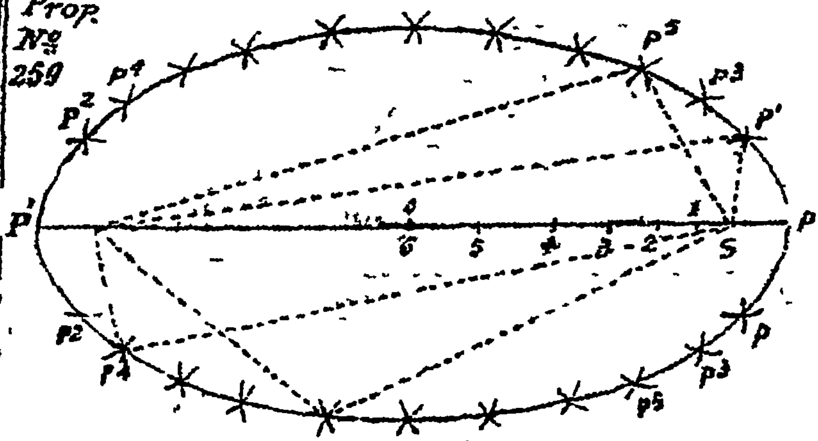
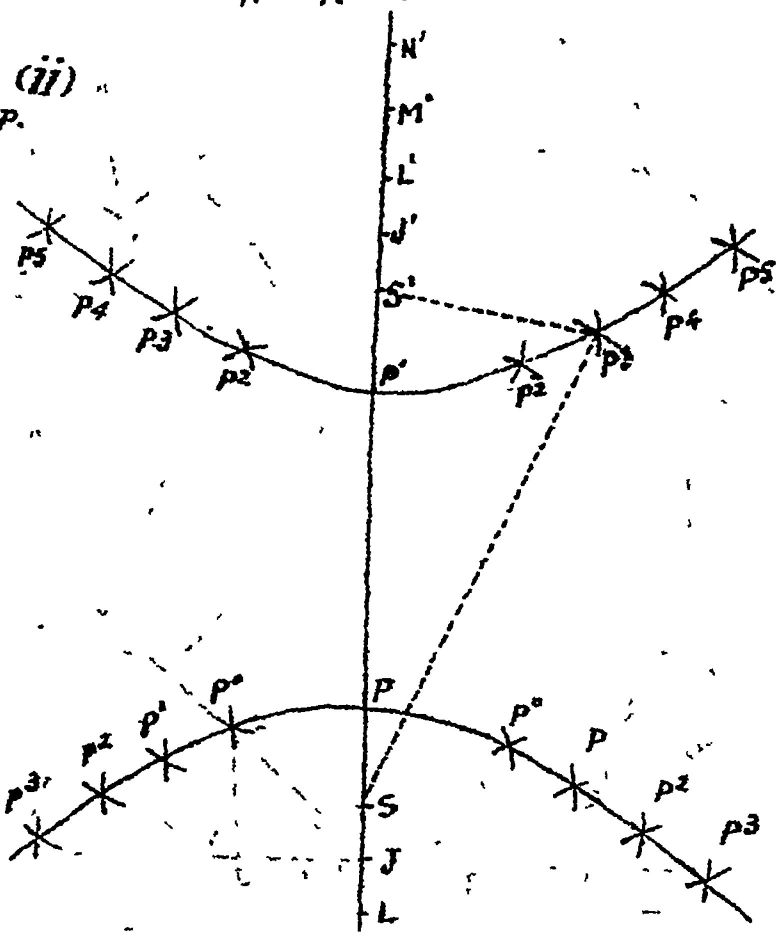
(i)

Prop. No. 259



(ii)

Prop. No. 260



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miscellanious.

Exer. Prop. № 260. 261.

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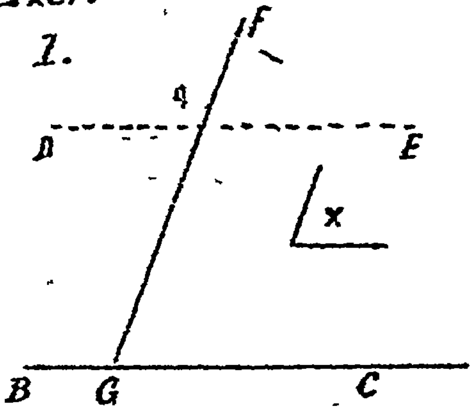

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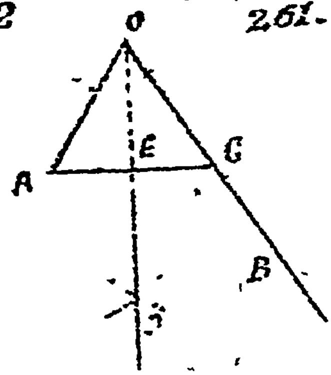

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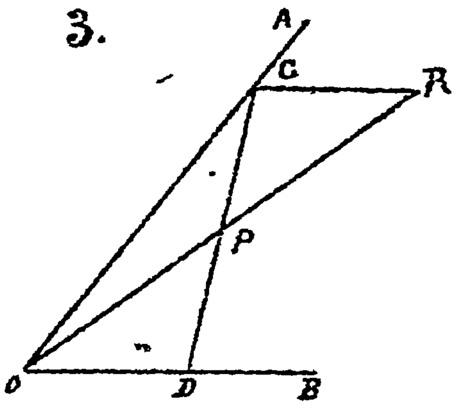

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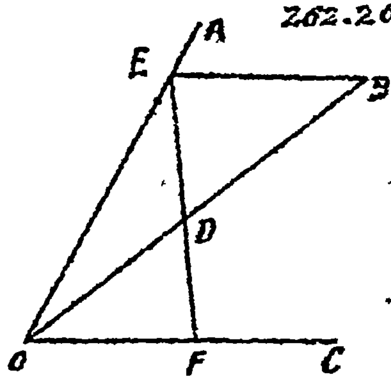
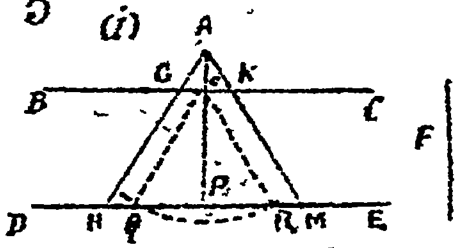


5 (i)



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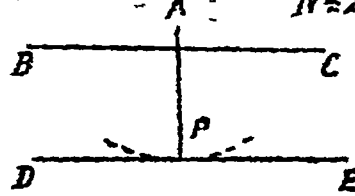

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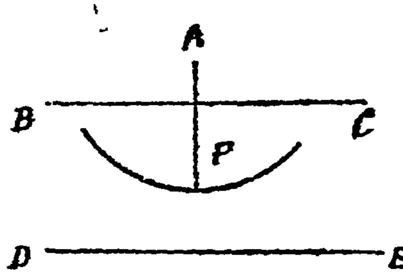

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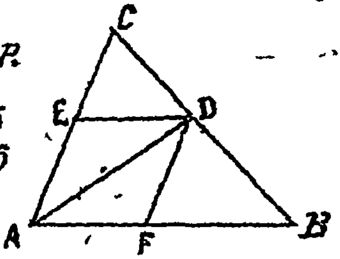

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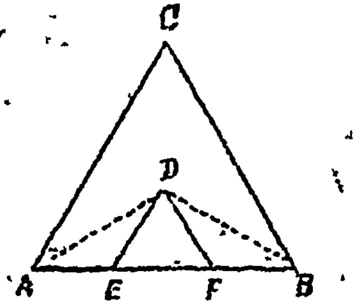

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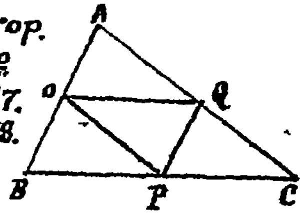

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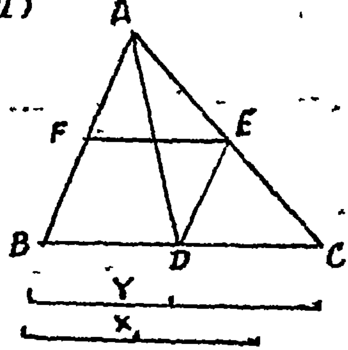

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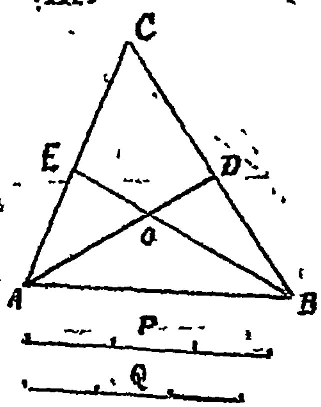

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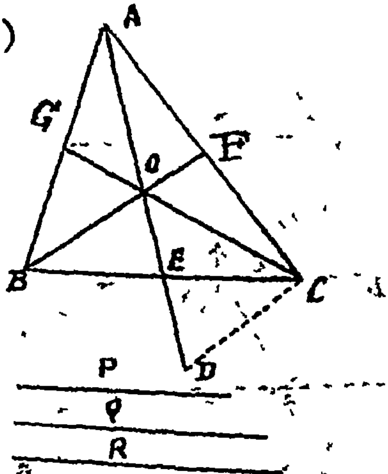

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On tables of length and area.

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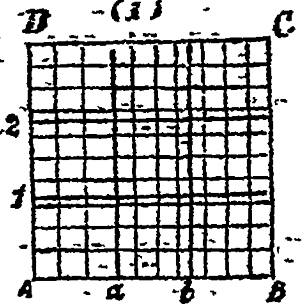

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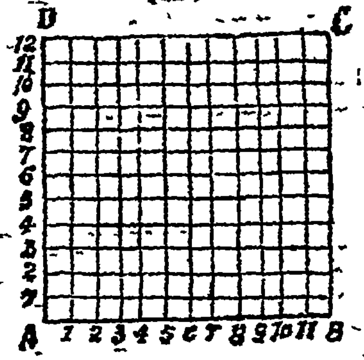

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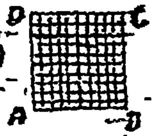

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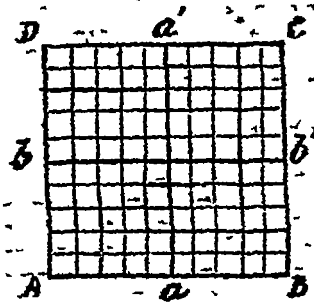

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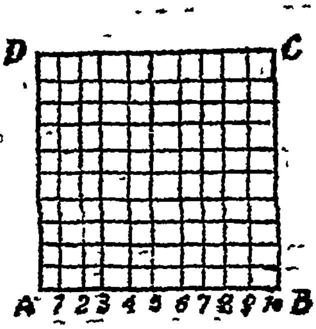

Prop. №. 275 276.

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Exer 1

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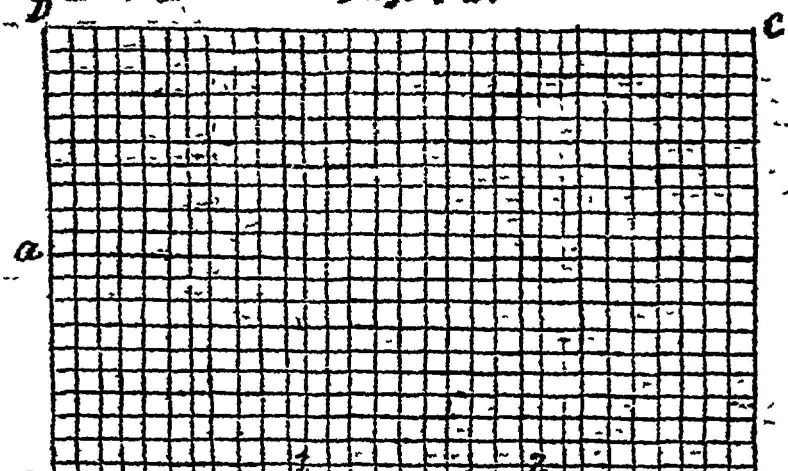

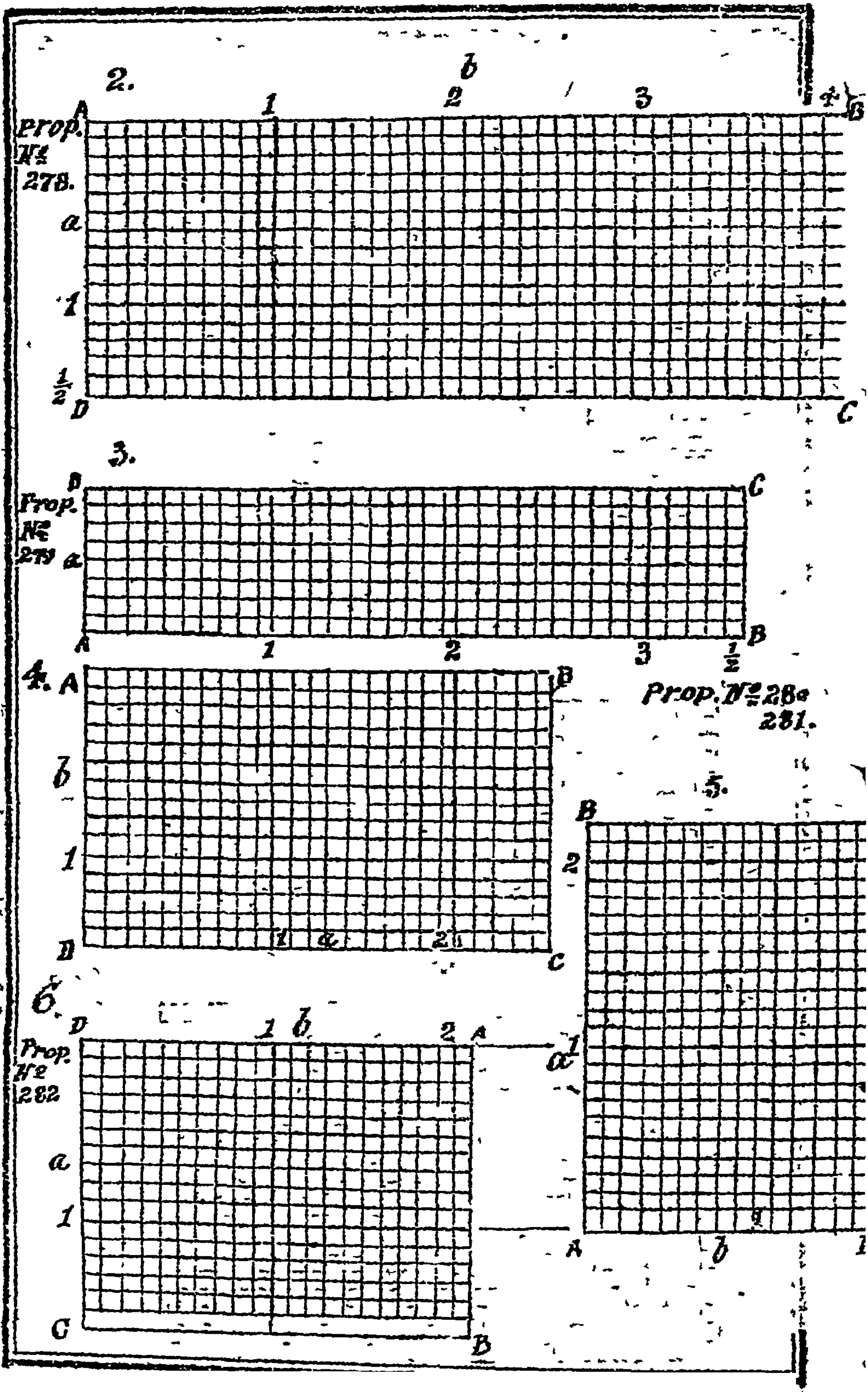
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b
A 1 2 3 4 B
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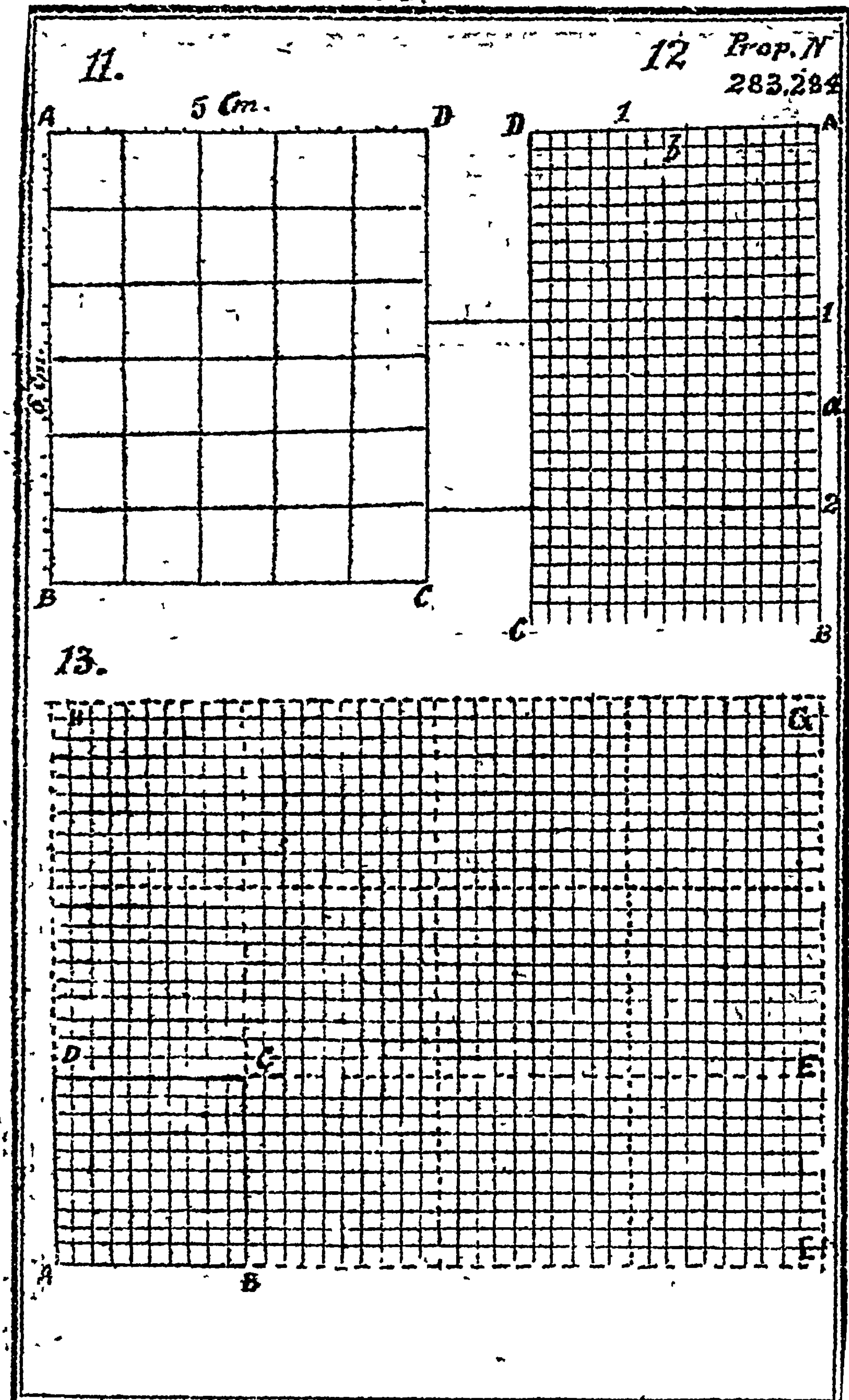
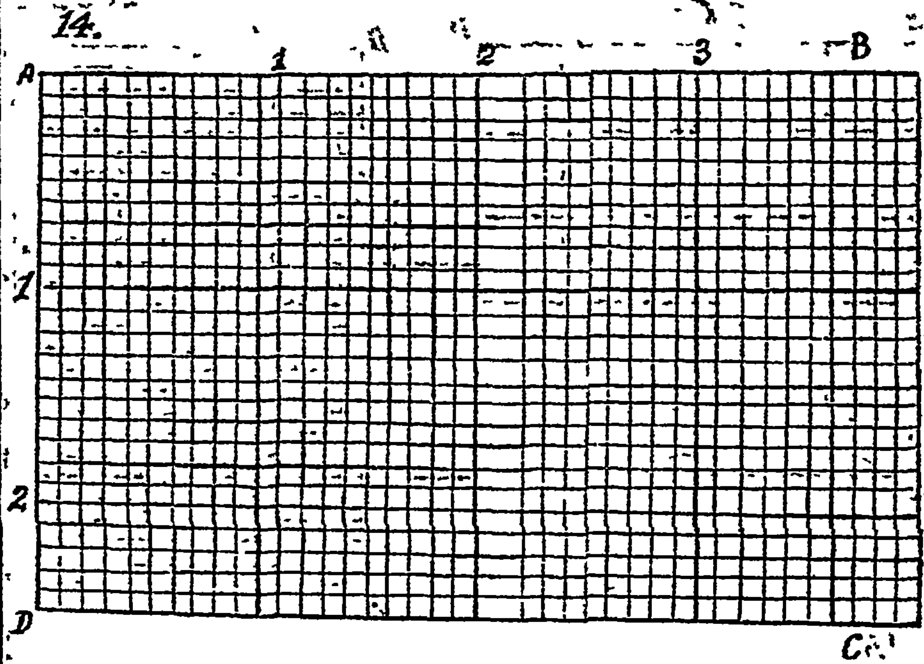
a
1
1/2
D C
3.
B C
Prop. No. 279
a
A 1 2 3 1/2 B
4. A B
b
1
D 1 a 2 C
Prop. No. 280
281.
5.
B
2
a 1
A b
6. D 1 b 2 A
Prop. No. 282
a
1
C B


11.
5 Cm.
A
D
B
C
12
Prop. N
283.284
D
1
A
b
1
a
2
C
B
13.
G
D
C
E
A
B
F

Prop. № 286.



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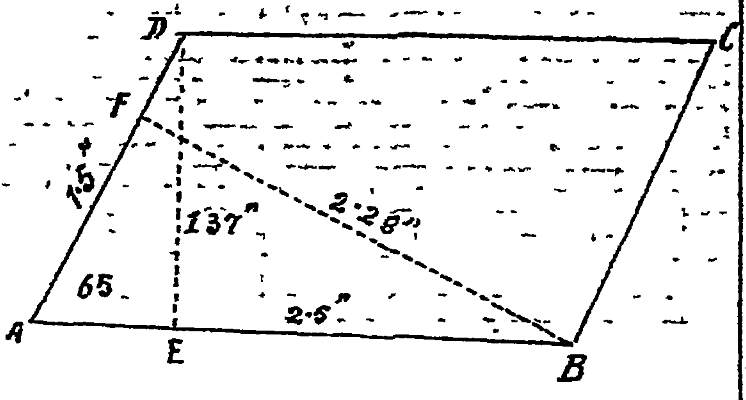
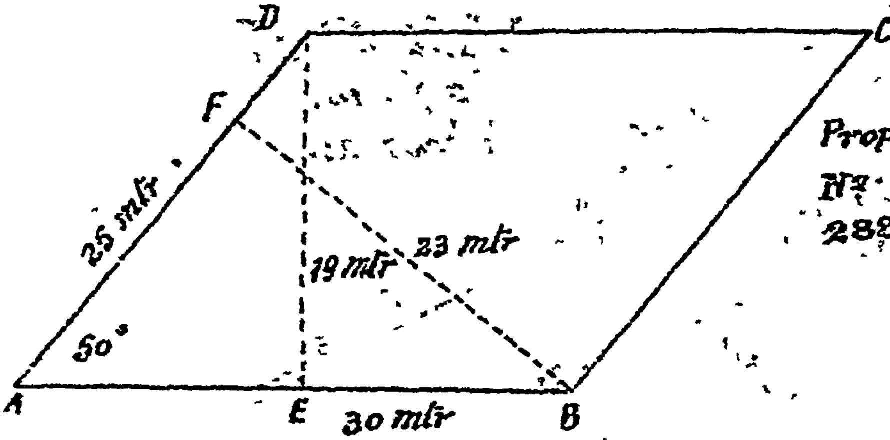
Theor 24.

Exer.

Prop. 2

№ 287.





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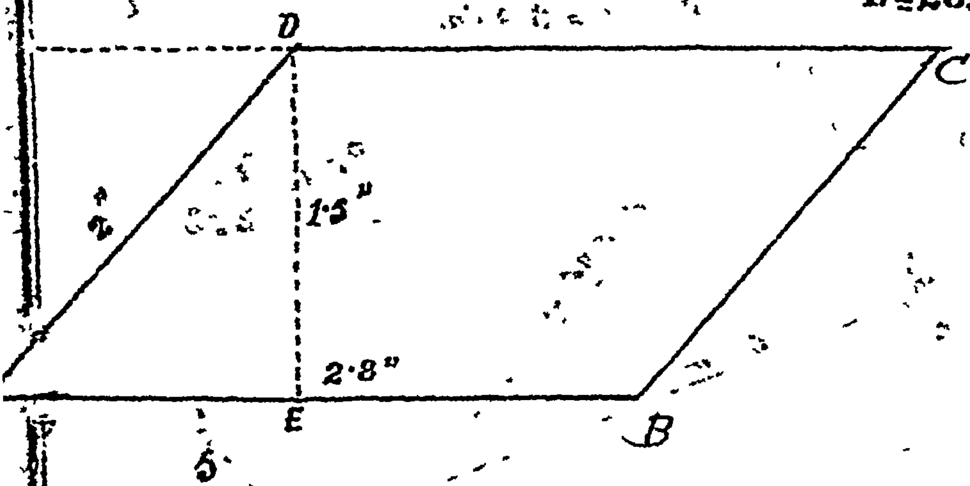

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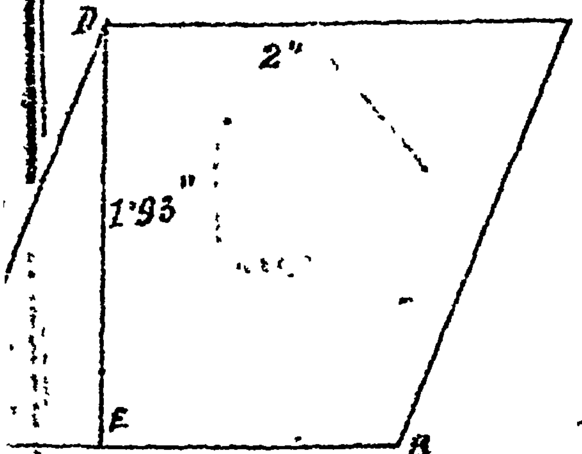

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Theor 25.

Exer
2 (i)

Prop
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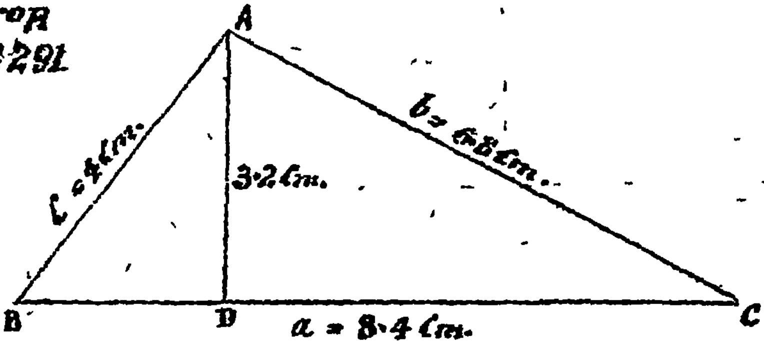

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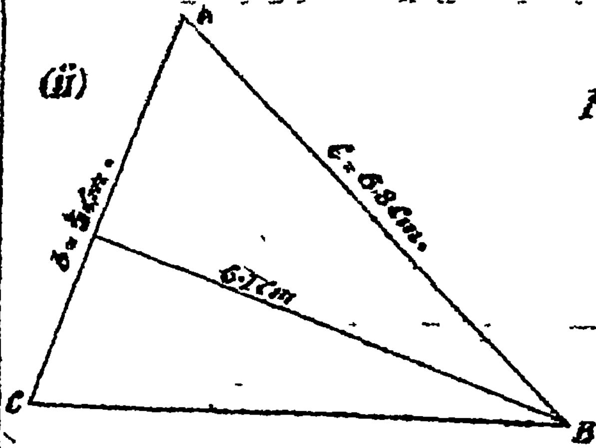

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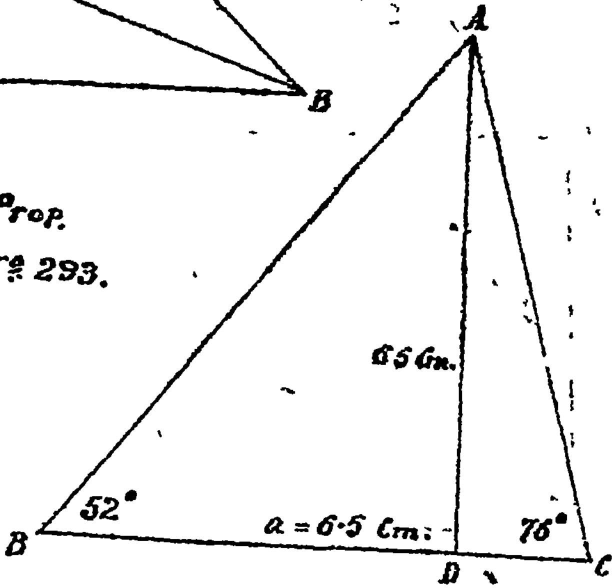

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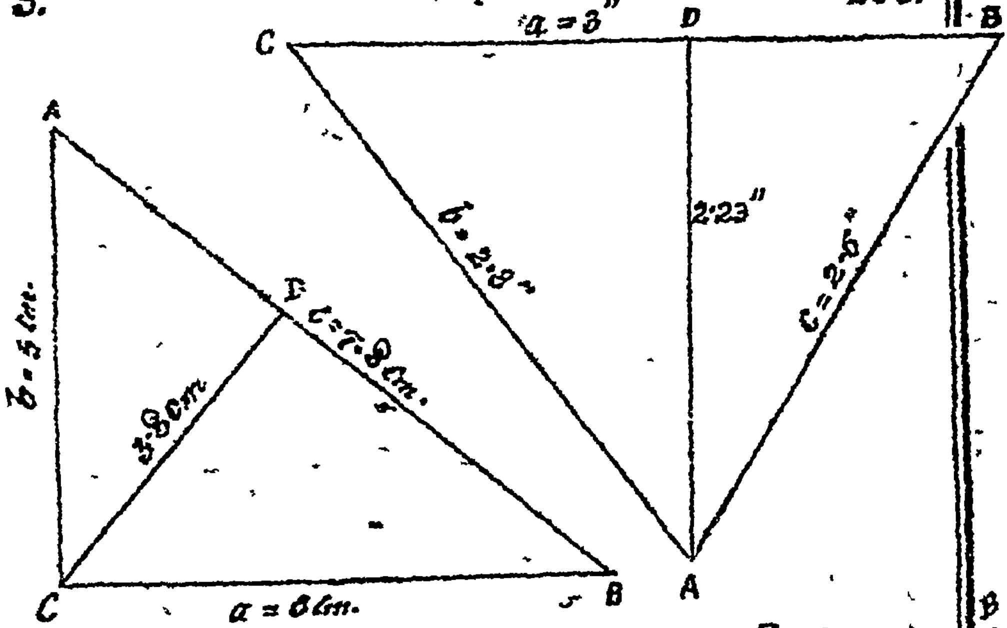
3.

6.



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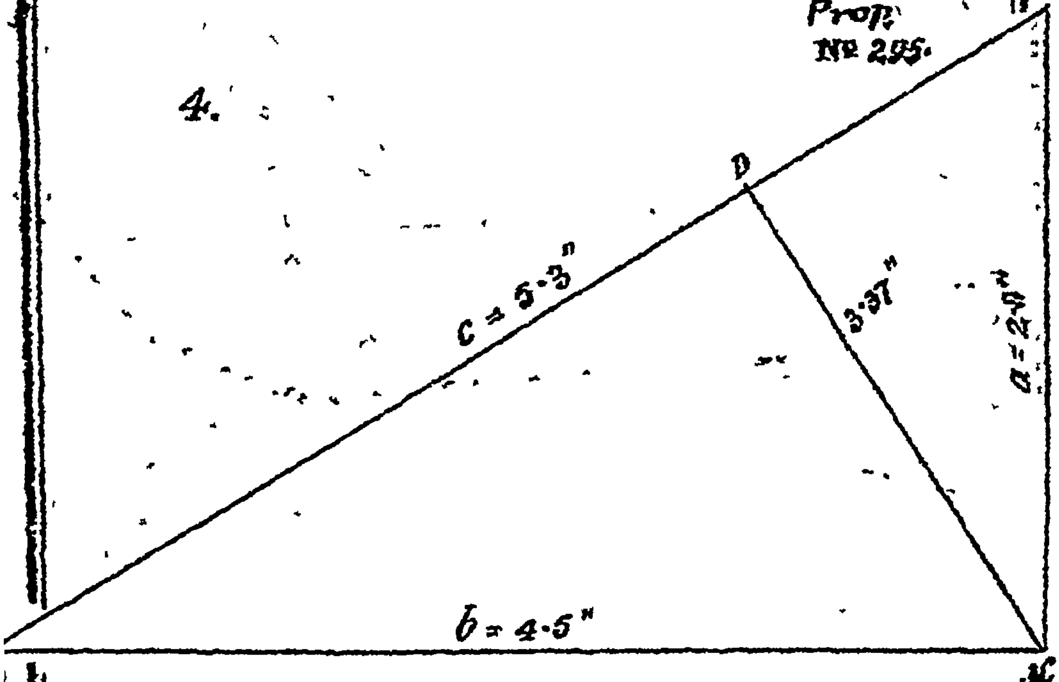

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Area of a Triangle.

Exer 1

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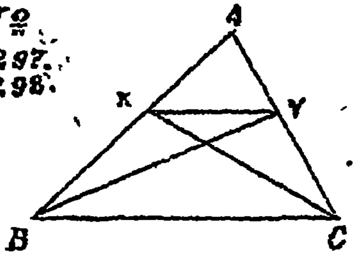

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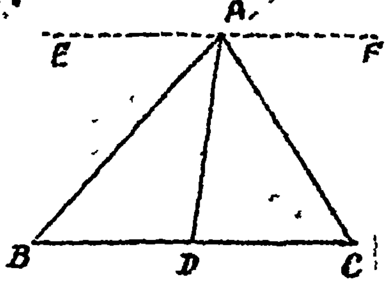

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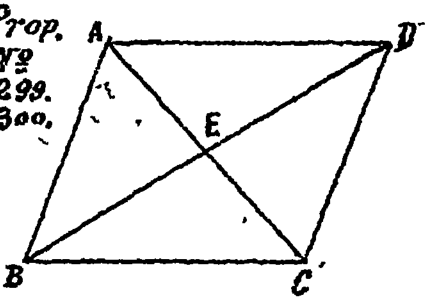

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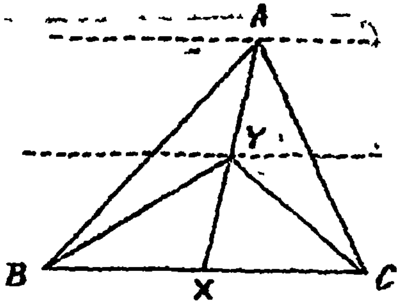

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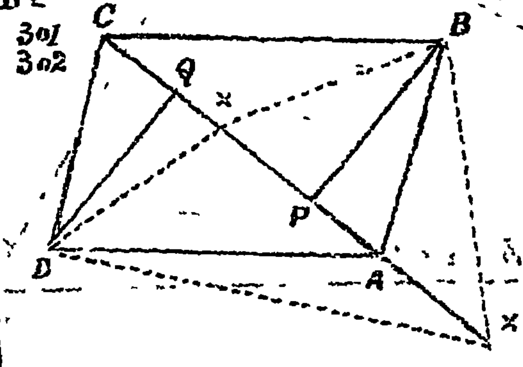
Prop Nº 301 302



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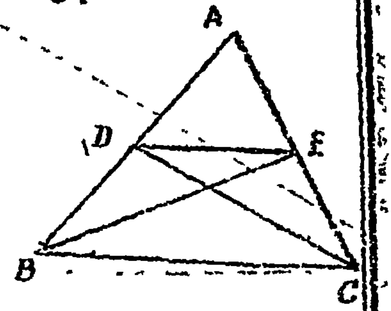

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303
304

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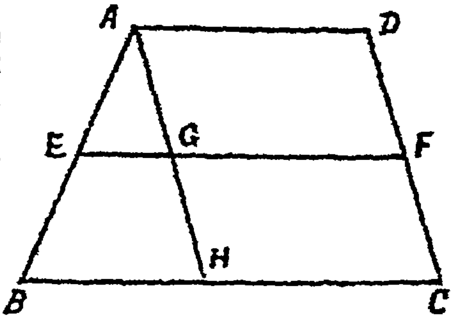

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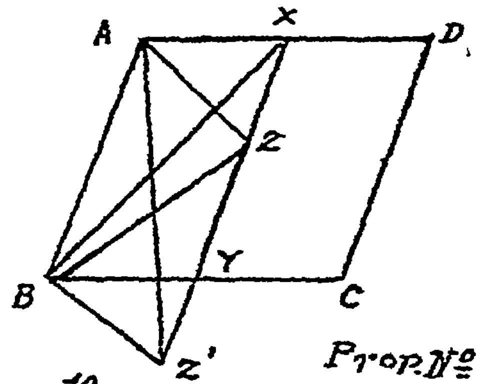

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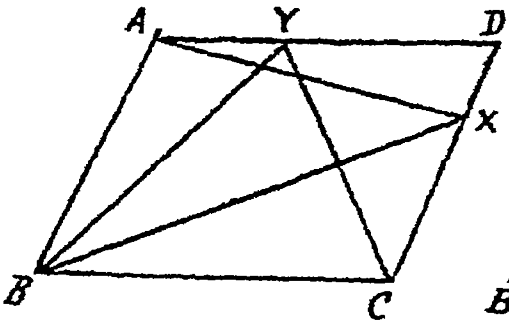

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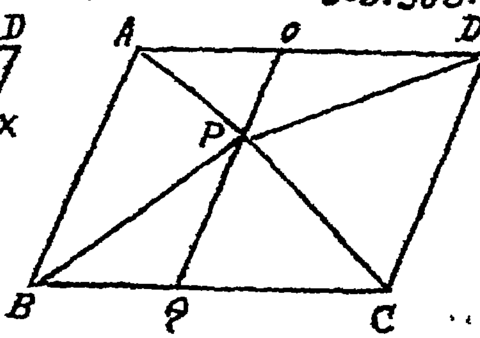

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on area of triangles.

Exer

1.

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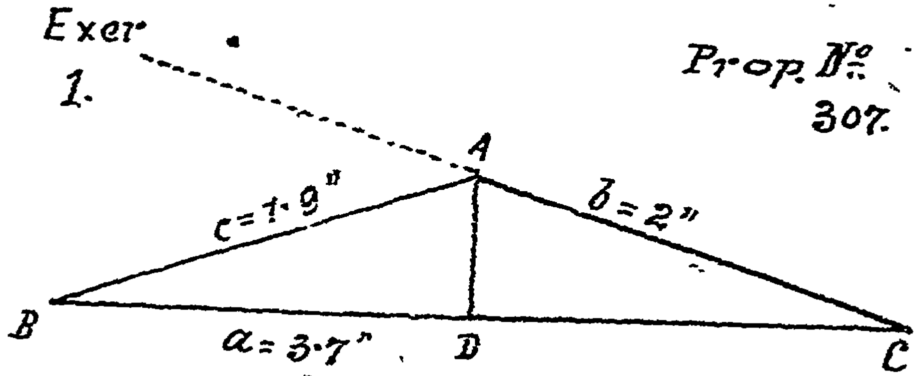

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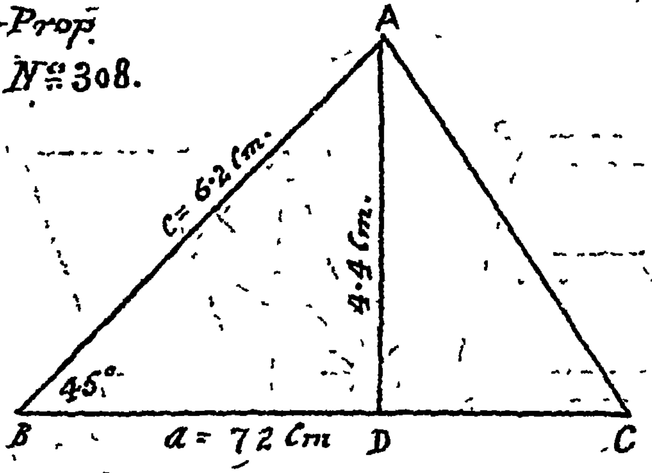

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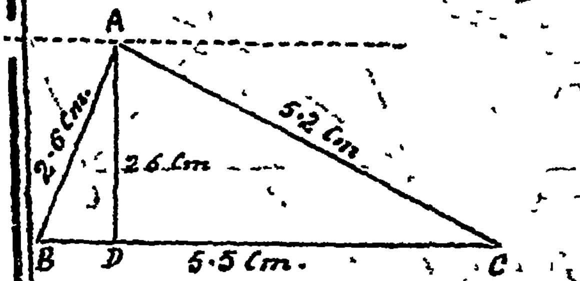

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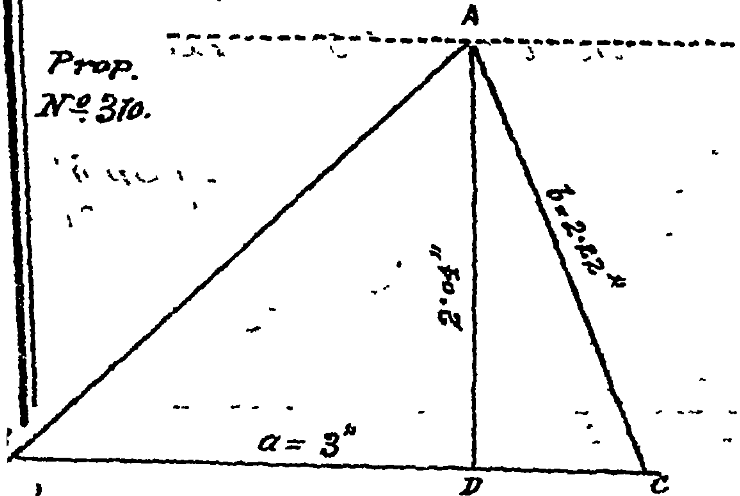

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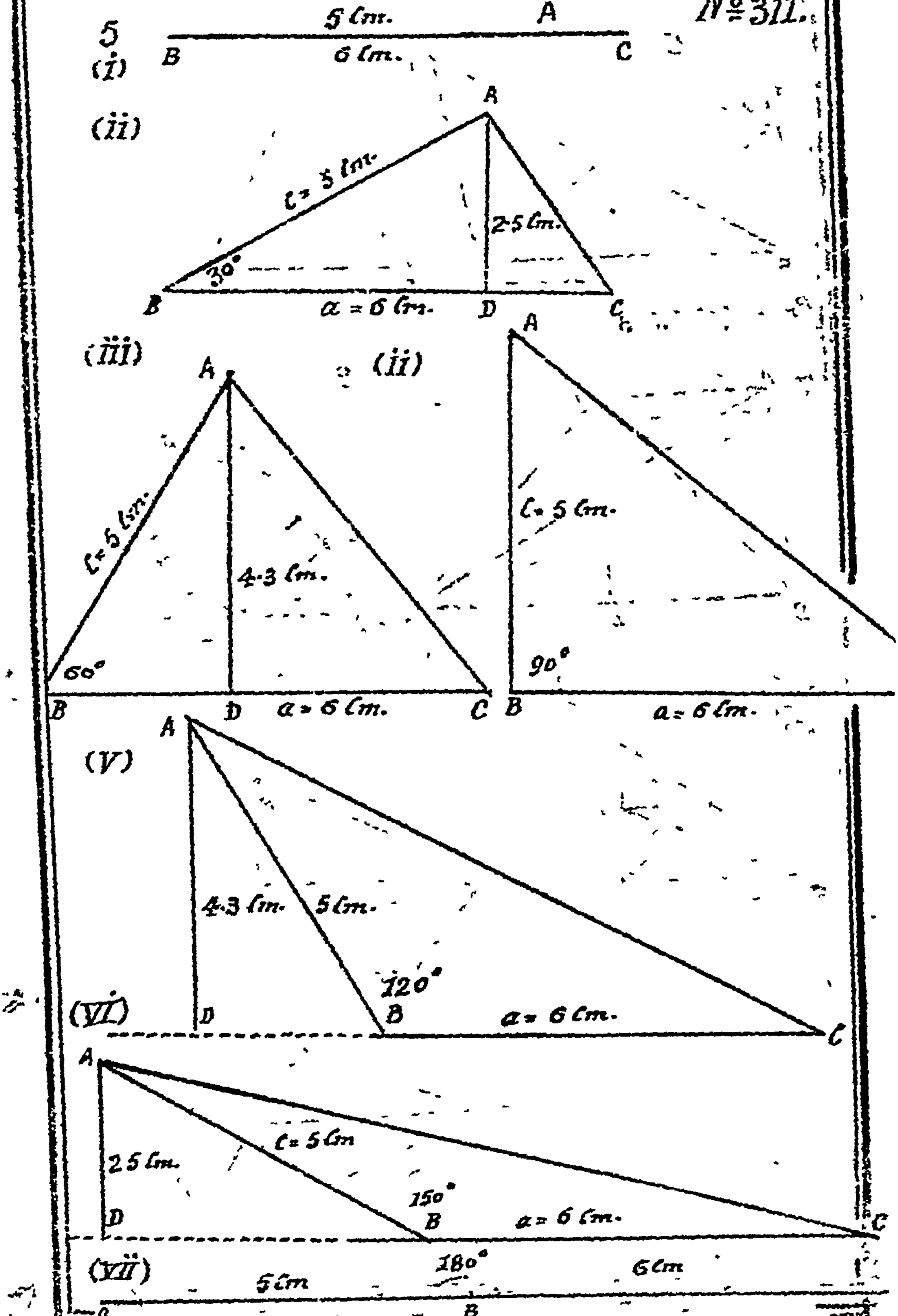
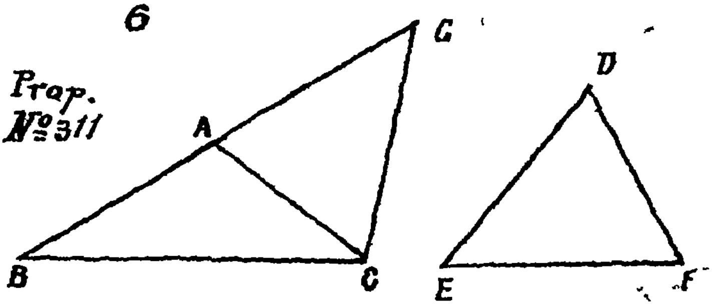

5
(i)
5 Cm.
A
B
6 Cm.
C
(ii)
A
c = 5 Cm.
2·5 Cm.
30°
B
a = 6 Cm.
D
C
(iii)
A
(iv)
A
c = 5 Cm.
4·3 Cm.
60°
B
D
a = 6 Cm.
C
c = 5 Cm.
90°
B
a = 6 Cm.
(v)
A
4·3 Cm.
5 Cm.
120°
(vi)
D
B
a = 6 Cm.
C
A
c = 5 Cm
2·5 Cm.
150°
D
B
a = 6 Cm.
C
(vii)
180°
5 Cm
6 Cm
B
B

THEORETICALLY.

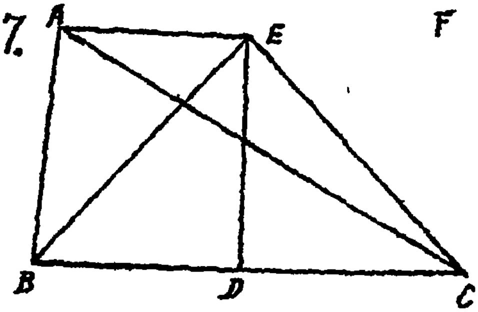
6

Prop. No. 311



Prop. No. 312. 313.

7.



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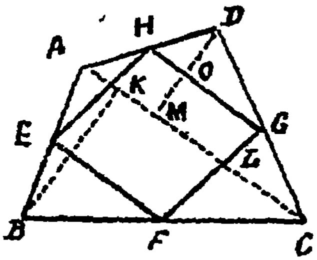

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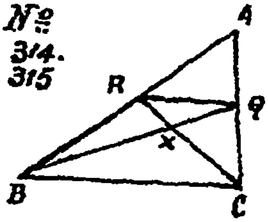

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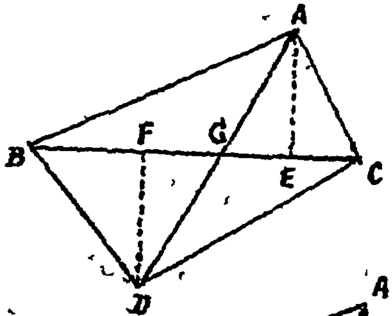

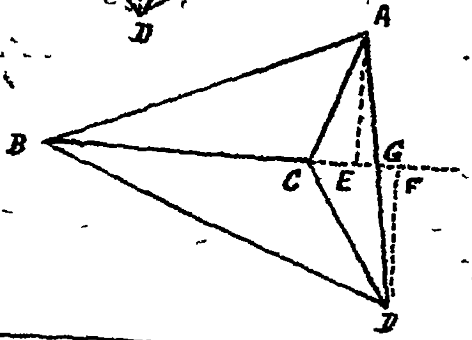

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Exer.

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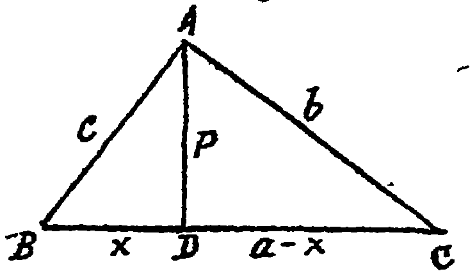

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Theor 28.

Exer.

4

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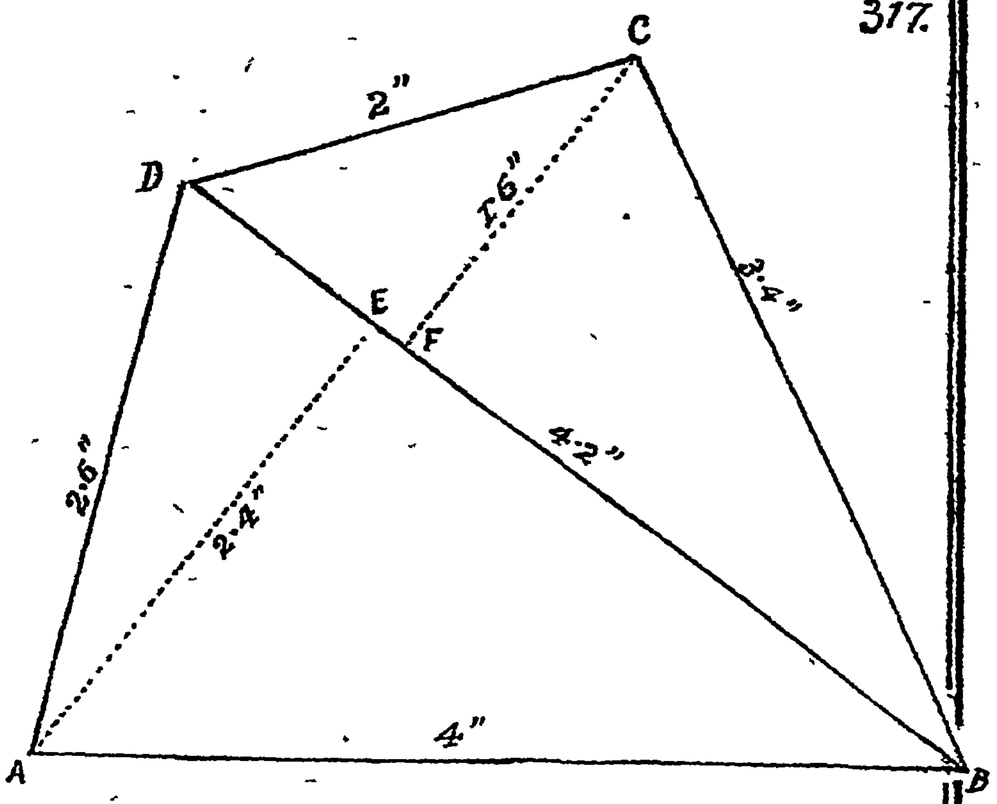

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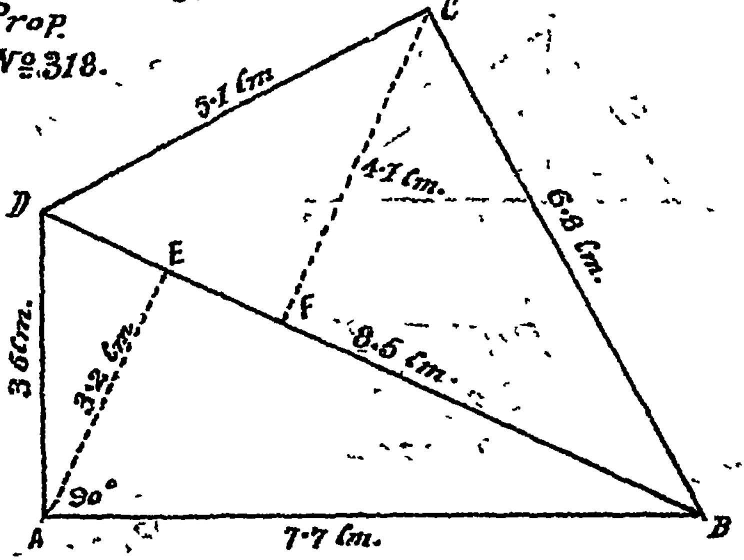

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Prop.
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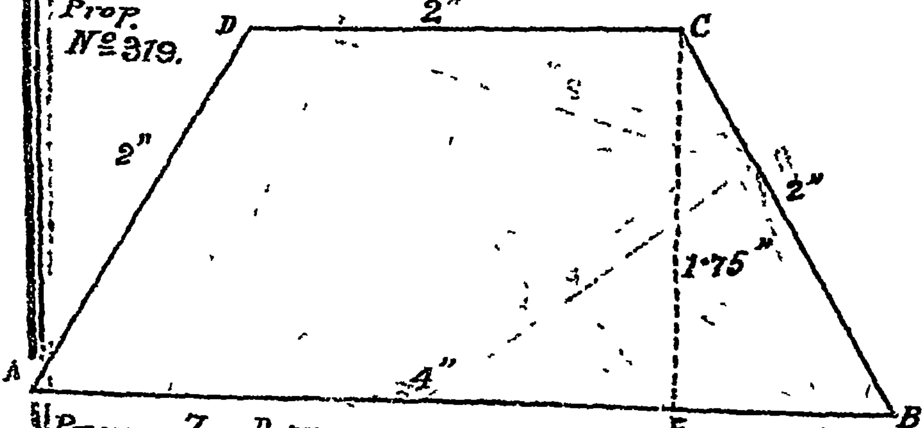

Prop.
No. 320.

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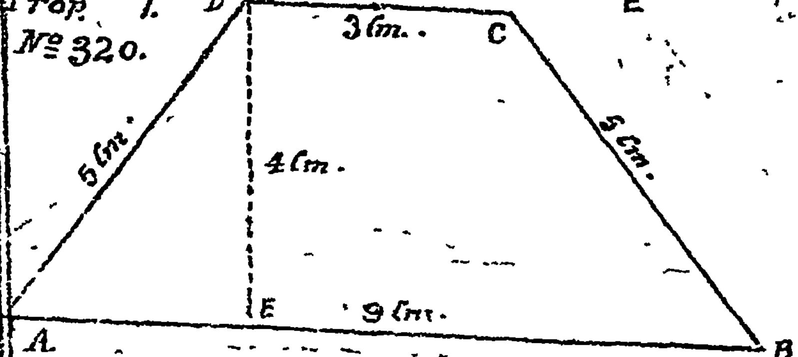

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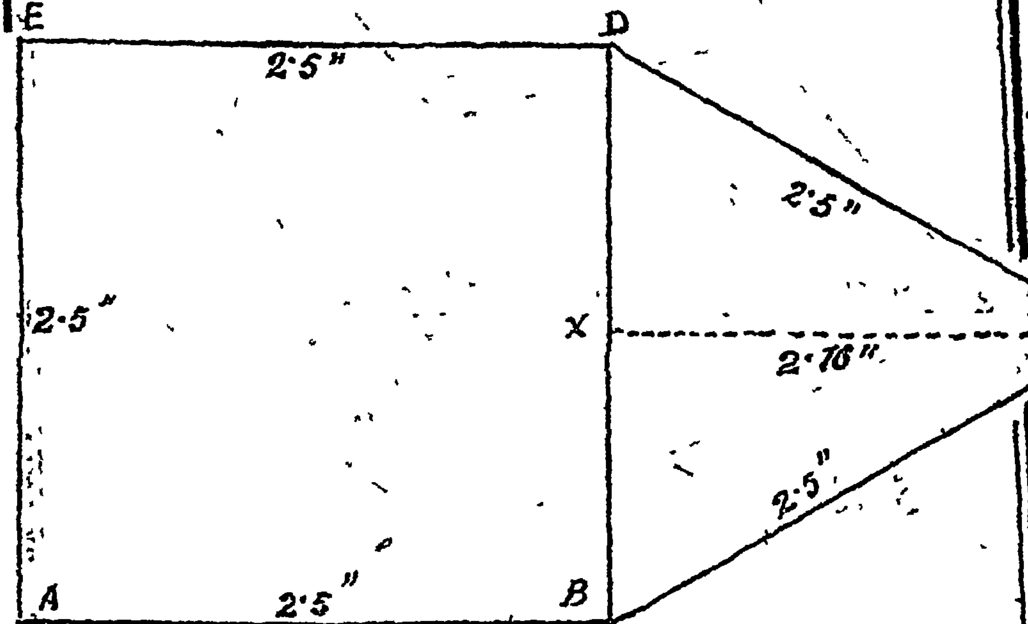
2. (i)



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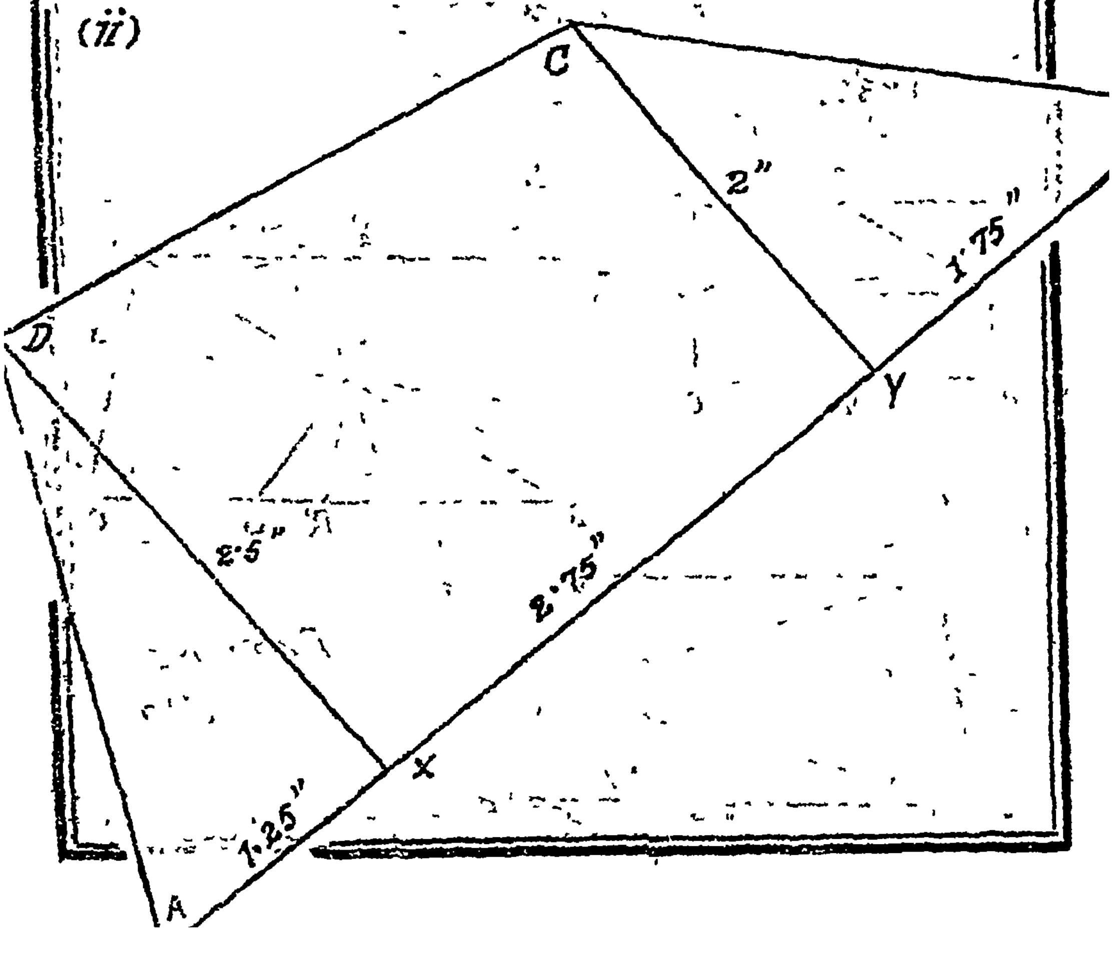

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No. 323.

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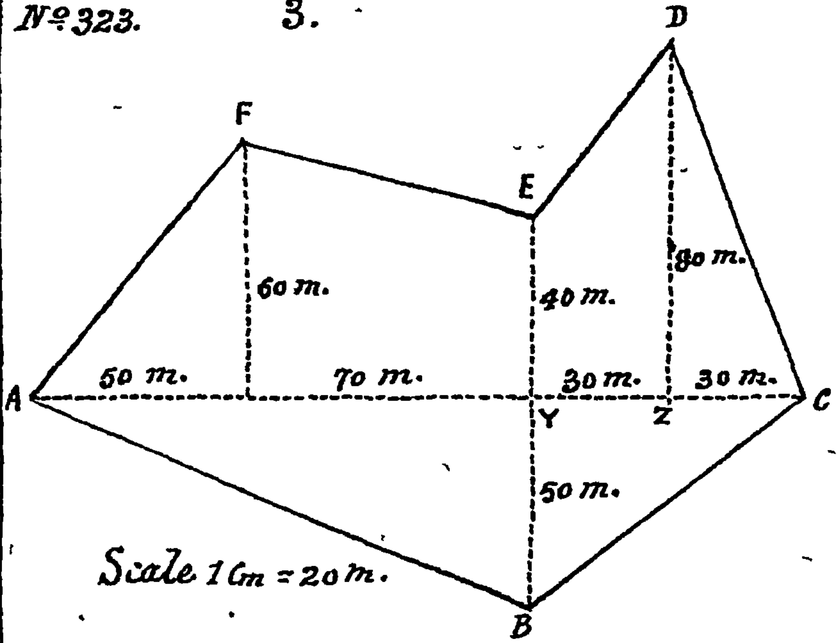

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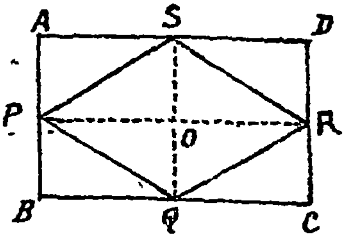
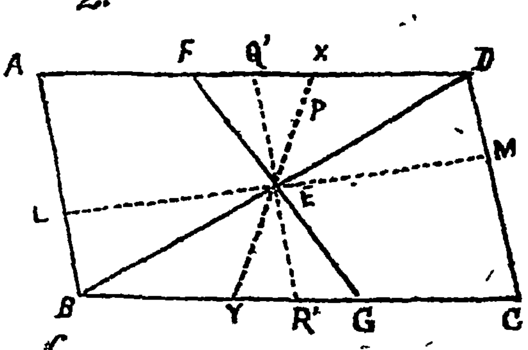
Prop No. 324 Page 116.

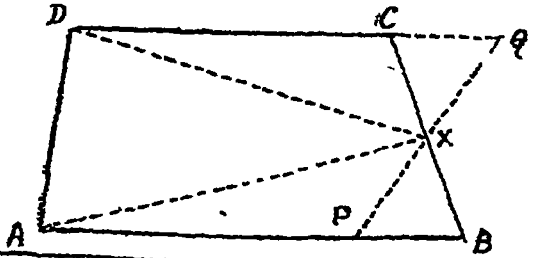
325. Exar-1



2.



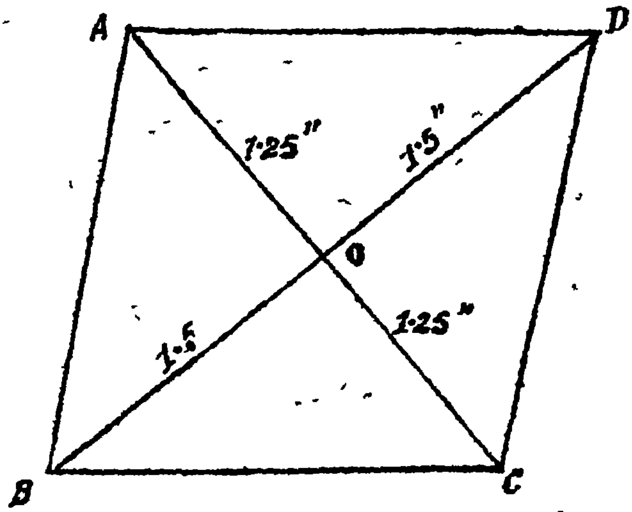
3



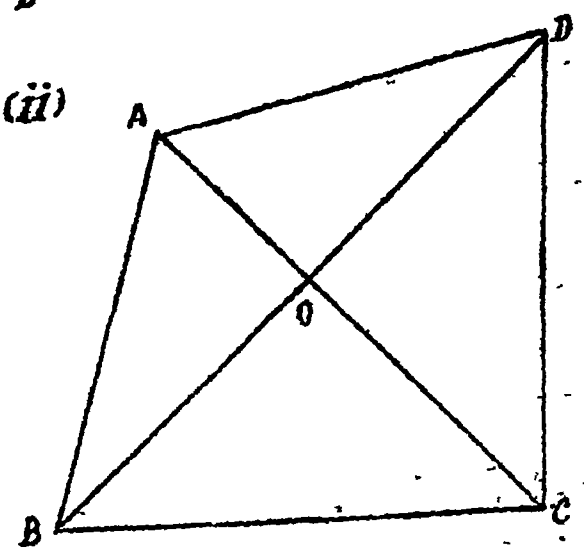
Prop. No.
326.

Prop.
No. 327.

4. (i)



(ii)



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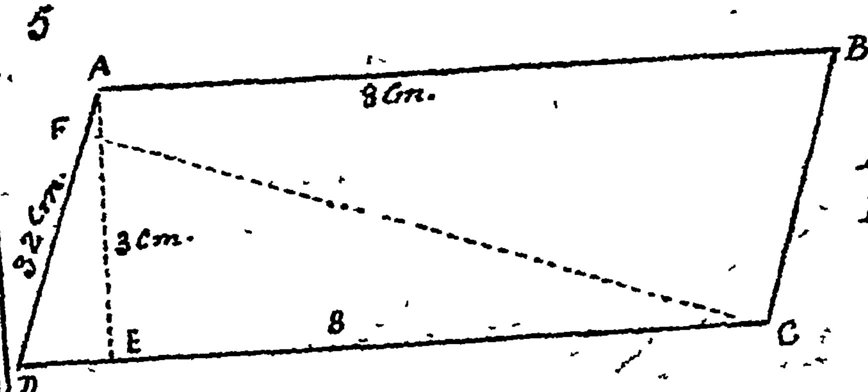
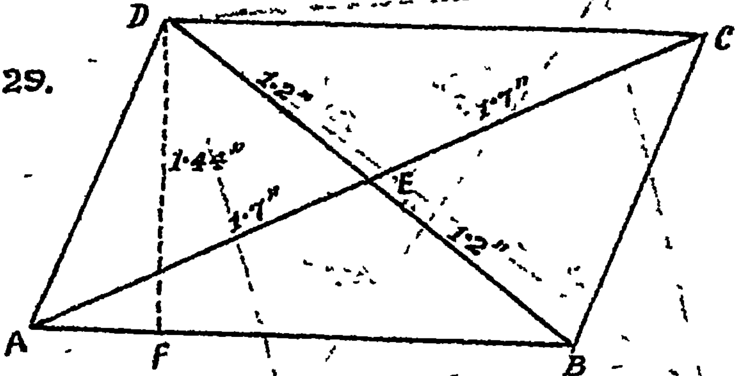


Prop.
No. 328

6.

Prop.
№ 329.



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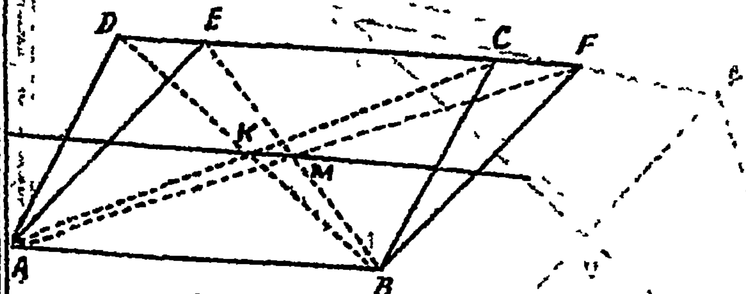
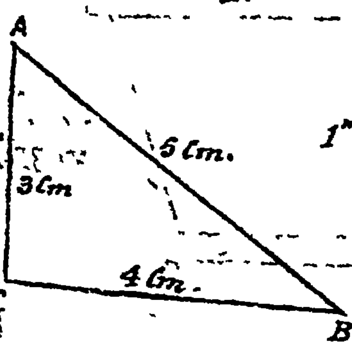


PART II

Page 117.

Prop.
№ 330.331

Exer
1.



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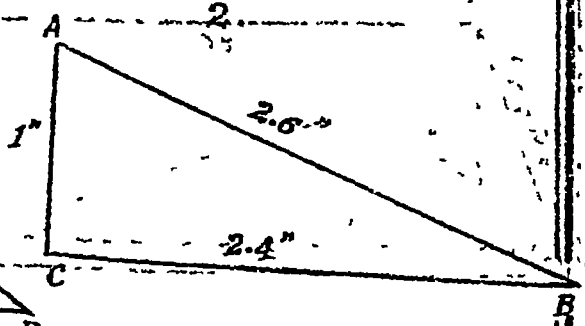

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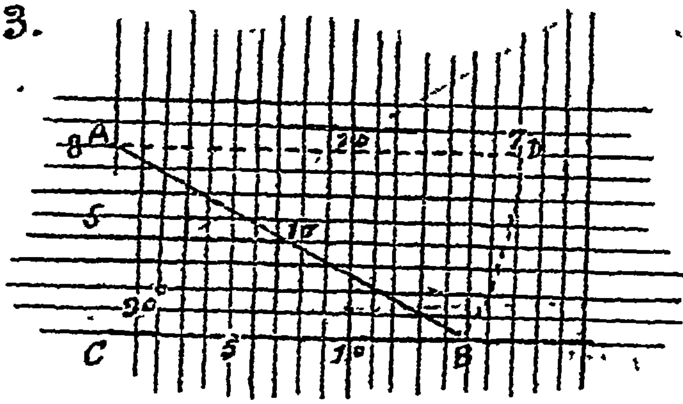


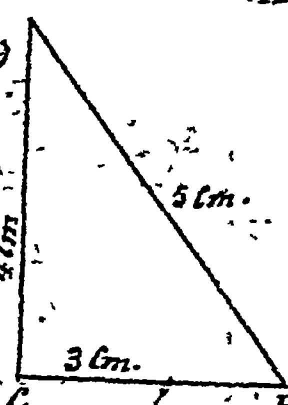
Scale 1" = 10 units.

Prop.
No 332.

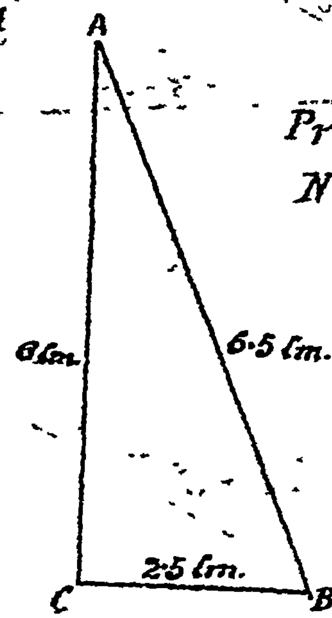
PART II

Page 121

Exer
1. (i)

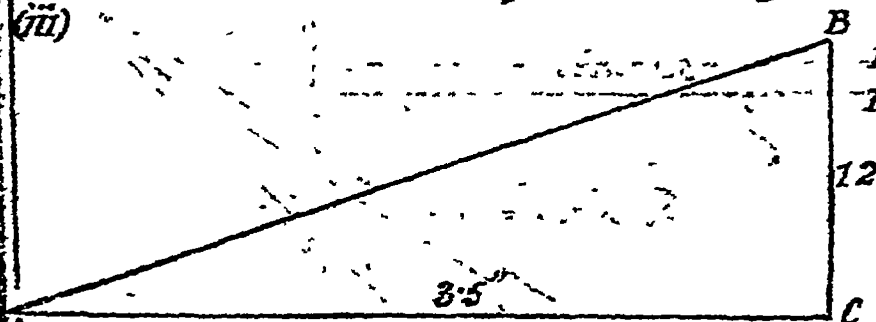


(ii)



Prop.
No 333
334.

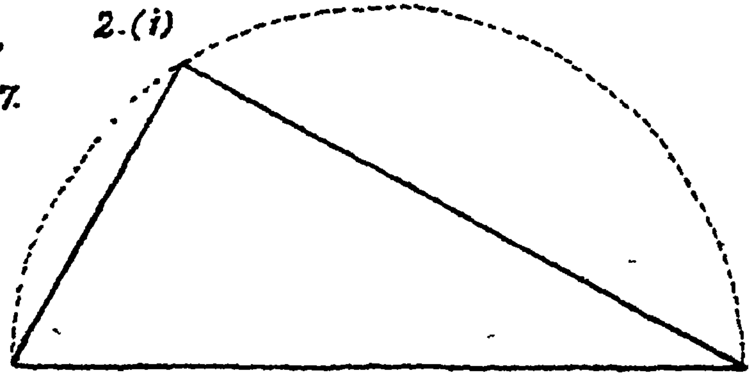
(iii)



Prop
No 335.

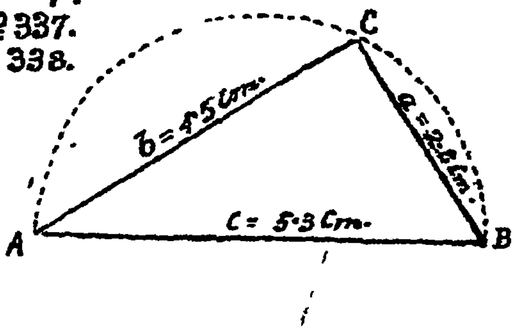
Prop
No 337.

2.(i)



(ii)

Prop.
No 337.
338.



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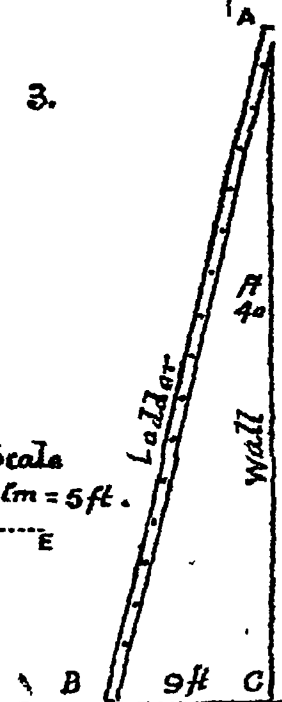
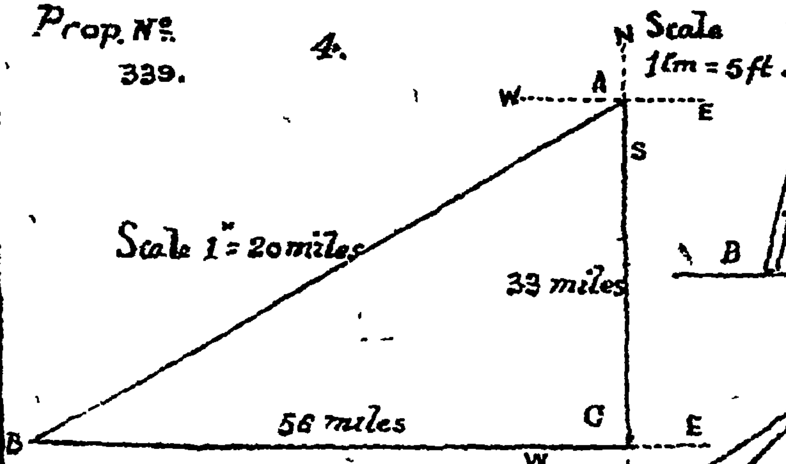


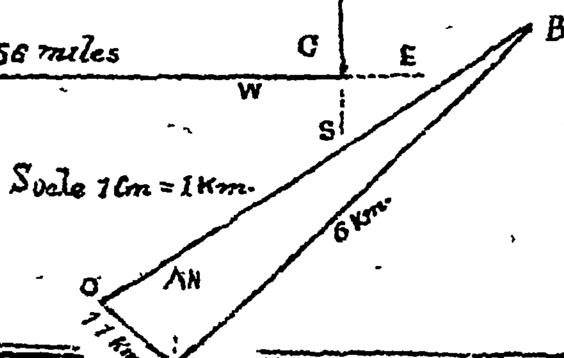
Prop. No.
339.

4.



Prop
No 340.

5



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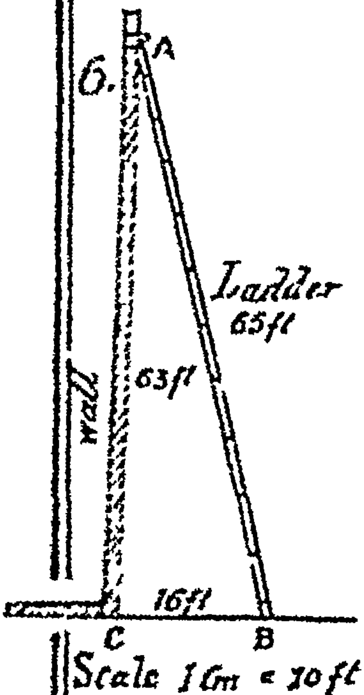

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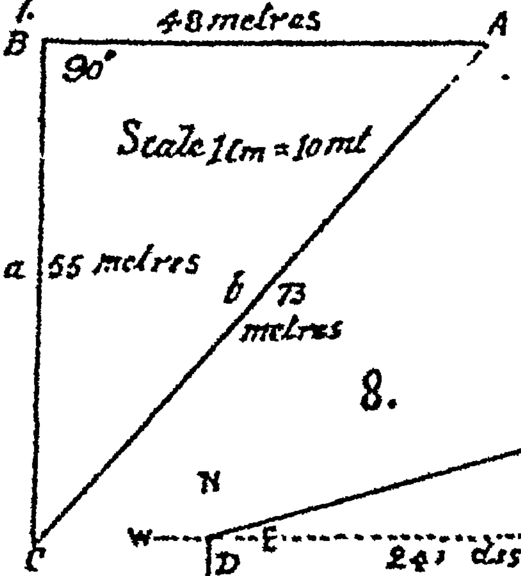
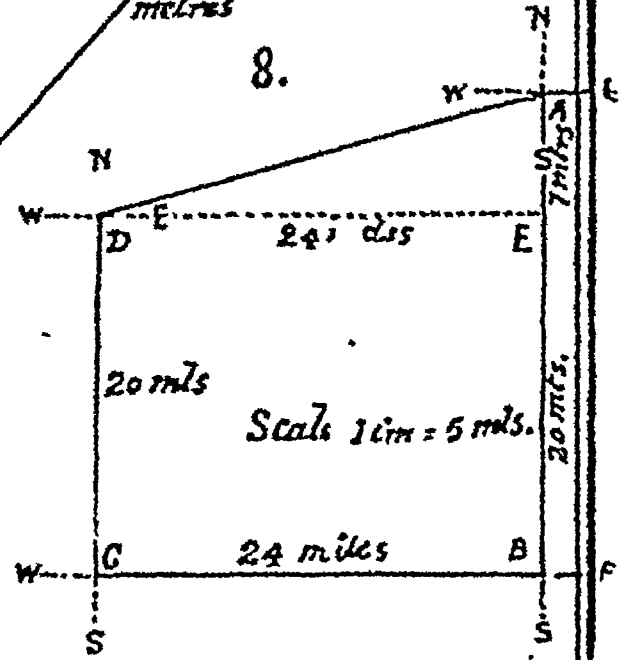


Prop.
Nº
341
342
343.

8.



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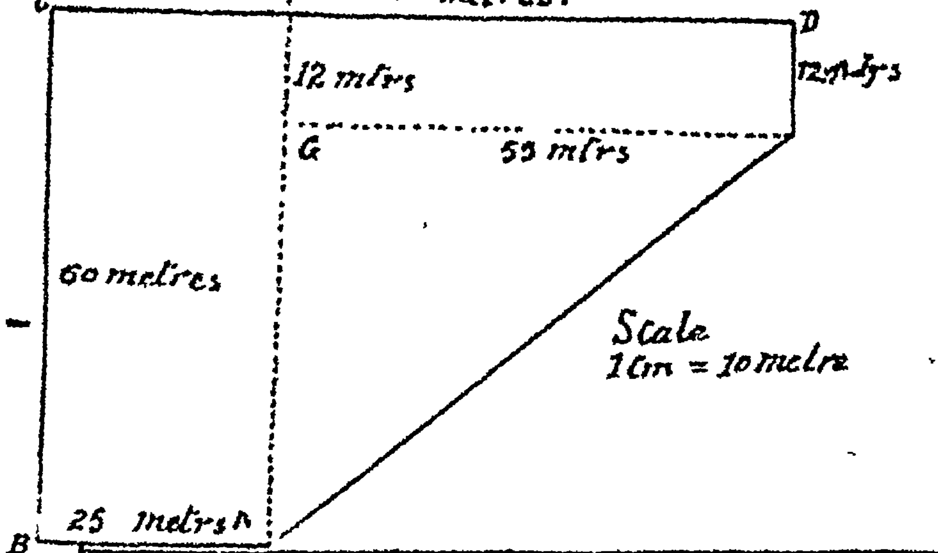
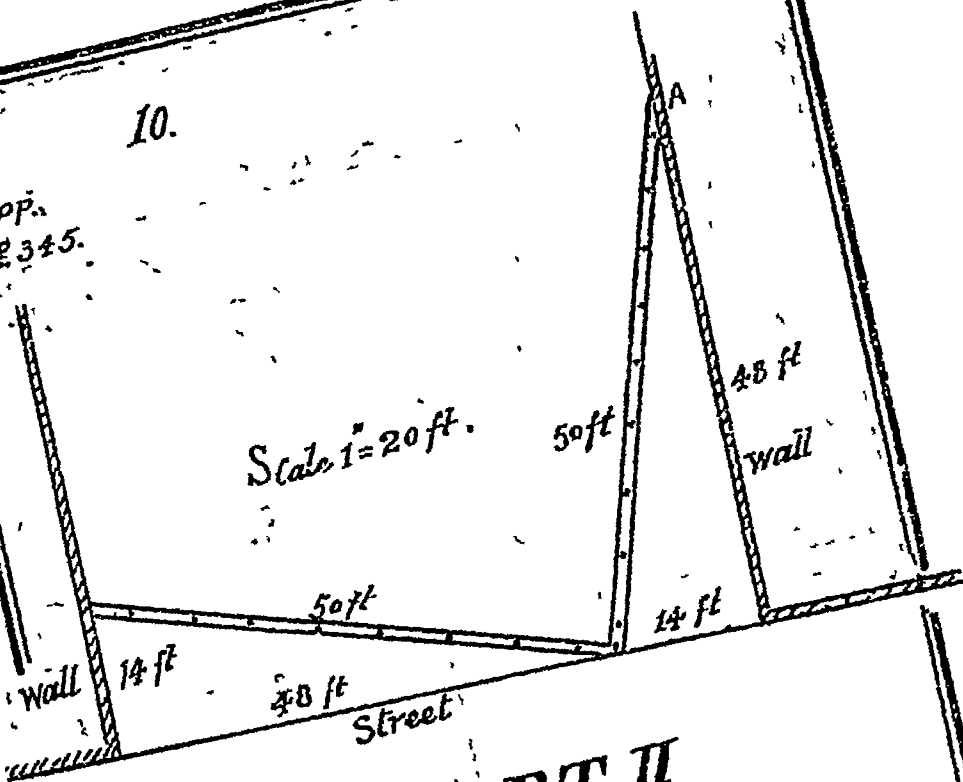


Prop.
Nº
344.

10.

Prop.
No. 345.



PART II

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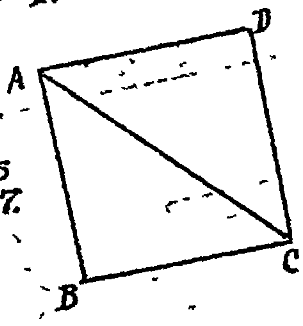
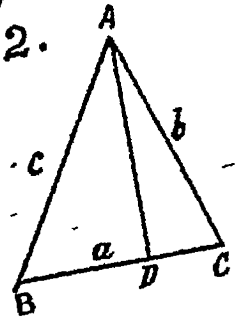
Exer 1.

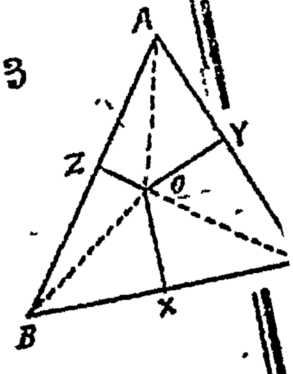
Prop.
No
346
347.



2.

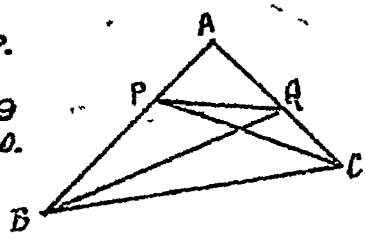


3

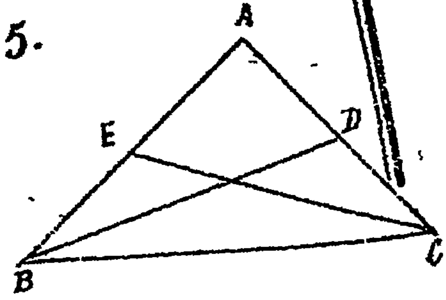


4.

Prop.
No.
349
350.

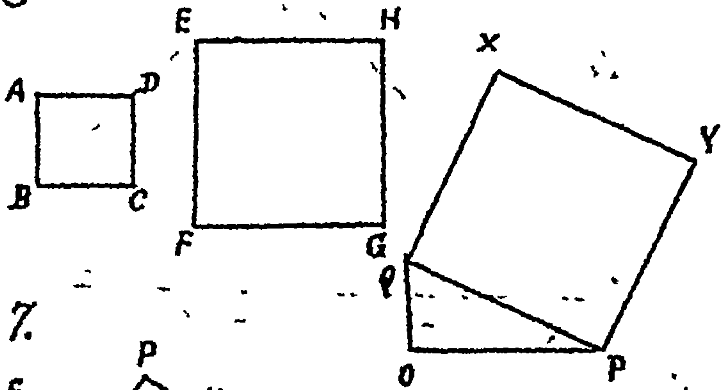


5.



6

Prop. No. 351.



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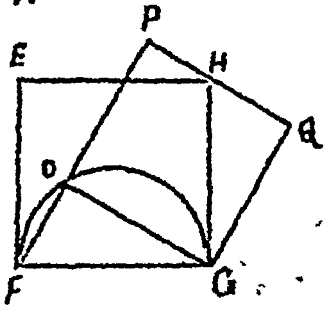
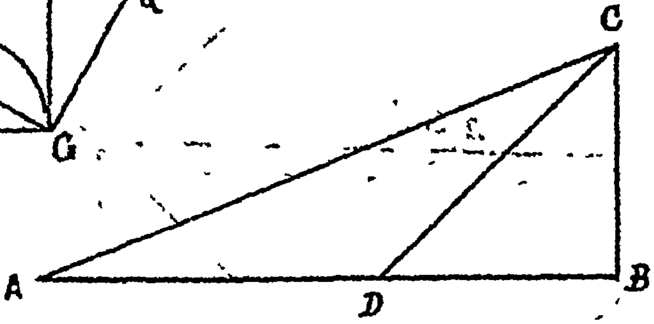


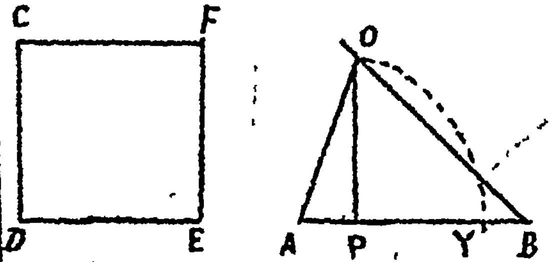
Prop. No. 252. 253

8.



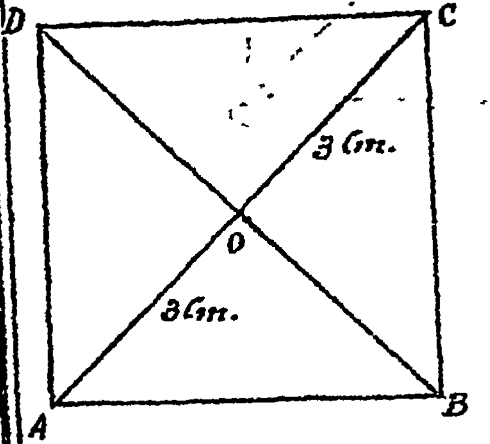
9.

Prop. No. 354 355.



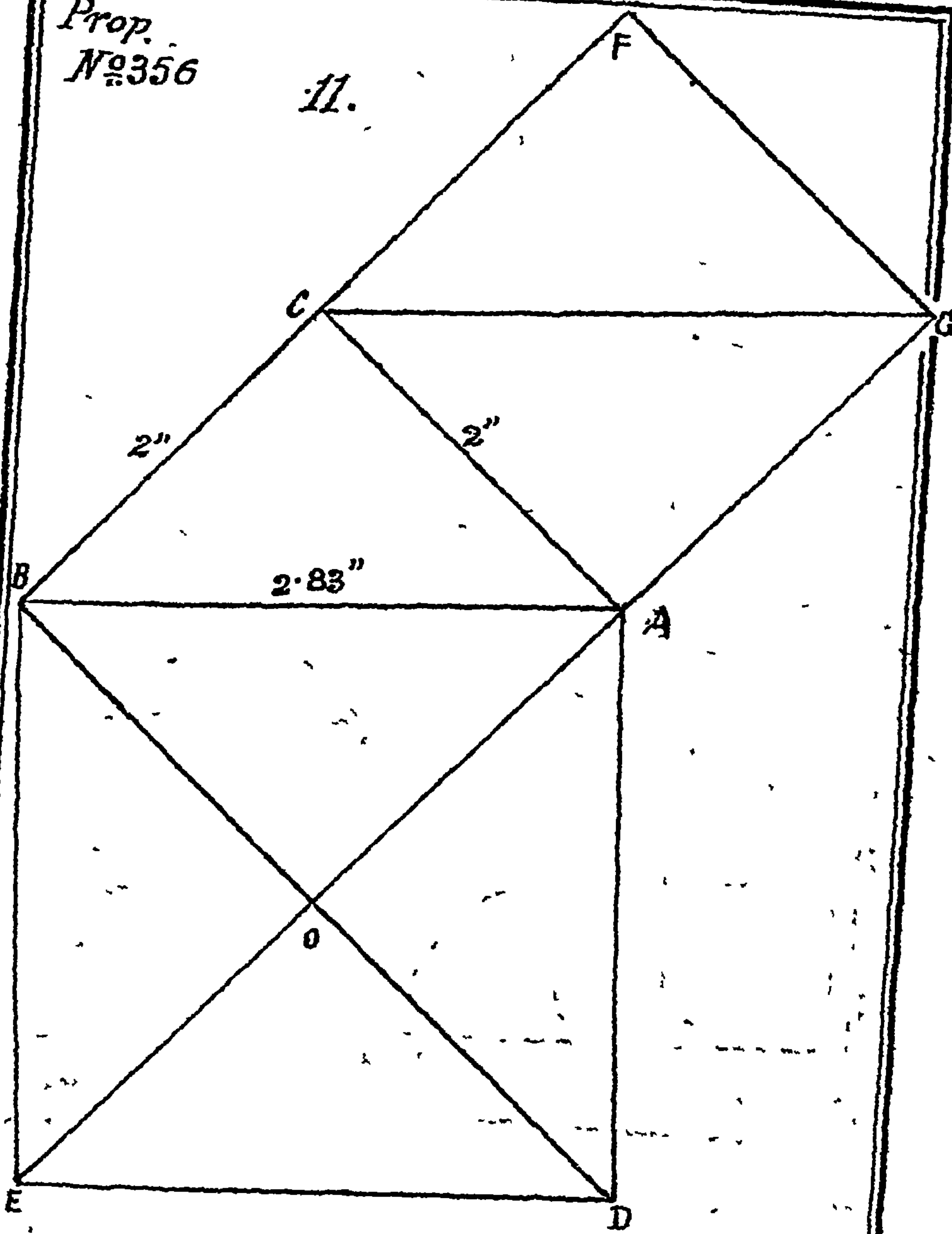
12.

Prop. No. 357



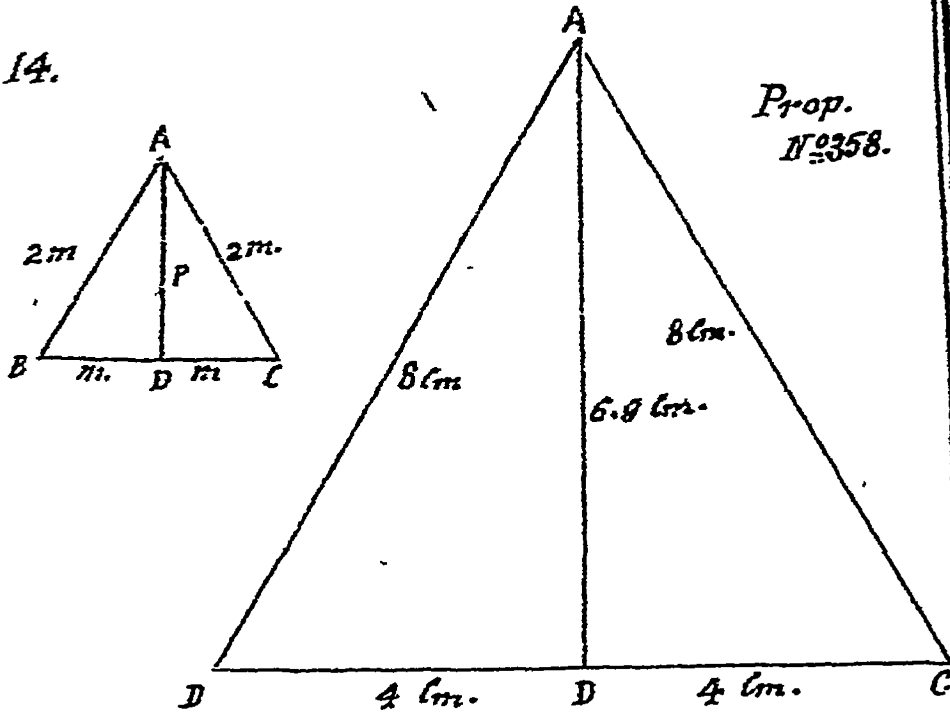
Prop.
No. 356

11.



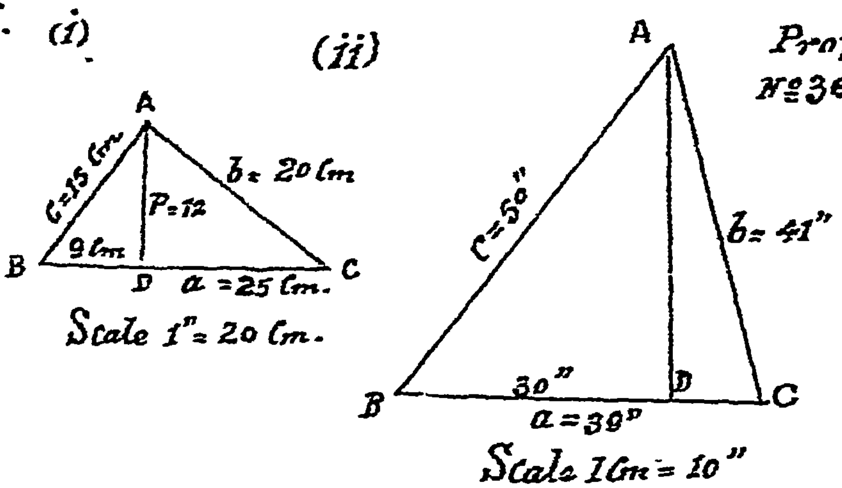
14.

Prop. No. 358.



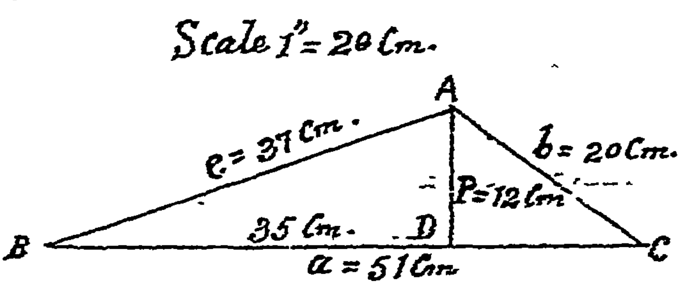
16.

Prop. No. 360.



17.

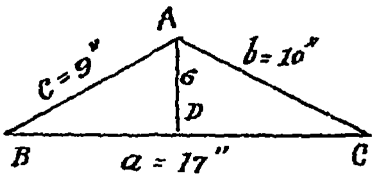
Prop. No. 361.



Prop No. 362.363.

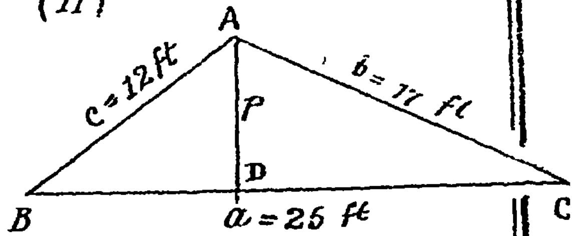
18

(i)



Scale 1" = 10"

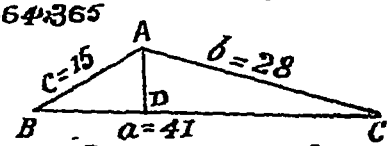
(ii)



Scale 1" = 10 ft

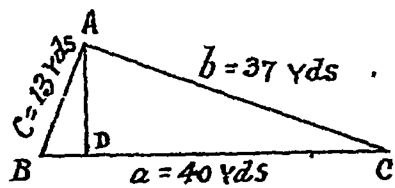
Prop No. 364.365

(iii)



Scale 1 Cm = 10 Cm

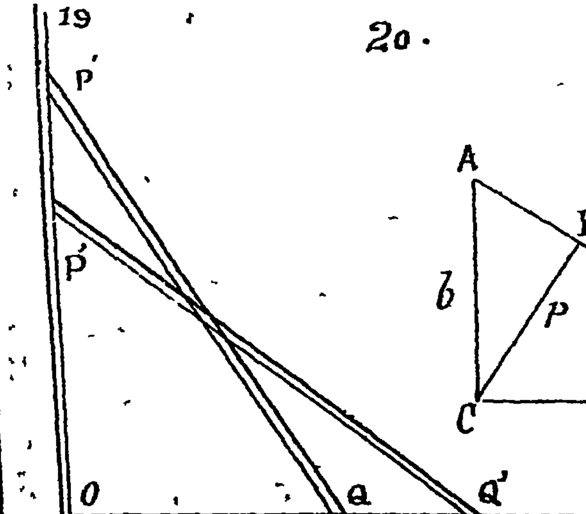
(iv)



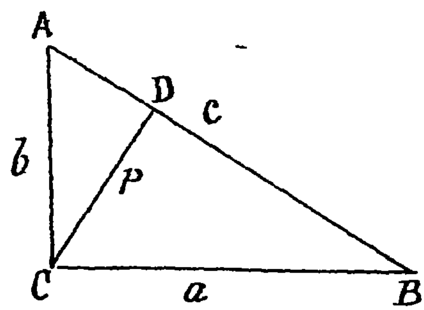
Scale 1 Cm = 10 yds

Prop. No. 366.
367

19



20.

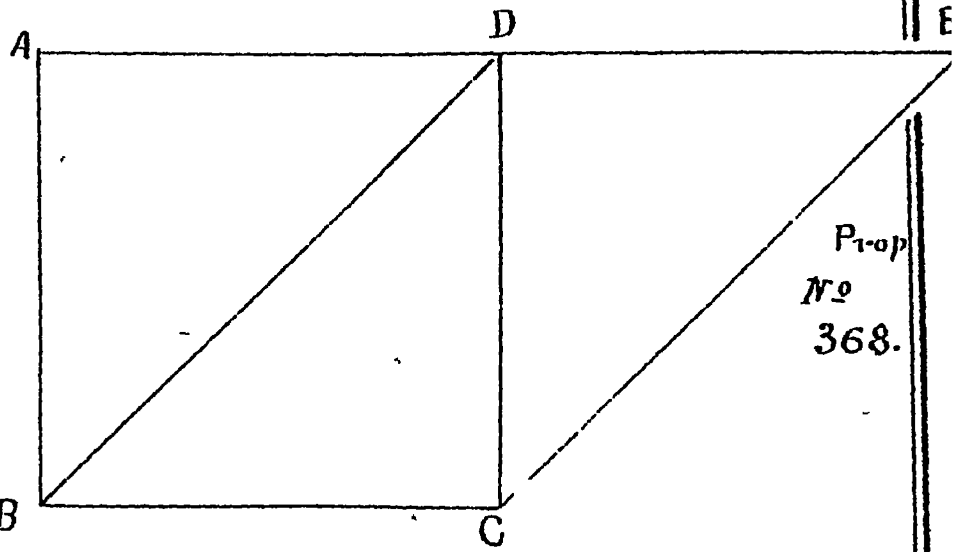


PART II

PAGE 127

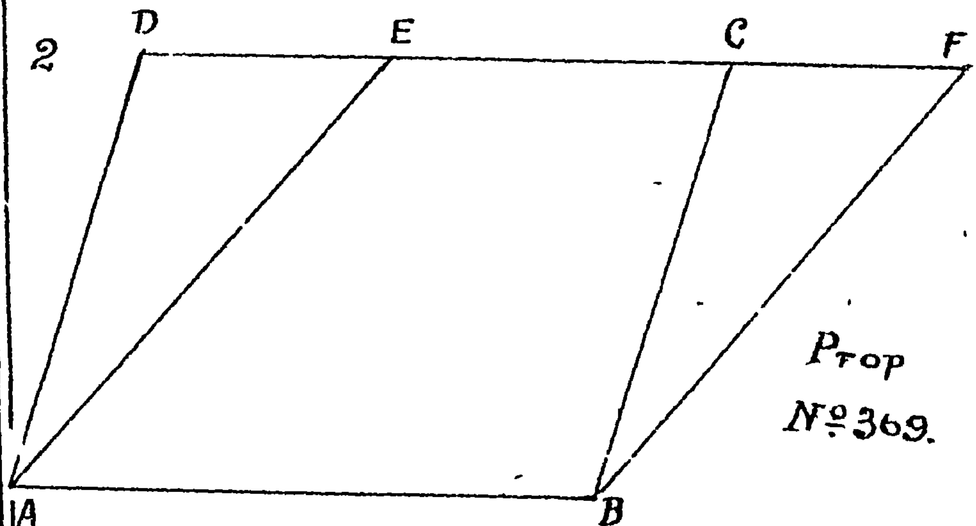
Exer.

1



Prop
No
368.

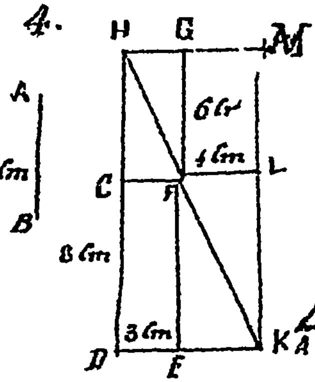
2



Prop
No 369.

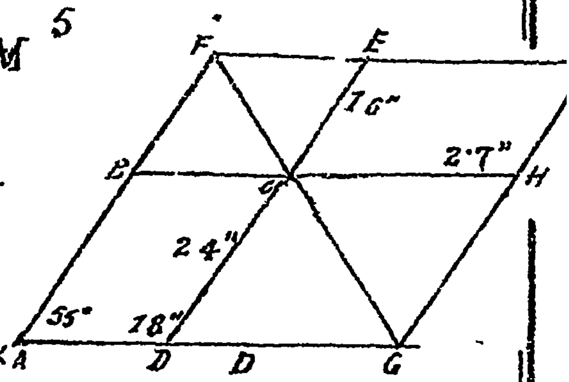
Prop.
No.
371
372

4.



Scale 1" = 10 Cm.

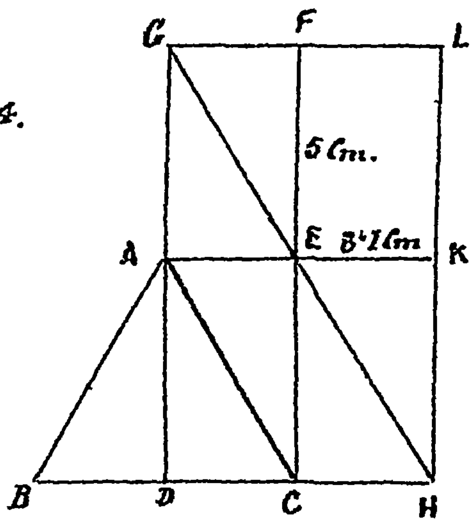
5



Scale 1 Cm = 1".

6.

Prop.
No. 3,74.



$a = 6$ Cm.

PART II.

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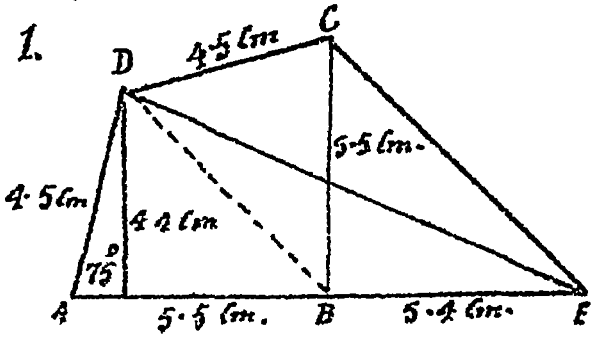
Prob. 18-19.

Exer.

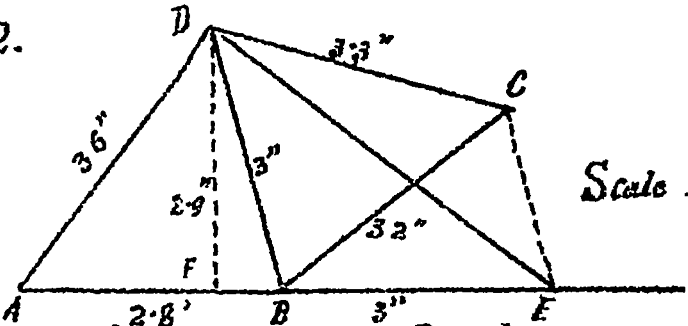
1.



Prop.
№ 375.

Scale 1" = 5 Cm.

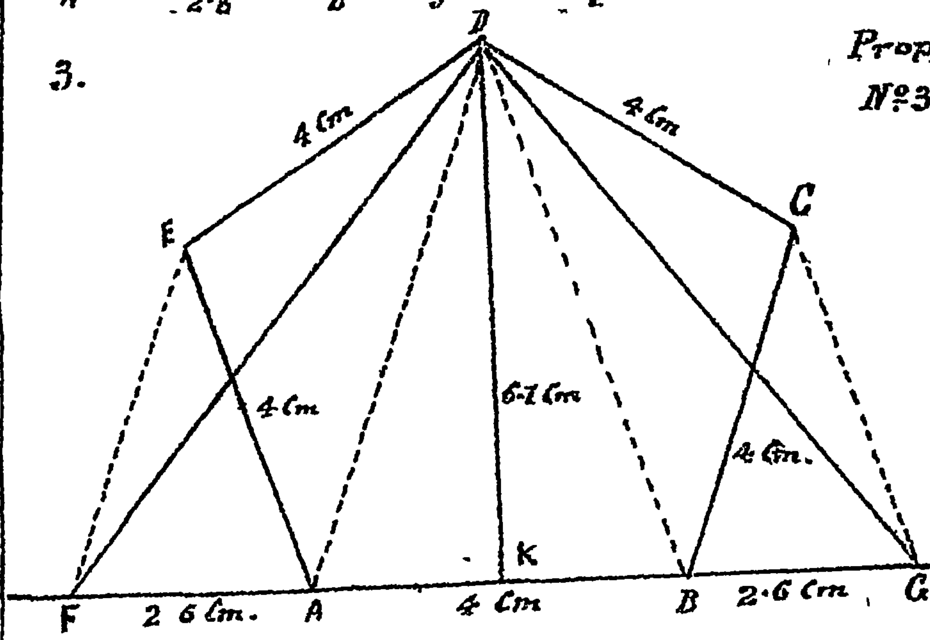
2.



Prop.
№ 376.

Scale 1Cm = 1"

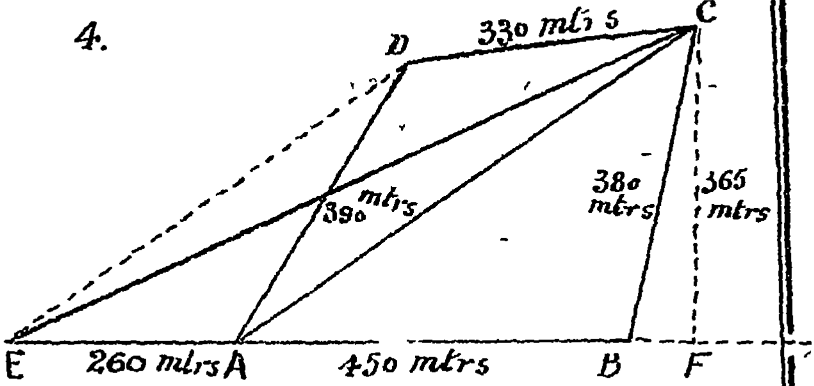
3.



Prop.
№ 377.

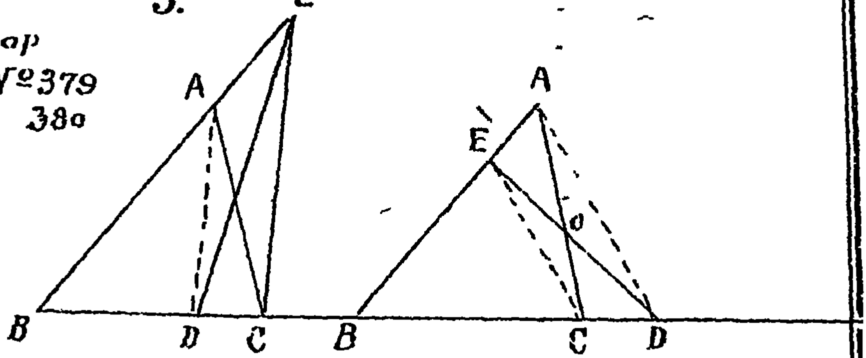
Prop. № 378

4.



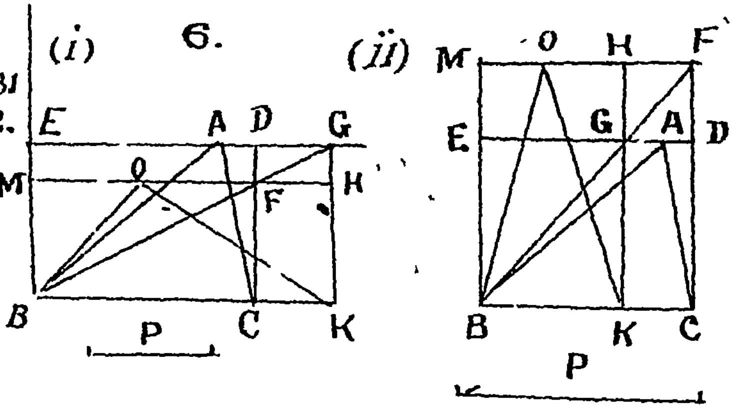
Prop № 379 380

5.



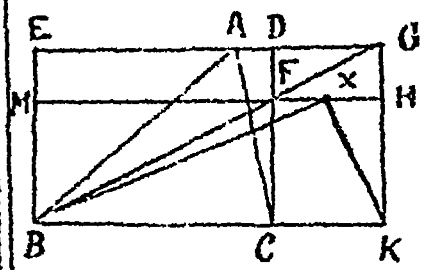
Prop № 381 382.

6.

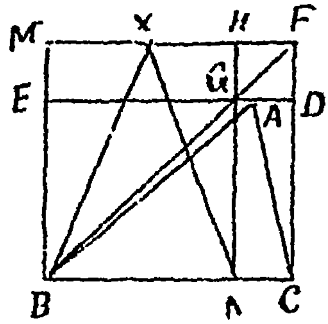


Prop Nº 383. 384

7. (*i*)

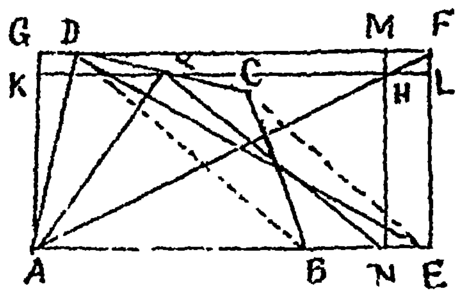


(*ii*)

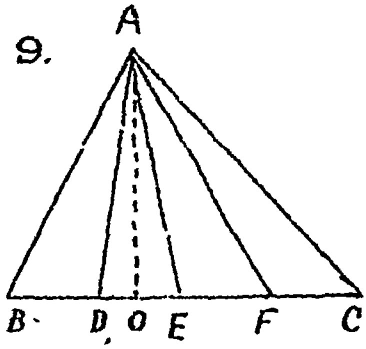


8

Prop. Nº 385.

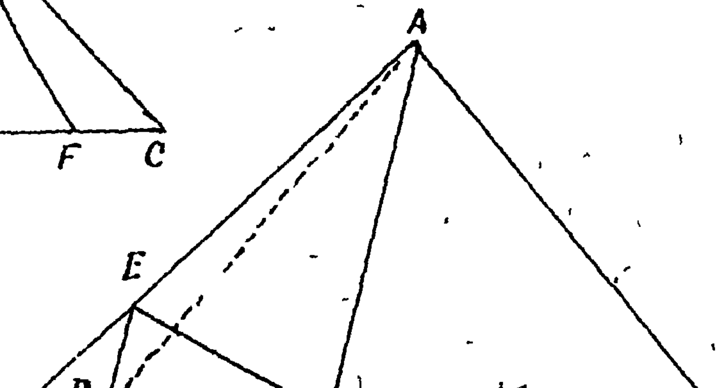


9.



12.

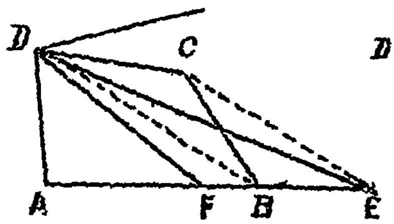
Prop. Nº 386. 387.

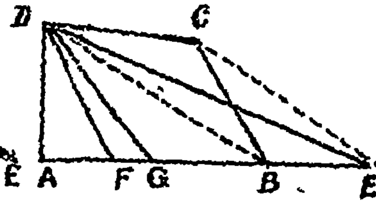


Prop. Nº 388. 389.

13

14.



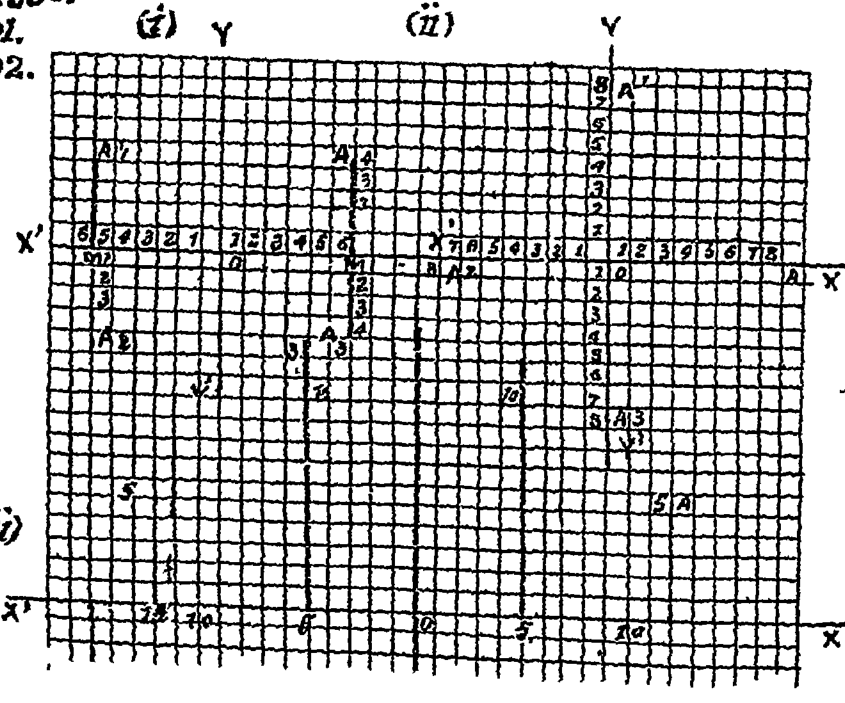


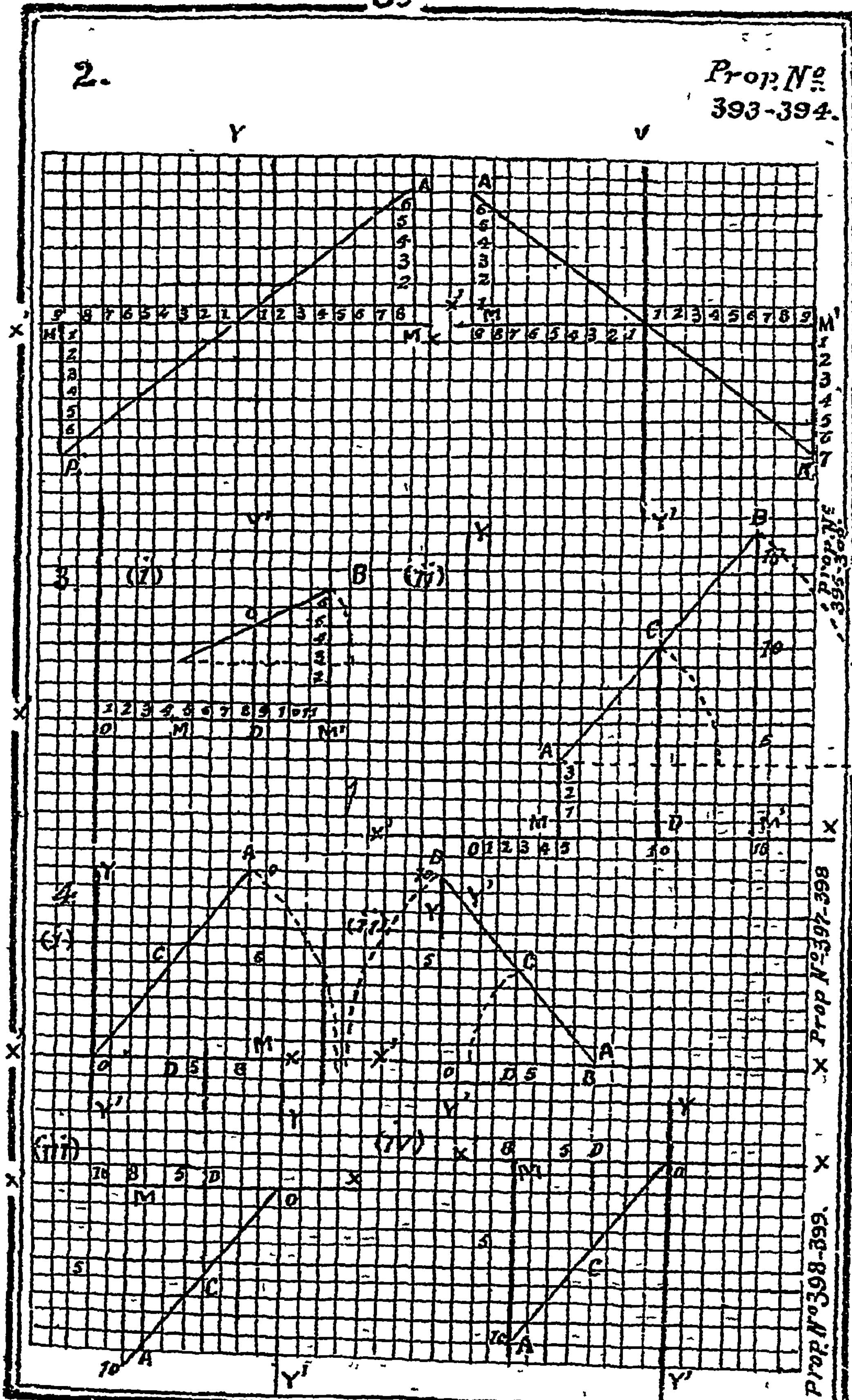
Here $AF = \frac{1}{4} AE$
" $AG = \frac{1}{3} AE$
So on.

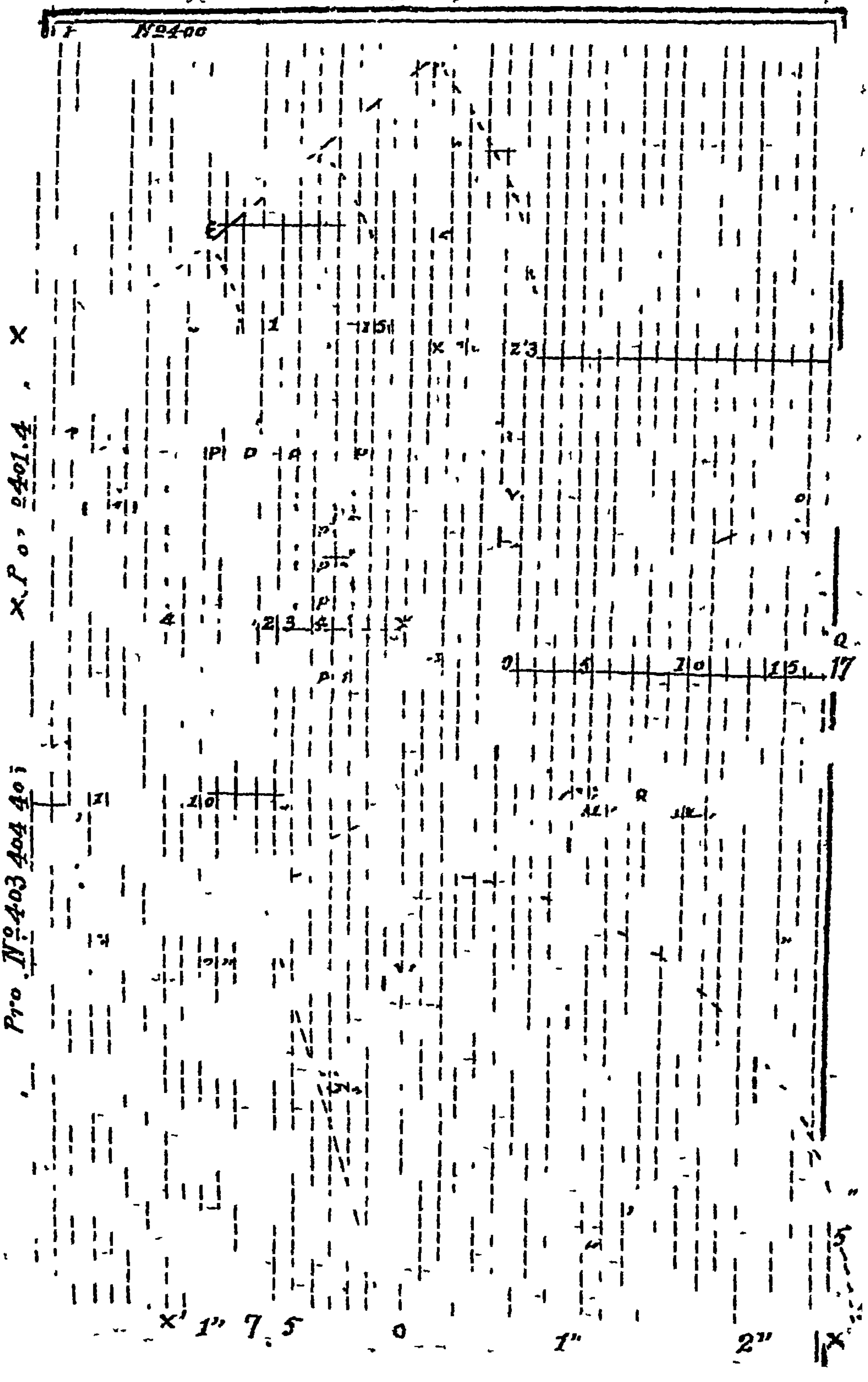
PART II

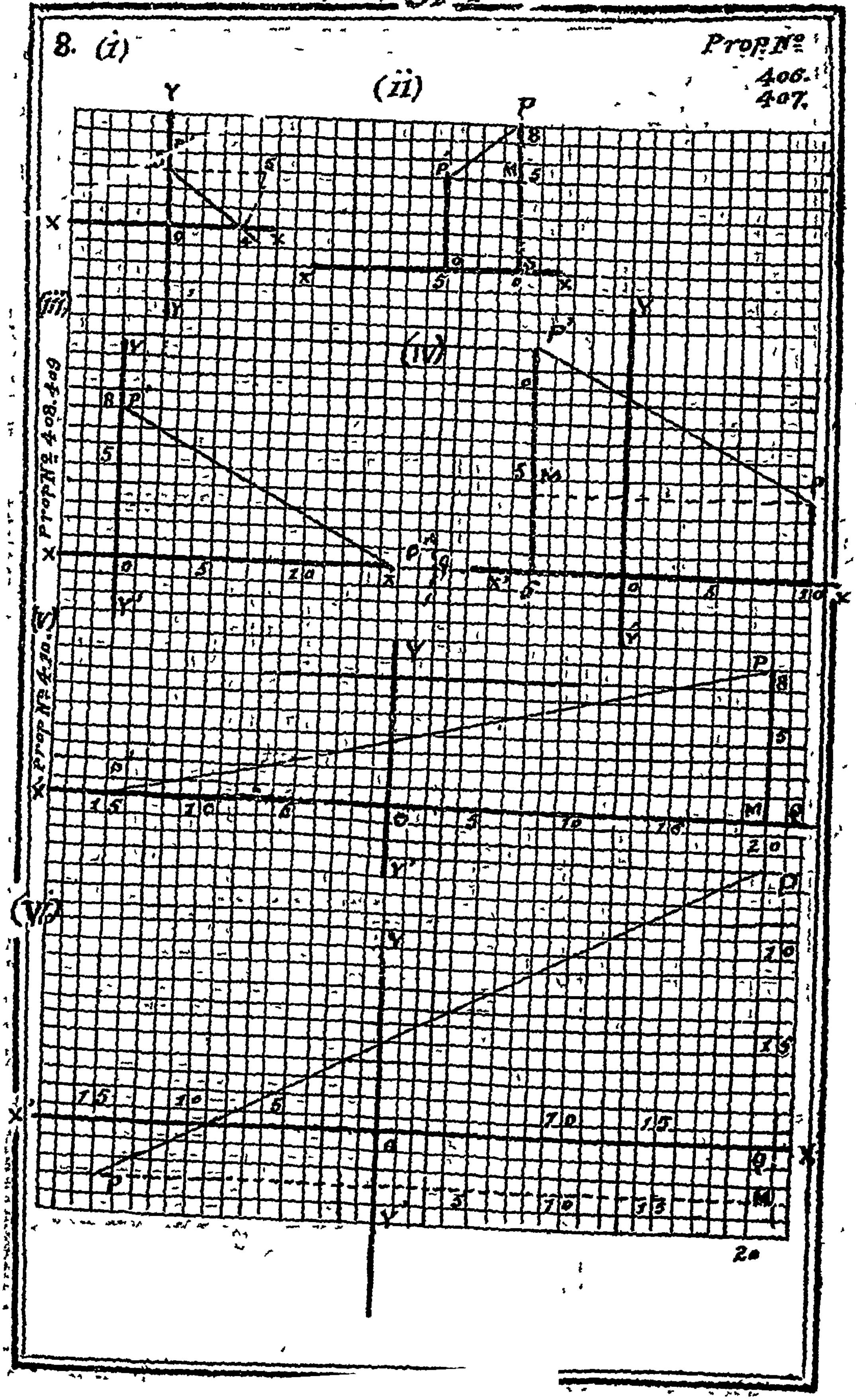
Prop. Nº 390. 391. 392.

Exer 1 Page 134.

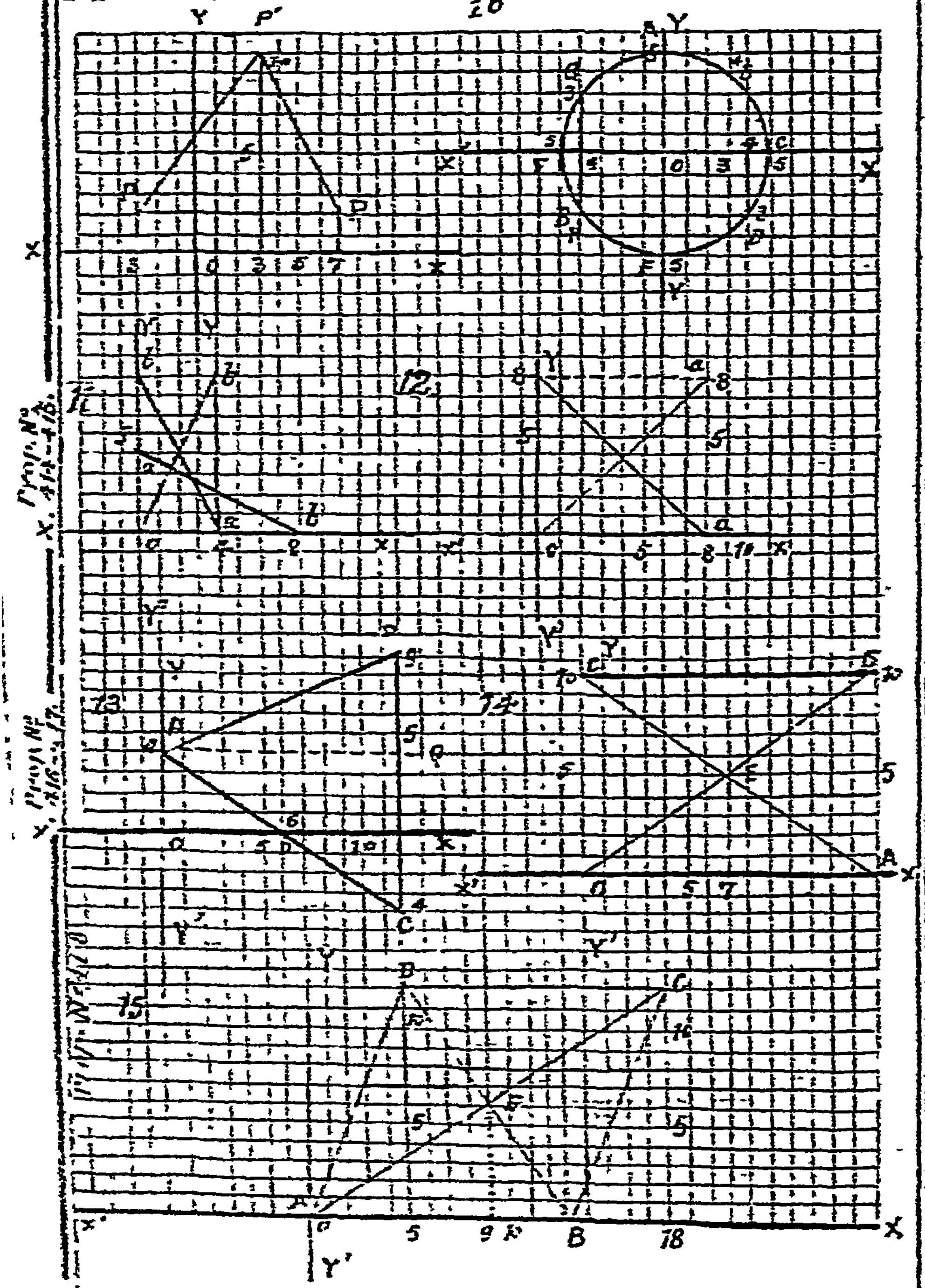



2.
Prop. No. 393-394.
3
(I)
(II)
Prop. No. 395-396.
4
(I)
(II)
Prop. No. 397-398
(III)
(IV)
Prop. No. 398-399.

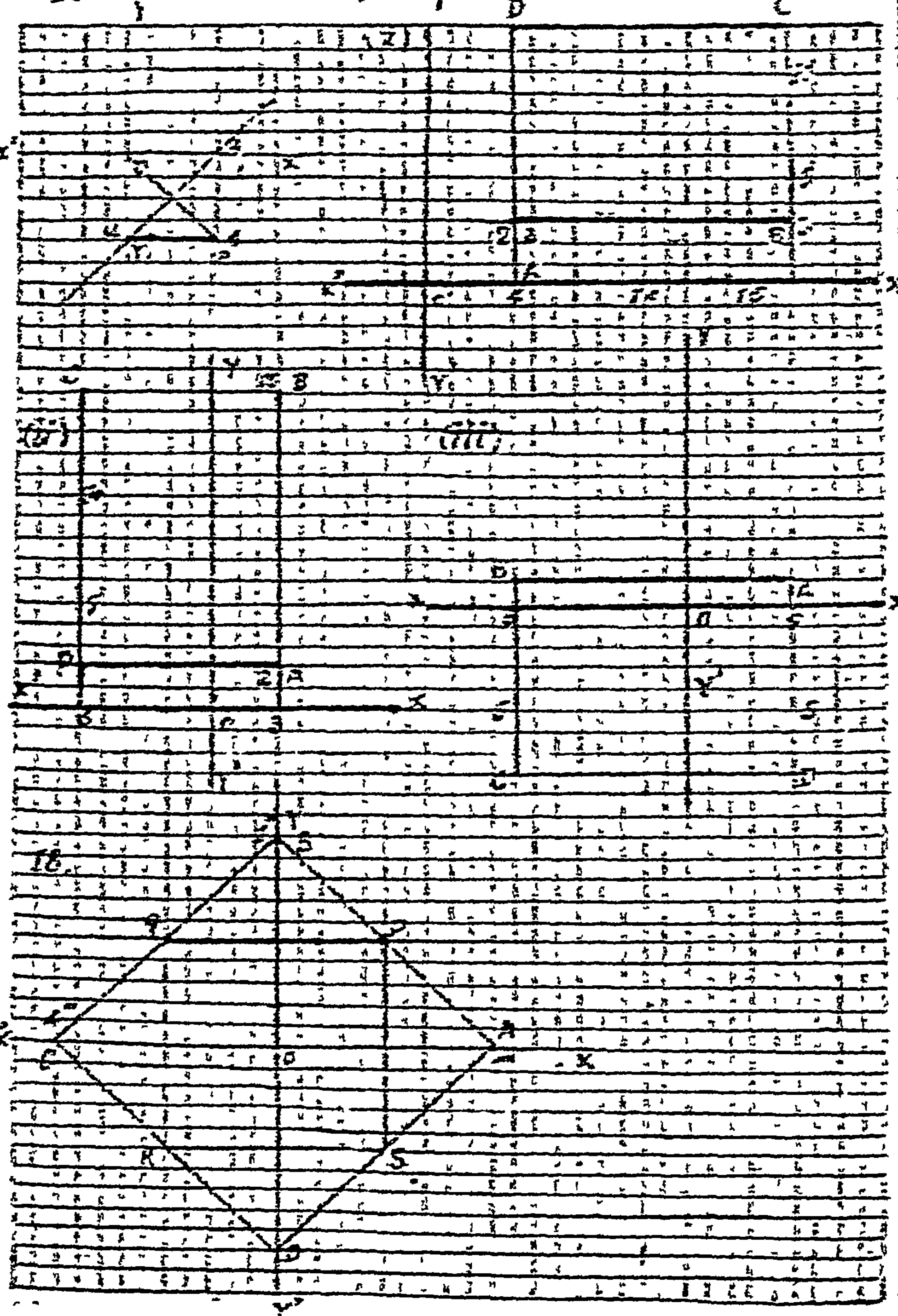

No 400
Pro No 403 404 401
X, P o, No 401.4, X
0 5 10 15 17
X' 1" 7.5 0 1" 2" X'


8. (i)
Prop. No
406.
407.
(ii)
(iii)
(IV)
(V)
(VI)
Prop. No 408. 409
Prop. No 410.

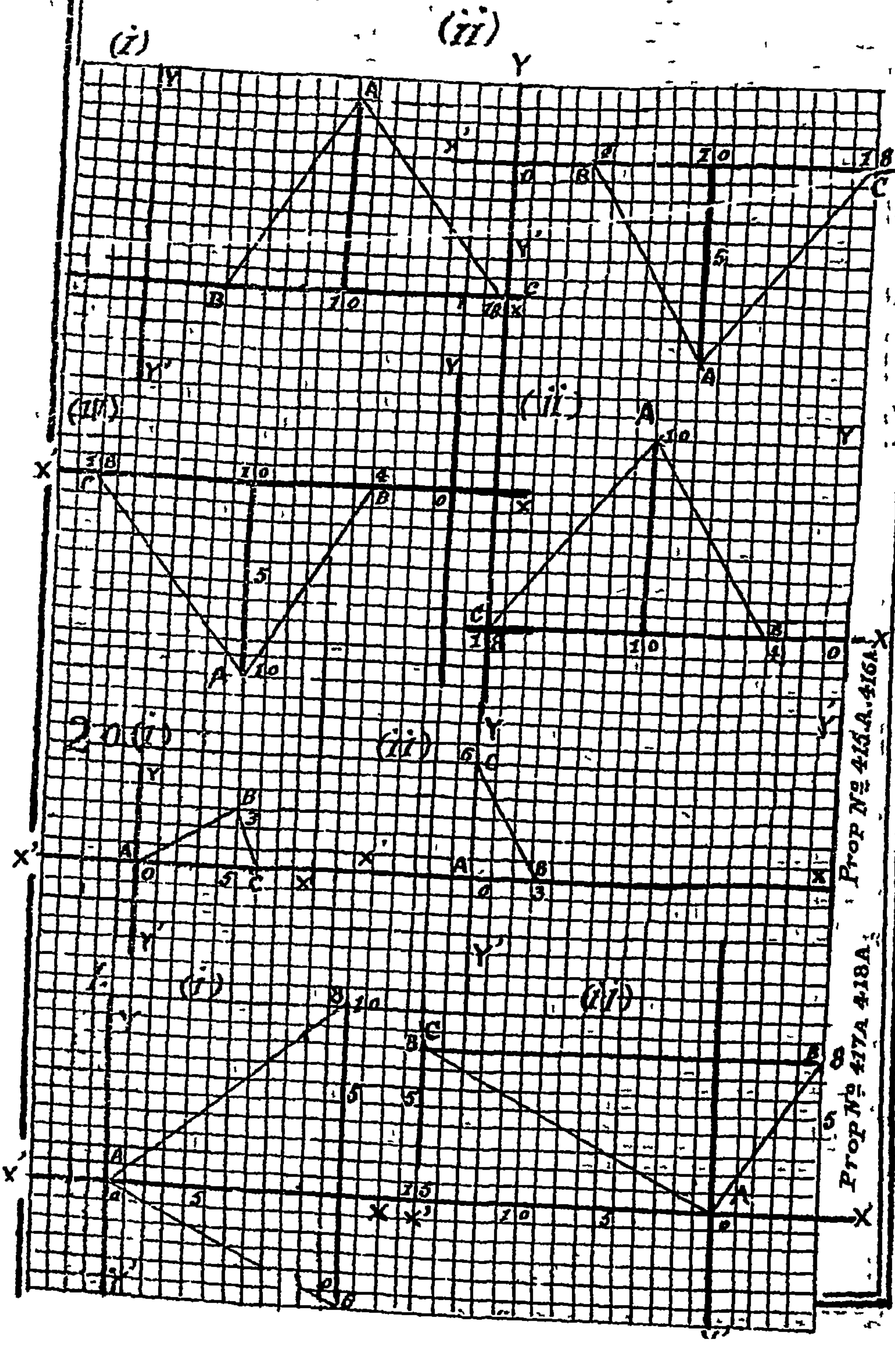
Prop.
Nº 412.413.



16. 17. 18.



19.



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420.

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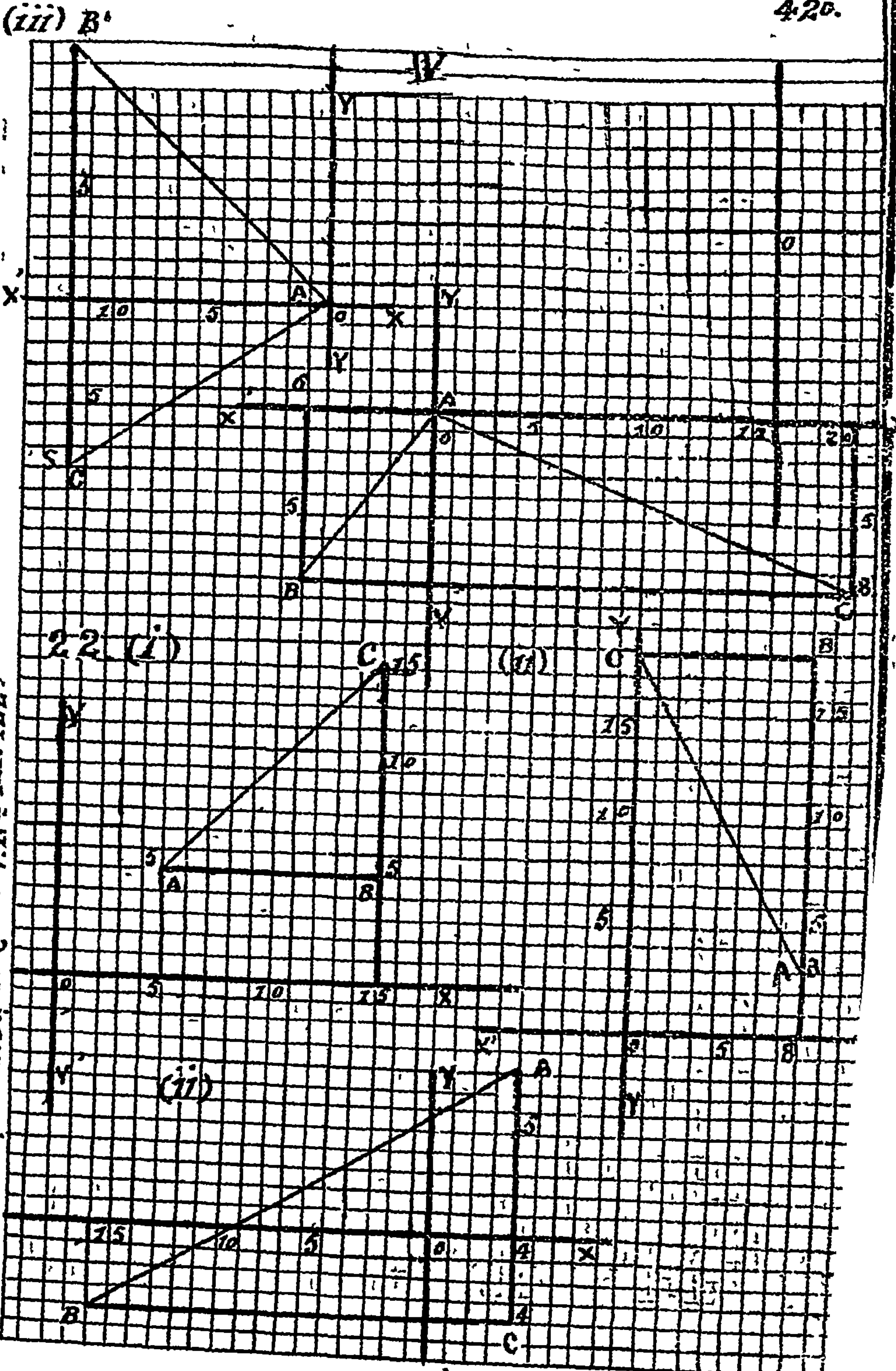
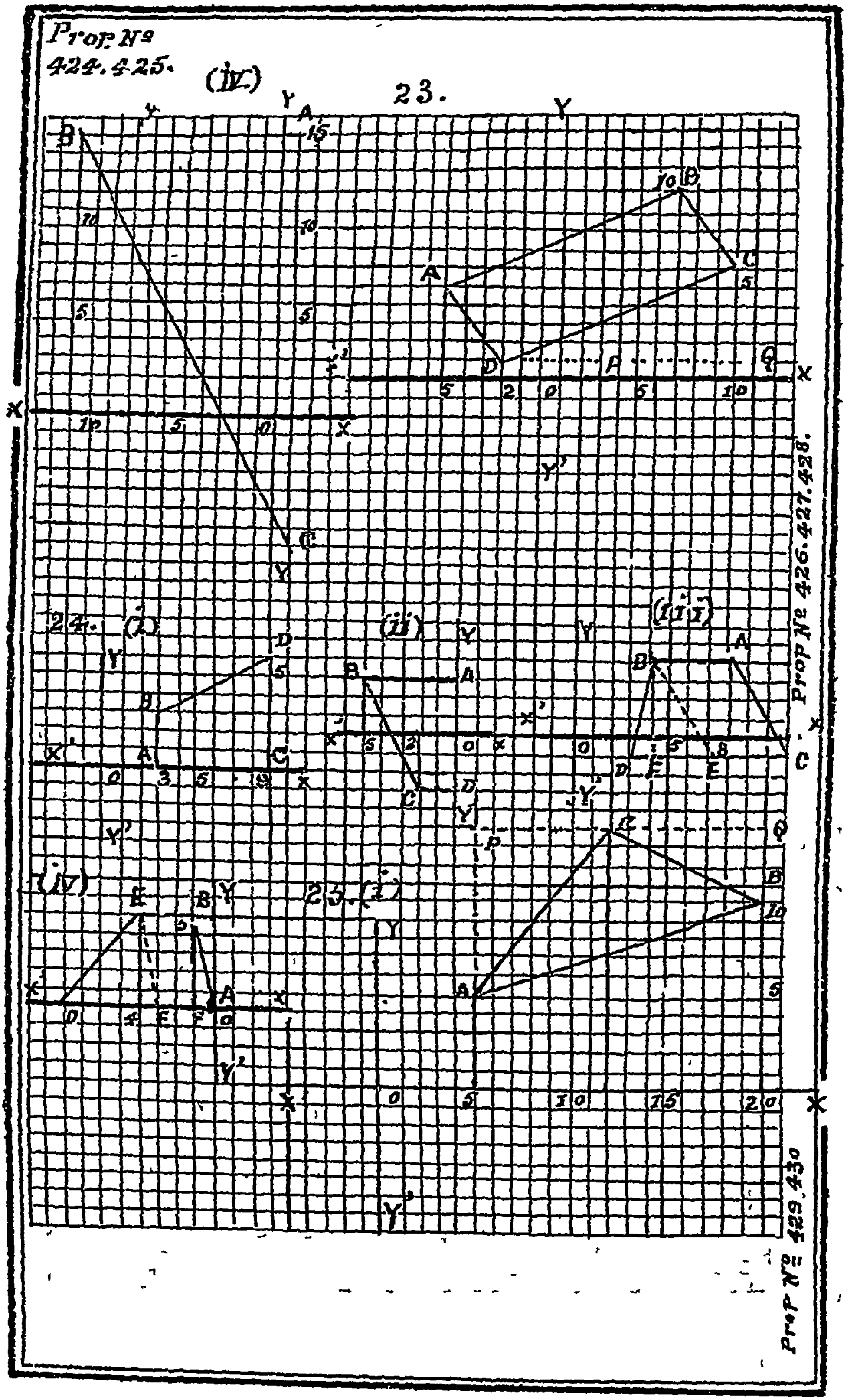
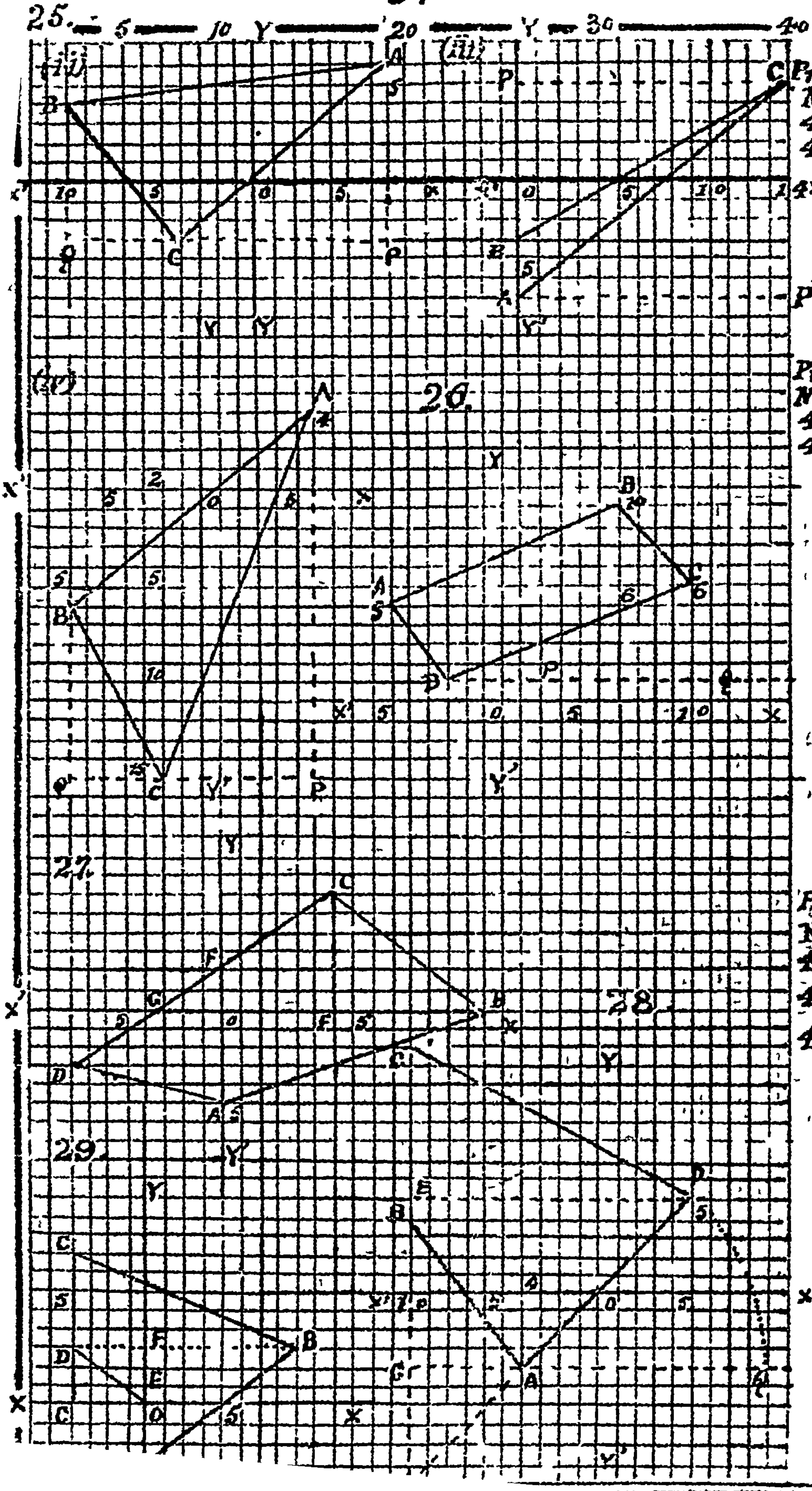


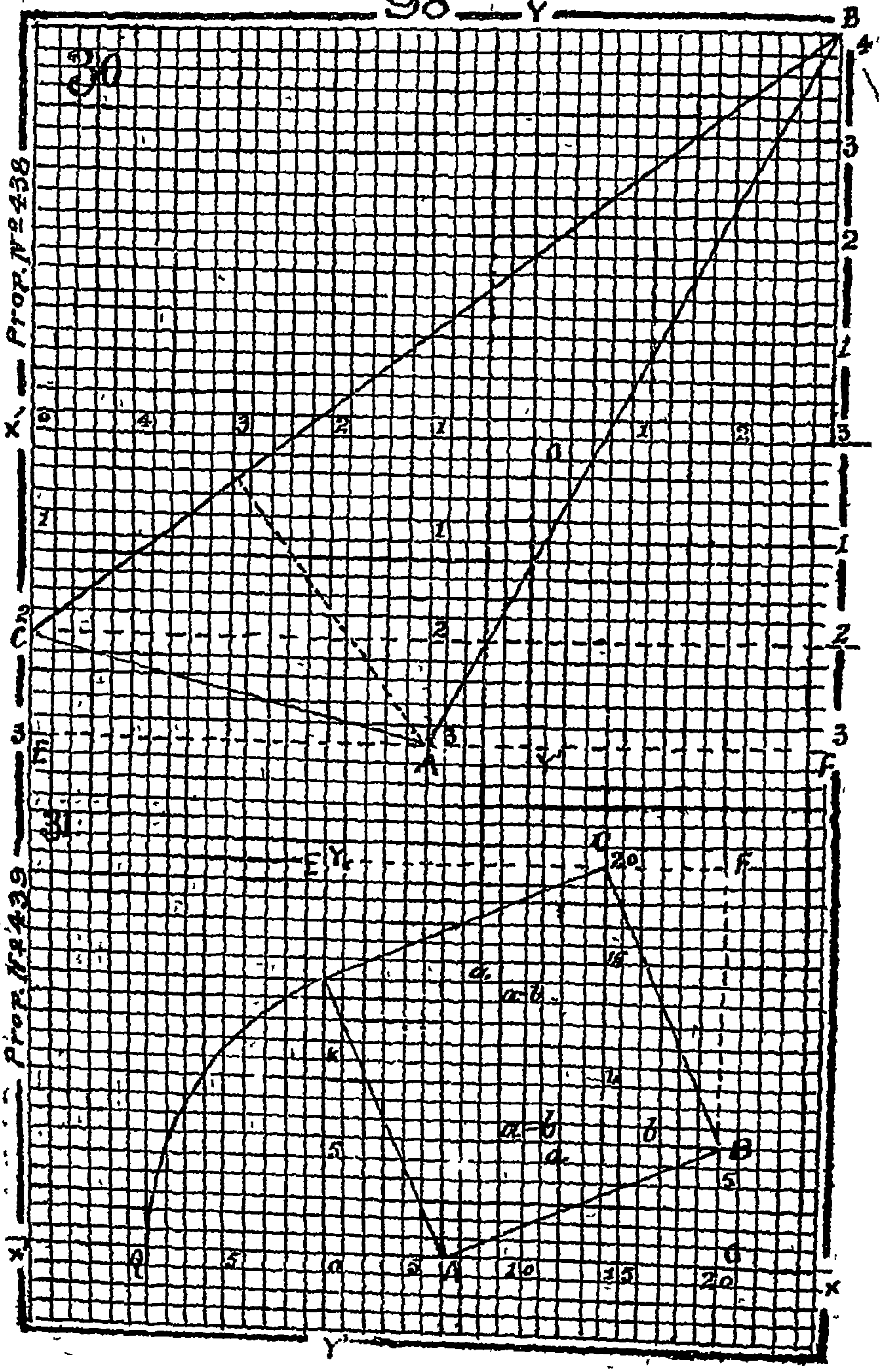

Prop. No
424. 425.
(IV)
23.
24. (i)
(ii)
(iii)
Prop No 426. 427. 428.
(IV)
23. (i)
Prop No 429. 430



Prop. No 431 432.

Prop. No 433. 434

Prop. No 435. 436 437


30
Prop. Nº 438
31
Prop. Nº 439

PART. II.

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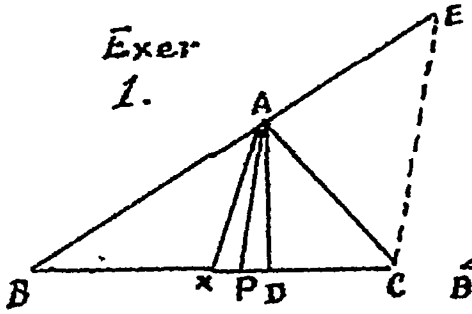
Exer

1.



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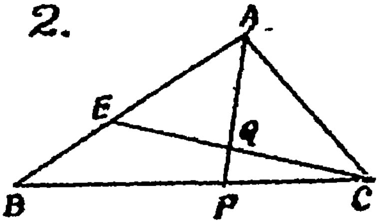
2.



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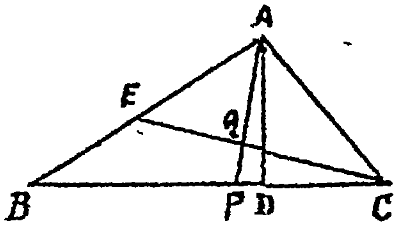

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No 442
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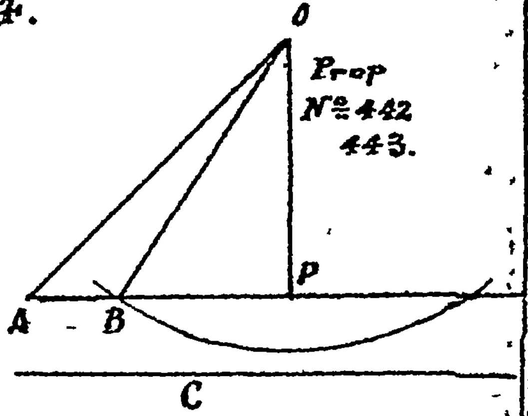
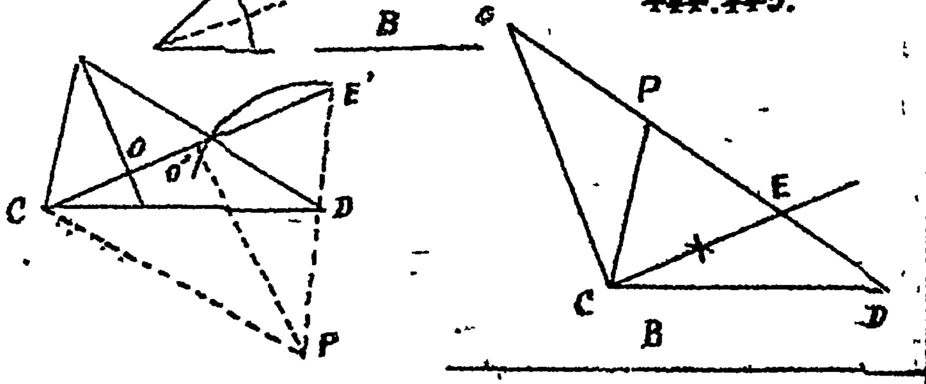
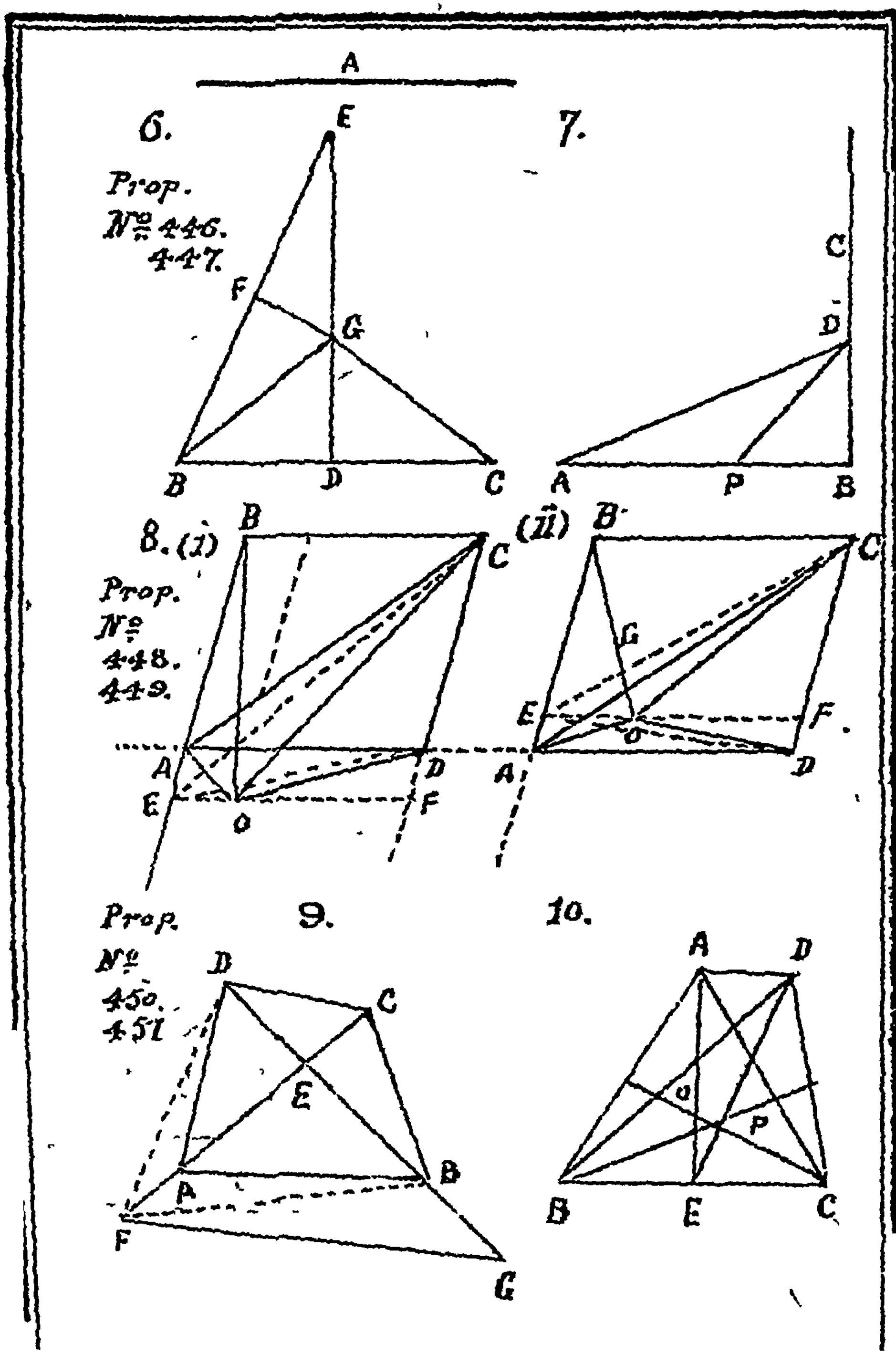


5. (i)

(ii)

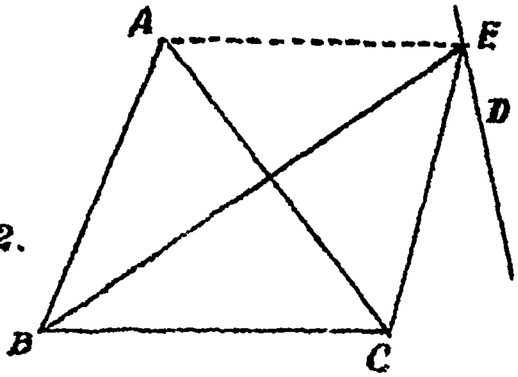
Prop. No
444.445.




A
6.
Prop.
Nº 446.
447.
E
F
G
B
D
C
7.
C
D
A
P
B
8. (I)
Prop.
Nº
448.
449.
B
C
A
D
E
O
F
(II)
B
C
G
E
F
O
A
D
Prop.
Nº
450.
451.
9.
D
C
E
A
B
F
G
10.
A
D
O
P
B
E
C

11.

Prop.
N°. 452.



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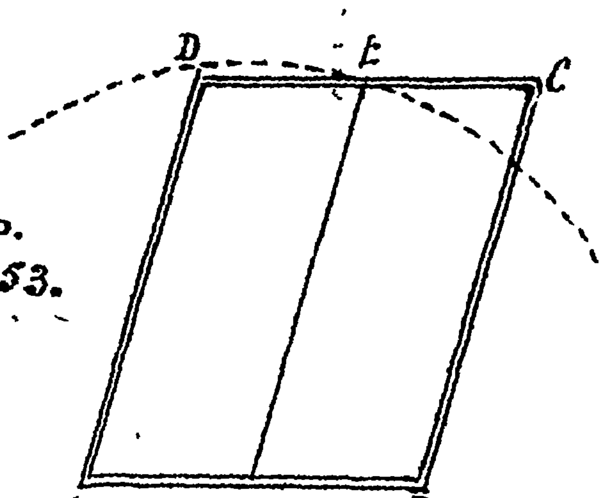
Prop.
N°. 453.



Finish Parts I & II.